MATH 211 Test #2 ◊ April 11, 2023

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Directions You must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified. All you will need is something to write with.

1. (10 points) You have two fair 6-sided dice, a black one and a white one. You toss them both. Write out the sample space S, and circle the event $E \subseteq S$ of the two dice adding to 5. Find $p(E)$.

11	21	31	41	51	61
12	22	32	42	52	62
13	23	33	43	53	63
14	24	34	44	54	64
15	25	35	45	55	65
16	26	36	46	56	66

S

E

$$p(E) = \frac{|E|}{|S|} = \frac{4}{36} = \boxed{\frac{1}{9}} = \boxed{11.\bar{1}\%}$$

2. (10 points) Toss a coin 12 times in a row. What are the chances that exactly four of the tosses are tails?

S = set of length-12 lists (repitition ok) using H & T.

$$|S| = 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2 = 2^{12}$$

E: "Exactly 4 of the 12 tosses are T."

To make an outcome in E, choose 4 of 12 spots for T & fill rest with H. Thus $|E| = \binom{12}{4}$

$$p(E) = \frac{|E|}{|S|} = \frac{\binom{12}{4}}{2^{12}} = \frac{\frac{12\cdot11\cdot10\cdot9}{4\cdot3\cdot2\cdot1}}{2^{12}} = \frac{11\cdot5\cdot9}{2^{12}} =$$

3. (10 points) A 5-card hand is dealt off a shuffled standard 52-card deck.
What is the probability that not all of the cards are hearts?

S = set of 5-card subsets of 52-card deck

$|S| = \binom{52}{5}$

Let E be the event "All cards are hearts"

$|E| = \binom{13}{5}$

We seek $p(\bar{E}) = 1 - p(E) = 1 - \frac{|E|}{|S|} = \boxed{1 - \frac{\binom{13}{5}}{\binom{52}{5}}}$

$= \boxed{1 - \frac{P(13,5)}{P(52,5)}}$

4. (10 points) A 4-card hand is dealt off a shuffled standard 52-card deck.
What is the probability that all four cards are black or none of them are clubs?

S = set of 4-element subsets of 52-card deck

$|S| = \binom{52}{4}$

Events:

A: All 4 cards are black. $|A| = \binom{26}{4}$

B: None of the 4 cards is a club. $|B| = \binom{39}{4}$

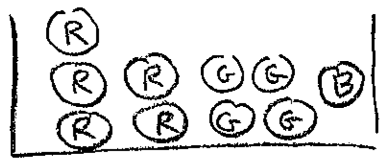
$A \cap B$: All 4 cards are spades $|A \cap B| = \binom{13}{4}$

Ans: $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$= \frac{|A|}{|S|} + \frac{|B|}{|S|} - \frac{|A \cap B|}{|S|}$

$= \boxed{\frac{\binom{26}{4}}{\binom{52}{4}} + \frac{\binom{39}{4}}{\binom{52}{4}} - \frac{\binom{13}{4}}{\binom{52}{4}}}$

5. (10 points) A box contains 5 red balls, 4 green balls and 1 blue ball. You reach in and remove two balls, one after the other. What is the probability that one of the balls is blue?



$S = \{ RR, RG, RB, GG, GR, GB, BR, BG \}$

$E = \{ RB, GB, BR, BG \}$

$$p(E) = p(RB) + p(GB) + p(BR) + p(BG)$$

$$= \frac{5}{10} \cdot \frac{1}{9} + \frac{4}{10} \cdot \frac{1}{9} + \frac{1}{10} \cdot \frac{5}{9} + \frac{1}{10} \cdot \frac{4}{9}$$

$$= \frac{5}{90} + \frac{4}{90} + \frac{5}{90} + \frac{4}{90} = \boxed{\frac{18}{90}} = \frac{2}{10} = \boxed{20\%}$$

6. (10 points) Suppose $A, B \subseteq S$ are two events in the sample space S of some experiment. Suppose $p(A) = 25\%$, $p(A|B) = 50\%$ and $p(B|A) = 40\%$.

(a) $p(\overline{A}) = 1 - p(A) = 1 - .25 = .75 = \boxed{75\%}$

(b) Are A and B independent or dependent?

Dependent because $p(A) = .25 \neq .50 = p(A|B)$.

(c) $p(A \cap B) = p(A) \cdot p(B|A) = (.25)(.4) = .1 = \boxed{10\%}$

(d) $p(B) =$

Formula: $p(A \cap B) = p(B) p(A|B)$

By above this is $0.1 = p(B) \cdot .5$

Thus $p(B) = \frac{0.1}{0.5} = \frac{1}{5} = \boxed{20\%}$

(e) $p(A \cup B) =$

$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

$$= .25 + .20 - .10 = \boxed{35\%}$$

7. (10 points) A woman has four children (no twins). Consider the following events:
   A: She has two girls and two boys.
   B: Her oldest child is a boy.
   Are events A and B independent, dependent, or is there not enough information to say for sure?

S = set of length-4 lists made from G & B

$|S| = 2^4 = 16$

$A = \{GGBB, GBGB, GBBG, BGGB, BGBG, BBGG\}$

$B = \{BBBB, BBBG, BBGB, BBGG, BGBB, BGBG, BGGB, BGGG\}$

$A \cap B = \{BBGG \ BGBG \ BGGB\}$

$P(B) = \frac{|B|}{|S|} = \frac{8}{16} = 50\%$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{|A \cap B|}{|S|}}{\frac{|A|}{|S|}} = \frac{\frac{3}{16}}{\frac{6}{16}} = \frac{3}{6} = 50\%$

Because $P(B) = P(B|A)$ events are independent

8. (10 points) Give the output for the following chunk of pseudocode.

```
y := 2
for n := 1 to 5 do
    y := 10 - y
    output y
end
output y
```

Iteration	y	output
0	2	—
1	8	8
2	2	2
3	8	8
4	2	2
5	8	8

$\boxed{\text{Output: } 8 \ 2 \ 8 \ 2 \ 8 \ 8}$

9. (10 points) What does the following algorithm do?

Algorithm

Input: A natural number $n \in \mathbb{N}$
Output: ?
begin
  while $(n > 1)$ **do**
    $n := n - 2$
  end
  if $(n = 1)$ **then**
    output "Yes"
  else
    output "No"
  end
end

Outputs "Yes" if n is odd
Outputs "No" if n is even

10. (10 points) Write an algorithm whose input is a positive integer n and whose output is the first n terms of the sequence $3, 6, 12, 24, 48, 96, \ldots$

Algorithm

input n
output 1st n terms of 3 6 12, 24, ...
begin
  y := 3
  for i := 1 to n do
    output y
    y := y · 2
  end
end