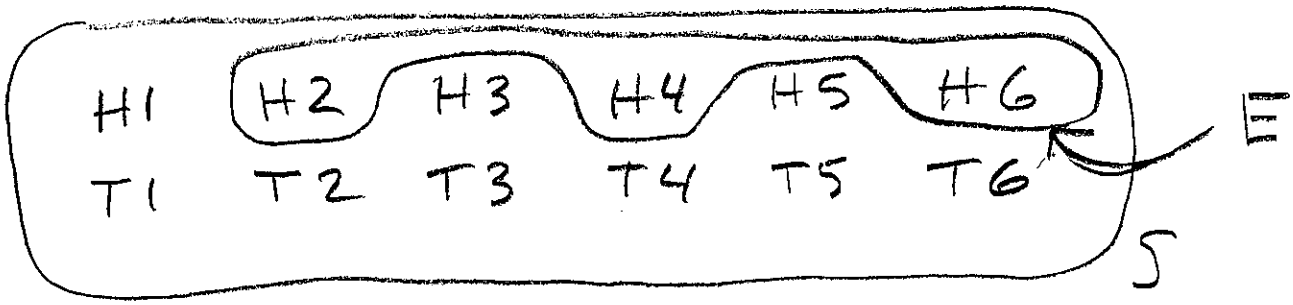
MATH 211 Test #2 ♡ April 11, 2023

Name: Richard R. Hammack Score:_____

Directions You must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified. All you will need is something to write with.

1. (10 points) Toss a coin and then roll a 6-sided dice. Write out the sample space S for this experiment. Consider the event E : *The coin is heads and the dice is even.* Circle E in S. Find $p(E)$.



$$P(E) = \frac{|E|}{|S|} = \frac{3}{12} = \frac{1}{4} = 25\%$$

2. (10 points) Toss a fair 6-sided dice 5 times in a row.
What are the chances that at least one of the tosses is odd?

S = set of length-5 lists made from 1, 2, 3, 4, 5, 6

$|S| = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$

Let E be the event "all tosses were even"

Thus $|E| = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

Answer: $P(\overline{E}) = 1 - P(E) = 1 - \frac{|E|}{|S|} = 1 - \frac{3^5}{6^5}$

$= 1 - \left(\frac{3}{6}\right)^5 = 1 - \left(\frac{1}{2}\right)^5 = 1 - \frac{1}{32} = \frac{31}{32}$

3. (10 points) The top card and the bottom card of a shuffled 52-card deck are removed. You win \$1 if the top card is black or the bottom card is a club. What are your chances of winning?

A: Top card is black

B: Bottom card is a club

Ans $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(B)\,P(A|B)$

$= \frac{1}{2} + \frac{1}{4} - \frac{1}{4} \cdot \frac{25}{51}$

$= \frac{1}{2} + \frac{1}{4} - \frac{25}{204}$

$= 62.74\%$

4. (10 points) A 6-card hand is dealt off a shuffled standard 52-card deck. What is the probability that not all of the cards are red?

S = set of 6-card subsets of 52-card deck

$|S| = \binom{52}{6}$

E = event of all six cards being red

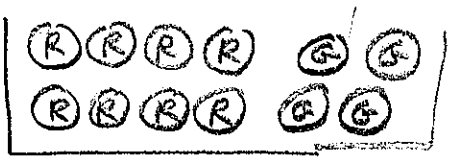
$|E| = \binom{26}{6}$

Ans $P(\bar{E}) = 1 - P(E) = 1 - \frac{|E|}{|S|}$

$= \boxed{1 - \frac{\binom{26}{6}}{\binom{52}{6}}} = \boxed{1 - \frac{\frac{26!}{20!\,6!}}{\frac{52!}{46!\,6!}}}$

5. (10 points) A box contains 8 red balls and 4 green balls. You reach in and remove two balls, one after the other. What is the probability that the two balls have different colors?

$S = \{RR, RG, GR, GG\}$



Events A: 1st draw is red
B: 2nd draw is red

Seek $P(\{RG, GR\}) = P(RG) + P(GR)$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) \cdot P(\bar{B}|A) + P(\bar{A}) \cdot P(B|\bar{A})$

$= \frac{8}{12} \cdot \frac{4}{11} + \frac{4}{12} \cdot \frac{8}{11} = \frac{8}{33} + \frac{8}{33} = \boxed{\frac{16}{33}}$

6. (10 points) Suppose $A, B \subseteq S$ are two events in the sample space S of some experiment. Suppose $p(A) = 50\%$, $p(B) = 60\%$ and $p(A|B) = 50\%$.

(a) $p(A \cap B) =$ $P(B)P(A|B) = (.6)(.5) = .3 = \boxed{30\%}$

(b) $p(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$
$= .5 + .6 - .3 = \boxed{80\%}$

(c) $p(B|A) =$ $\frac{P(A \cap B)}{P(A)} = \frac{.3}{.5} = \frac{3}{5} = \boxed{60\%}$

(d) $p(\overline{B}) =$ $1 - P(B) = 1 - .6 = 0.4 = \boxed{40\%}$

7. (10 points) A coin is tossed four times in a row, and there are more heads than tails. What is the probability that the first toss is a head?

Consider events:

A: "More heads than tails" ← {A = {HHHH, HHHT, HHTH, HTHH, THHH}}

B: "First toss is H"

Seek $p(B|A)$.

Note that if A has happened, then the first toss is a H in 4 out of 5 outcomes in A, so

$$p(B|A) = \frac{4}{5} = 80\%$$

8. (10 points) Give the output for the following chunk of pseudocode.

```
y := 3
for n := 1 to 6 do
    output y
    y := 10 − y
end
```

iteration	output	y
0	–	3
1	3	7
2	7	3
3	3	7
4	7	3
5	3	7
6	7	3

Output: 3 7 3 7 3 7

9. (10 points) What does the following algorithm do?

Algorithm

Input: A list $X = (x_1, x_2, \ldots, x_n)$ of integers
Output: ?
begin
  sum := 0
  for $k := 1$ **to** n **do**
    sum := sum + x_k
  end
  sum := $\frac{\text{sum}}{n}$
  output sum
end

It adds up the entries of X and then divides by the number of entries.

Thus it computes the average of the entries of X

10. (10 points) Write an algorithm whose input is a positive integer n and whose output is the first n terms of the sequence $1, 2, 4, 8, 16, 32, 64 \ldots$.

Algorithm

Input: n
Output: 1st n terms of 1, 2, 4, 8, 16 ...

```
begin
    y := 1
    for i := 1 to n do
        output y
        y := y · 2
    end
end
```