

Name: \_\_\_\_\_

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Score: \_\_\_\_\_

**Directions** Except in a problem designated **short answer**, you must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified, as in, for example,  $7^{15} - 7!$ . All you will need is something to write with. Scratch paper will be provided.

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1. (9 points) **Short answer.**

(a) Write  $\left\{ \frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \frac{36}{7}, \dots \right\}$  in set-builder notation.

(b) Write the set  $\{2n^2 : n \in \mathbb{Z}, 0 \leq 3n \leq 9\}$  by listing its elements between braces.

(c) Write the set  $\mathcal{P}(\{4\} \cap \mathbb{Z}) \times (\{0, 1, 2\} - \mathbb{N})$  by listing its elements between braces.

2. (12 points) **Short answer.** Suppose  $A = \{1, 3, 4, 6, 9\}$  and  $B = \{4, 5, 6, 8, 9\}$  are two sets in a universal set  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(a)  $A \cap \overline{B} =$

(b)  $\overline{\emptyset} =$

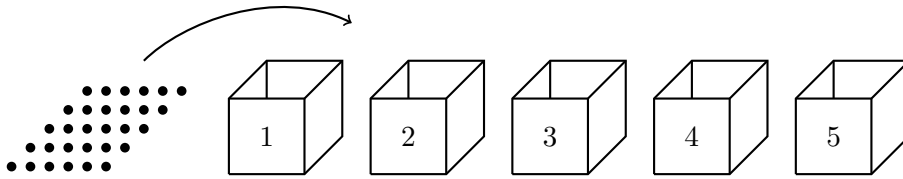
(c)  $\overline{U} \times A =$

(d)  $A - (A \times A) =$

3. (9 points) Write a truth table for  $P \Rightarrow Q$  and  $\sim (P) \vee Q$ . Based on this, say whether or not these two expressions are logically equivalent.



5. (8 points) In how many ways can you put 30 identical balls in five different boxes?



6. (8 points) Consider the length-6 lists made from the symbols  $A, B, C, D, E, F$ , with **no repetition**. How many such lists have the property that the letters  $A, B, C$  occur in alphabetical order? (For example,  $ABCDEF$  or  $ADBECF$ , or  $FEDABC$ , but **not**  $BACFED$ , etc.)

7. (10 points)

(a) A row of Pascal's triangle begins as  $1 \ 8 \ 28 \ 56 \ 70 \dots$

Write out the entire row. (You can do this without working out the triangle from top down.)

(b) Using part (a) above, and the binomial theorem, expand  $(x - 1)^8$ .

(Please simplify to the extent possible — it's easy to do!)

8. (8 points) Explain why  $2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} + \binom{n}{5} + \dots + \binom{n}{n}$ .

9. (8 points) Consider lists of length 5 made from the letters  $L, I, S, T, E, D$ , with **repetition allowed**. How many such lists have the property that the list begins with  $T$  **or** the last two entries are vowels?

10. (8 points) You deal seven cards from a 52-card deck and line them up in a row. How many possible 7-card lineups are there where no two cards of the same color are next to one another? (A deck has 26 red cards and 26 black cards.)