MATH 211 | Test #2 ♣ | November 16, 2017

Name:____________________ | R. Hammack | Score:______

Directions No calculators. Please put all phones, etc., away.

1. (4 points) Complete the following truth tables.

P	Q	$P \Rightarrow Q$
T	T	
T	F	
F	T	
F	F	

Q	R	$Q \Leftrightarrow R$
T	T	
T	F	
F	T	
F	F	

2. (12 points) Complete the truth table to decide if $P \Rightarrow (Q \wedge R)$ and $(\sim P) \vee (Q \Leftrightarrow R)$ are logically equivalent.

P	Q	R	
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Are they logically equivalent? Why or why not?

3. (6 points) Suppose the statement $(P \vee \sim P) \Leftrightarrow (P \wedge Q \wedge \sim R)$ is **true**.
Find the truth values of P, Q and R. (This can be done without a truth table.)

4. (12 points) This problem concerns the following statement.
$P:$ For each $n \in \mathbb{Z}$, there exists a number $m \in \mathbb{Z}$ for which $n + m = 0$.

(a) Is the statement P true or false? **Explain**.

(b) Write the statement P in symbolic form.

(c) Form the negation $\sim P$ of your answer from (b), and simplify.

(d) Write the negation $\sim P$ as an English sentence.
(The sentence may use mathematical symbols.)

5. (6 points) Complete the first and last lines of each of the following proof outlines.

Proposition: If P, then Q. **Proof:** (Direct) Suppose ______ $\vdots$ Therefore ______ . $\blacksquare$	**Proposition:** If P, then Q. **Proof:** (Contrapositive) Suppose ______ $\vdots$ Therefore ______ . $\blacksquare$	**Proposition:** If P, then Q. **Proof:** (Contradiction) Suppose ______ $\vdots$ Therefore ______ . $\blacksquare$

6. (15 points) Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$.
Prove: If $a \equiv b \pmod{n}$, then $a^2 \equiv b^2 \pmod{n}$. [Use direct proof.]

7. (15 points) Suppose $a \in \mathbb{Z}$. **Prove:** If $100 \nmid a^2$, then a is odd or $5 \nmid a$. [Use contrapositive.]

8. (15 points) **Prove:** If $a \in \mathbb{Z}$, then $4 \nmid (a^2 - 3)$. [Contradiction may be easiest.]

9. (15 points) **Prove:** If $n \in \mathbb{N}$, then $1 + (-1)^n(2n-1)$ is a multiple of 4. [Try cases.]