

**SECTION 2.1** (Let me know if you see any typos and I'll correct them. -RH)

4. (a)  $A + B = \begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$

(b)  $A - B = \begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix}$

(c)  $2A = \begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix}$

(d)  $2A - B = \begin{bmatrix} 2 & 5 & -2 \\ 1 & -3 & 10 \end{bmatrix}$

12. (a)  $AB = [12]$

(b)  $BA = \begin{bmatrix} 6 & 4 & 2 \\ 9 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

18. (a)  $AB$  (Can't do—matrices don't match)

(b)  $BA = \begin{bmatrix} 37 & 78 & 51 & -104 & 124 \\ 16 & 26 & 28 & -42 & 56 \end{bmatrix}$

22. The system becomes  $\begin{bmatrix} 2 & 2 \\ -6 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -21 \end{bmatrix}$

To find  $x_1$  and  $x_2$ , we solve the system in the usual way.  $\begin{bmatrix} 2 & 2 & 7 \\ -6 & -6 & -21 \end{bmatrix} R_2 + 3R_1 \rightarrow R_2$

$\begin{bmatrix} 2 & 2 & 7 \\ 0 & 0 & 0 \end{bmatrix} 1/2R_1 \rightarrow R_1 \begin{bmatrix} 1 & 1 & 7/2 \\ 0 & 0 & 0 \end{bmatrix}$  Solution:  $x_1 = 7/2 - t$ ,  $x_2 = t$ , where  $t$  is any real number.

30. Solve the matrix equation for  $a, b, c$  and  $d$ .

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 17 \\ 4 & -1 \end{bmatrix}$$

First, multiply and get  $\begin{bmatrix} 2a + 3b & a + b \\ 2c + 3d & c + d \end{bmatrix} = \begin{bmatrix} 3 & 17 \\ 4 & -1 \end{bmatrix}$

The fact that the corresponding entries are equal gives the following system in variables  $a, b, c$  and  $d$ .

$$\begin{cases} 2a + 3b & = & 3 \\ a + b & = & 17 \\ & 2c + 3d & = & 4 \\ & c + d & = & -1 \end{cases}$$

Solving this in the usual way,

$$\begin{bmatrix} 2 & 3 & 0 & 0 & 3 \\ 1 & 1 & 0 & 0 & 17 \\ 0 & 0 & 2 & 3 & 4 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \quad \begin{array}{l} R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_4 \end{array} \quad \begin{bmatrix} 1 & 1 & 0 & 0 & 17 \\ 2 & 3 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 2 & 3 & 4 \end{bmatrix} \quad \begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_4 - 2R_3 \rightarrow R_4 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 0 & -31 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix} \quad \begin{array}{l} R_1 - R_2 \rightarrow R_1 \\ R_3 - R_4 \rightarrow R_3 \end{array} \quad \begin{bmatrix} 1 & 0 & 0 & 0 & 48 \\ 0 & 1 & 0 & 0 & -31 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

The solutions are  $a = 48, b = -31, c = -7, d = 6$ , so the matrix we seek is  $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 48 & -31 \\ -7 & 6 \end{bmatrix}$ .