MATH 310, Section 3.1 Solutions

20. Expanding along the third row, $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & 4 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} -1 & 3 \\ 4 & 4 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix} = \boxed{\mathbf{2}}$

28. Expanding along the third row (which is all 0's) you can see that the answer is $\boxed{\mathbf{0}}$.

30. The first row has the most 0's, so let's expand along it.

$$\begin{vmatrix} 3 & 0 & 7 & 0 \\ 2 & 6 & 11 & 12 \\ 4 & 1 & -1 & 2 \\ 1 & 5 & 2 & 10 \end{vmatrix} = 3\begin{vmatrix} 6 & 11 & 12 \\ 1 & -1 & 2 \\ 5 & 2 & 10 \end{vmatrix} + 7\begin{vmatrix} 2 & 6 & 12 \\ 4 & 1 & 2 \\ 1 & 5 & 10 \end{vmatrix} =$$

$$3\left(-\begin{vmatrix} 11 & 12 \\ 2 & 10 \end{vmatrix} - \begin{vmatrix} 6 & 12 \\ 5 & 10 \end{vmatrix} - 2\begin{vmatrix} 6 & 11 \\ 5 & 2 \end{vmatrix}\right) + 7\left(-4\begin{vmatrix} 6 & 12 \\ 5 & 10 \end{vmatrix} + \begin{vmatrix} 2 & 12 \\ 1 & 10 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 1 & 5 \end{vmatrix}\right) =$$

$$3(-(110-24) - (60-60) - 2(12-55)) + 7(-4(60-60) + (20-12) - 2(10-6)) =$$

$$3(-86+86) + 7(8-8)) = \boxed{\mathbf{0}}$$

42. Because this matrix is triangular, its determinant will be the product of its diagonal entries, that is $4 \cdot \frac{1}{2} \cdot 3 \cdot (-2) = \boxed{\mathbf{-12}}$

46. Because this matrix is triangular, its determinant will be the product of its diagonal entries, that is $7 \cdot \frac{1}{4} \cdot 2 \cdot (-1) \cdot (-2) = \boxed{\mathbf{7}}$

Section 3.2 Solutions

30. Let's start by knocking out those 2's in the 5th column to create a column with a lot of zeros, and then expand along that last column. (Thus, start with $R_3 - 2R_1 \to R_3$ and $R_5 - 2R_1 \to R_5$.)

$$\begin{vmatrix} 3 & -2 & 4 & 3 & 1 \\ -1 & 0 & 2 & 1 & 0 \\ 5 & -1 & 0 & 3 & 2 \\ 4 & 7 & -8 & 0 & 0 \\ 1 & 2 & 3 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -2 & 4 & 3 & 1 \\ -1 & 0 & 2 & 1 & 0 \\ -1 & 3 & -8 & -3 & 0 \\ 4 & 7 & -8 & 0 & 0 \\ -5 & 6 & -5 & -6 & 0 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 2 & 1 \\ -1 & 3 & -8 & -3 \\ 4 & 7 & -8 & 0 \\ -5 & 6 & -5 & -6 \end{vmatrix}$$

Now add appropriate multiples of the first row to the lower rows to get zeros in the first column, and then expand along that column.

$$= \begin{vmatrix} -1 & 0 & 2 & 1 \\ 0 & 3 & -10 & -4 \\ 0 & 7 & 0 & 4 \\ 0 & 6 & -15 & -11 \end{vmatrix} = -\begin{vmatrix} 3 & -10 & -4 \\ 7 & 0 & 4 \\ 6 & -15 & -11 \end{vmatrix} = -\left(-7\begin{vmatrix} -10 & -4 \\ -15 & -11 \end{vmatrix} - 4\begin{vmatrix} 3 & -10 \\ 6 & -15 \end{vmatrix}\right)$$

$$-(-7(110-60) - 4(-45+60)) = \boxed{\mathbf{410}}$$