LINEAR ALGEBRA
Section 2.2 (Let me know if you see any typos and I'll correct them. –RH)

12. $C(BC) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \left(\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 3 & -1 \end{bmatrix}$

20. $A^4 = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Section 2.3 Solutions

10. $\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 3 & 7 & 9 & 0 & 1 & 0 \\ -1 & -4 & -7 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} R_2 - 3R_1 \to R_2 \\ R_3 + R_1 \to R_3 \end{array} \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & -2 & -5 & 1 & 0 & 1 \end{array}\right] \begin{array}{c} R_1 - 2R_2 \to R_1 \\ R_3 + R_2 \to R_3 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & 7 & -2 & 0 \\ 0 & 1 & 3 & -3 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right] \begin{array}{c} R_1 + 4R_3 \to R_1 \\ R_2 - 3R_3 \to R_2 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -13 & 6 & 4 \\ 0 & 1 & 0 & 12 & -5 & -3 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array}\right]$

Therefore $A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

14. $\left[\begin{array}{ccc|ccc} 3 & 2 & 5 & 1 & 0 & 0 \\ 2 & 2 & 4 & 0 & 1 & 0 \\ -4 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \; R_1 - R_2 \to R_1 \; \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 2 & 2 & 4 & 0 & 1 & 0 \\ -4 & 4 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} R_2 - 2R_1 \to R_2 \\ R_3 + 4R_1 \to R_3 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & -2 & 3 & 0 \\ 0 & 4 & 4 & 4 & -4 & 1 \end{array}\right] \; R_3 - 2R_2 \to R_3 \; \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & -2 & 3 & 0 \\ 0 & 0 & 0 & 8 & -10 & 1 \end{array}\right] \begin{array}{c} \frac{1}{2}R_2 \to R_2 \\ \frac{1}{8}R_3 \to R_3 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & -\frac{5}{4} & \frac{1}{8} \end{array}\right] \begin{array}{c} R_2 + R_3 \to R_2 \\ R_1 - R_3 \to R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & \frac{1}{4} & -\frac{1}{8} \\ 0 & 1 & 1 & 0 & \frac{1}{4} & -\frac{1}{8} \\ 0 & 0 & 0 & 1 & -\frac{5}{4} & \frac{1}{8} \end{array}\right]$

This is reduced row-echelon form. Since the identity matrix does not appear on the left, the matrix in this problem is **not invertible**.

32. (a) $(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 5/11 & 2/11 \\ 3/11 & -1/11 \end{bmatrix} \begin{bmatrix} -2/7 & 1/7 \\ 3/7 & 2/7 \end{bmatrix} = \begin{bmatrix} -4/77 & 9/77 \\ -9/77 & 1/77 \end{bmatrix}$

(b) $(A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2/7 & 1/7 \\ 3/7 & 2/7 \end{bmatrix}^T = \begin{bmatrix} -2/7 & 3/7 \\ 1/7 & 2/7 \end{bmatrix}$

(c) $A^{-2} = (A^{-1})^2 = \begin{bmatrix} -2/7 & 1/7 \\ 3/7 & 2/7 \end{bmatrix} \begin{bmatrix} -2/7 & 1/7 \\ 3/7 & 2/7 \end{bmatrix} = \begin{bmatrix} 1/7 & 0 \\ 0 & 1/7 \end{bmatrix}$

(d) $(2A)^{-1} = \frac{1}{2}A^{-1} = \frac{1}{2}\begin{bmatrix} -2/7 & 1/7 \\ 3/7 & 2/7 \end{bmatrix} = \begin{bmatrix} -2/14 & 1/14 \\ 3/14 & 2/14 \end{bmatrix} = \begin{bmatrix} -1/7 & 1/14 \\ 3/14 & 1/7 \end{bmatrix}$