MATH 211 Test #2 November 10, 2016

Name:____________________ R. Hammack Score:_______

Directions No calculators. Please put all phones, etc., away.

1. Short Answer:

   (a) Give at least one statement that is logically equivalent to $P \Rightarrow Q$.

   (b) State DeMorgan's Laws.

2. Write a truth table to decide if $P \Rightarrow \sim Q$ and $(\sim P) \vee (\sim Q)$ are logically equivalent.

3. Suppose the statement $((R \wedge S) \Rightarrow P) \Leftrightarrow (Q \wedge \sim Q)$ is **true**. Find the truth values of R, S and P. (This can be done without a truth table.)

4. This problem concerns the following statement.
$P:$ Given any $x \in \mathbb{R}$, there exists an element $y \in \mathbb{R}$ for which $xy = 1$.

(a) Is the statement P true or false? **Explain**.

(b) Write the statement P in symbolic form.

(c) Form the negation $\sim P$ of your answer from (b), and simplify.

(d) Write the negation $\sim P$ as an English sentence.
(The sentence may use mathematical symbols.)

5. A **geometric sequence** with ratio r is a sequence of numbers for which any term is r times the previous term. If the first term of the sequence is a, then the sequence is $a, ar, ar^2, ar^3, ar^4, ar^5 \ldots.$ Write an algorithm whose input is three numbers $a, r \in \mathbb{R}$, and $n \in \mathbb{N}$, and whose output is the first n terms of the geometric sequence with first term a and ratio r.

6. **Prove:** If a is an even integer, then a^2 is even. [Direct proof may be easiest.]

7. **Prove:** If a is an odd integer, then $a^2 + 3a + 5$ is odd. [Direct proof may be easiest.]

8. Suppose $n \in \mathbb{Z}$. **Prove:** If $3 \nmid n^2$, then $3 \nmid n$. [Contrapositive may be easiest.]

9. **Prove:** If $n \in \mathbb{Z}$, then $4 \nmid (n^2 + 2)$. [Contradiction may be easiest.]

10. Suppose $a, b, c \in \mathbb{Z}$ and $n \in \mathbb{N}$. **Prove:** If $a \equiv b \pmod{n}$ and $a \equiv c \pmod{n}$, then $c \equiv b \pmod{n}$.