**Directions** No calculators. Please put all phones, smart watches, etc., away.

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1. (16 points) This problem concerns the following statement.
   $P$ :    There is a subset $X$ of $\mathbb{N}$ for which $X \cap \mathbb{N} = \emptyset$.

   (a) Is the statement $P$ true or false? **Explain**.

   (b) Write the statement $P$ in symbolic form.

   (c) Form the negation $\neg P$ of your answer from (b), and simplify.

   (d) Write the negation $\neg P$ as an English sentence. (The sentence may use mathematical symbols.)

2. (6 points) Complete the first and last lines of each of the following proof outlines.

| **Proposition:** If $P$, then $Q$. | **Proposition:** If $P$, then $Q$. | **Proposition:** If $P$, then $Q$. |
|---|---|---|
| **Proof:** (Direct) | **Proof:** (Contrapositive) | **Proof:** (Contradiction) |
| Suppose _____ | Suppose _____ | Suppose _____ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| Therefore _____ .    ∎ | Therefore _____ .    ∎ | Therefore _____ .    ∎ |

3. (16 points) **Prove:** If $n \in \mathbb{Z}$, then $4|n^2$ or $4|(n^2+3)$.      [Use direct proof, with cases]

4. (16 points) Suppose $n \in \mathbb{Z}$. **Prove:** If $n^2$ is even, then $n$ is even.      [Use contrapositive.]

5. (16 points) Suppose $a, b, c \in \mathbb{Z}$.  **Prove:** If $a|b$ and $b|c$, then $a|c$.    [Use any appropriate method.]

6. (15 points) Prove or disprove:    If $a, b \in \mathbb{N}$, then $a + b < ab$.

7. (15 points)Prove or disprove:    Given $a, b, c \in \mathbb{Z}$, if $a|bc$, then $a|b$ or $a|c$.