MATH 211 | Test #3 ♠ | December 5, 2019

Name:____________________ R. Hammack Score:______

Directions No calculators. Please put all phones, etc., away.

---

1. (12 points) This problem concerns the following statement.
   P : There is a number $n \in \mathbb{Z}$ for which $m \mid n$ for every $m \in \mathbb{Z}$.

   (a) Is the statement P true or false? **Explain**.

   (b) Write the statement P in symbolic form.

   (c) Form the negation $\sim P$ of your answer from (b), and simplify.

   (d) Write the negation $\sim P$ as an English sentence.
   (The sentence may use mathematical symbols.)

2. (2 points) Complete the first and last lines of each of the following proof outlines.

Proposition: If P, then Q.
Proof: (Direct)
Suppose ______
⋮
Therefore ______ . ■

Proposition: If P, then Q.
Proof: (Contrapositive)
Suppose ______
⋮
Therefore ______ . ■

Proposition: If P, then Q.
Proof: (Contradiction)
Suppose ______
⋮
Therefore ______ . ■

3. (12 points) Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$.
Prove: If $a \equiv b \pmod{n}$, then $ab \equiv b^2 \pmod{n}$. [Use direct proof.]

4. (12 points) Suppose $a, b, c \in \mathbb{Z}$. **Prove:** If $a \nmid bc$, then $a \nmid b$ and $a \nmid c$. [Use contrapositive.]

5. (12 points) **Prove:** If $4 \,|\, (a^2 + b^2)$, then a and b are not both odd. [Use contradiction.]

6. (12 points) Suppose $a, b, c \in \mathbb{Z}$. **Prove:** If $a \,|\, b$ and $a \,|\, (b + c)$, then $a \,|\, c$.

7. (14 points) Suppose $n \in \mathbb{Z}$. Prove: $n^2 + 3$ is odd if and only if $n + 2$ is even.

8. (12 points) Prove or Disprove: There is a set X for which $\mathbb{N} \in X$ and $\mathbb{N} \subseteq X$.

9. (12 points) Prove or Disprove: For all $a, b \in \mathbb{Z}$, if $a \mid b$ and $b \mid a$ then $a = b$.