MATH 211 Test #1 ♣ March 14, 2023

Name: Richard R. Hammack Score:______

Directions Except in a problem designated **short answer**, you must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified, as in, for example, $7^{15} - 7!$. All you will need is something to write with. Scratch paper will be provided.

1. (9 points) **Short answer.**

   (a) Write $\{2, 4, 8, 16, 32, 64, \dots\}$ in set-builder notation. $\boxed{\{2^n : n \in \mathbb{N}\}}$

   (b) Write the set $\{X \subseteq \{3,4\} : |X| \le 1\}$ by listing its elements between braces. $\boxed{\{\phi, \{3\}, \{4\}\}}$

   (c) $\mathscr{P}(\{a,b\}) = \boxed{\{\phi, \{a\}, \{b\}, \{a,b\}\}}$

2. (12 points) **Short answer.** Suppose $A = \{3,4\}$ and $B = \{4,5\}$.

   (a) $A \times B = \boxed{\{(3,4), (3,5), (4,4), (4,5)\}}$

   (b) $B \times B = \boxed{\{(4,4), (4,5), (5,4), (5,5)\}}$

   (c) $(A \times B) \cap (B \times B) = \boxed{\{(4,4), (4,5)\}}$

   (d) $(B \times B) - (A \times B) = \boxed{\{(5,4), (5,5)\}}$

3. (9 points) Write a truth table for the expression $(P \Rightarrow Q) \vee \neg R$.

P	Q	R	$P \Rightarrow Q$	$\neg R$	$(P \Rightarrow Q) \vee \neg R$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

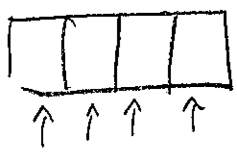
4. (20 points) Consider **length-4** lists made from the symbols A, B, C, D, with **repetition allowed.**

(a) How many such lists are there?

$4 \cdot 4 \cdot 4 \cdot 4$

Answer: $4^4 = \boxed{256}$

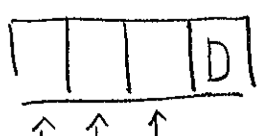
(b) How many such lists are there if the list does not contain an A?



$3 \cdot 3 \cdot 3 \cdot 3$

Answer: $3^4 = \boxed{81}$

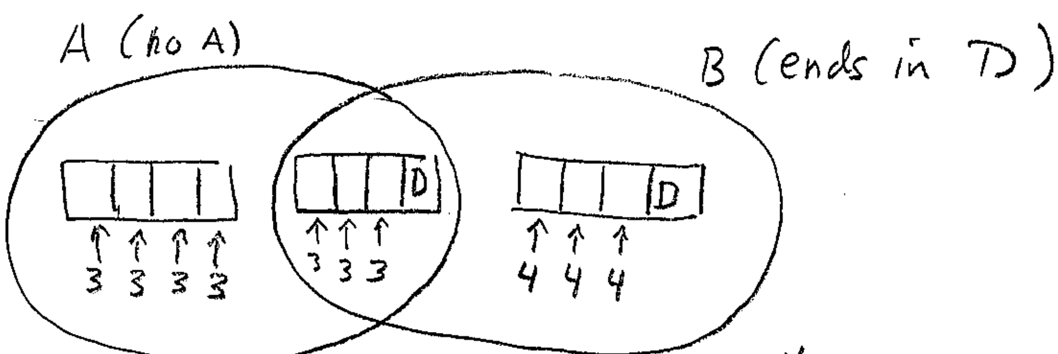
(c) How many such lists are there if the list does not contain an A **and** it ends with D?



$3 \cdot 3 \cdot 3$

Answer: $3^3 = \boxed{27}$

(d) How many such lists are there if the list does not contain an A **or** it ends with D?



Answer $|A \cup B| = |A| + |B| - |A \cap B| = 3^4 + 3^3 - 4^3 = 81 + 64 - 27$
$= 145 - 27 = \boxed{118}$

5. (8 points) Imagine tossing a 6-sided dice five times. A typical outcome can be described as a length-5 list such as 43461, meaning you rolled a 4 first, then 3, then 4, then 6, then 1.
How many outcomes are there in which not all tosses are odd? (e.g., 12134 or 22462, but not 31551)

All possible outcomes — All tosses are odd

6 6 6 6 6 − 3 3 3 3 3

By the subtraction principle, the answer is $6^5 - 3^5 = 7776 - 243 = \boxed{7533}$

6. (8 points) Consider the length-8 lists made from the symbols A, B, C, D, E, with repetition allowed. How many such lists are in alphabetical order? (For example, $AAABBBBC$ or $BBBDDDEE$, but **not** $BBAADADD$, etc.)

Imagine separating groups of letters by bars, such as AA|BB|cc|D|E or AAA|BB||DDD|
Such a configuration is modeled by a star and bar list **|**|**|** with 8 stars and 4 bars, so its length is $8+4=12$. Make such a list by selecting 4 out of 12 positions for bars and filling the rest with stars.
The number of such lists is $\binom{12}{4} = \frac{12!}{8!\,4!}$

$= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} = 55 \cdot 9 = \boxed{495}$

7. (10 points)

(a) How many subsets $X \subseteq \{0,1,2,3,4,5,6,7,8,9\}$ are there for which $|X| = 4$?

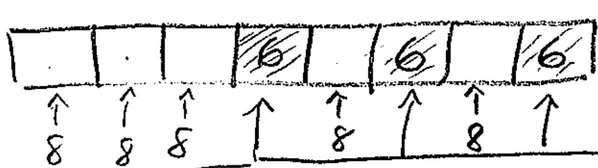
$$\binom{10}{4} = \frac{10!}{4!\,6!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2} = 10\cdot 3\cdot 7 = \boxed{210}$$

(b) Write Pascal's triangle to the 5th row and use it to expand $(x+y)^5$.

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

$$\boxed{(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5}$$

8. (8 points) How many 8-digit positive integers have no 0's and exactly three 6's?



Choose 3 of 8 positions for 6's

View such a 8-digit number as a length-8 list made from the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9.
First, choose 3 of 8 positions for the 6's.
Then there are 8 choices for the remaining 5 positions.

Answer: $\binom{8}{3} 8^5 = \frac{8!}{5!\,3!} 8^5 = \frac{8\cdot 7\cdot 6}{3!} 8^5 = 56\cdot 8^5 = \boxed{1835008}$

9. (8 points) A bag contains 20 pennies, 20 nickels, 20 dimes and 20 quarters. You reach in and take 10 coins. How many different outcomes are possible?

Model your outcomes like this:

pennies nickels dimes quarters

||***|* , with a * for each coin. List has 10 stars and 3 bars.

Total # of lists is

$$\binom{13}{3} = \frac{13!}{10!\,3!} = \frac{13\cdot 12\cdot 11}{3\cdot 2\cdot 1} = 13\cdot 2\cdot 11 = 13\cdot 22 = \boxed{286}$$

26
·6

10. (8 points) You deal five cards from a 52-card deck and line them up in a row. How many possible 5-card lineups are there where no two cards of the same color are next to one another?
(A deck has 26 red cards and 26 black cards.)

Two types of lineups:

R	B	R	B	R
26	26	25	25	24

B	R	B	R	B
26	26	25	25	24

By the addition principle, the answer is

$$2\cdot 26^2\cdot 25^2\cdot 24 = \boxed{20{,}280{,}000}$$