MATH 211 Test #1 ♡ October 5, 2017

Name:____________________ R. Hammack Score:______

Directions Except for those problems designated **short answer**, you must show and explain your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices. All you will need is something to write with. I will provide scratch paper.

1. (6 points) **Short answer.**

   (a) Write the set $\{2n : n \in \mathbb{Z},\ n^2 \leq 16\}$ by listing its elements between braces.

   (b) Write $\{\ldots -2, 3, 8, 13, 18, 23, 28, 33, \ldots\}$ in set-builder notation.

   (c) Write the set $\big(\{1,3\} \times \mathbb{Z}\big) \cap \big(\mathbb{Z} \times \{5,6\}\big)$ by listing its elements between braces.

2. (6 points) **Short answer.** Suppose A and B are sets for which $|A| = m$ and $|B| = n$. Find the following cardinalities.

   (a) $|A \times B| =$

   (b) $|\mathscr{P}(A \times B)| =$

   (c) $\big|\{X \in \mathscr{P}(A) \ : \ |X| = 5\}\big| =$

3. (8 points) **Short answer.** Suppose $A = \{1,3,4,6,9\}$ and $B = \{4,5,6,8,9\}$ are two sets in a universal set $U = \{1,2,3,4,5,6,7,8,9\}$

   (a) $A \cap B =$

   (b) $A - \overline{B} =$

   (c) $\overline{\emptyset} =$

   (d) $(A - B)^2 =$

4. (20 points) This question concerns **length-5** lists made from the letters A, B, C, D, E, F, G, H, I.

   (a) How many such lists have no repetition and end with a vowel?

   (b) How many such lists are there if repetition is allowed, and the list contains at least one A?

   (c) How many such lists are there if repetition is allowed, and the list contains *exactly* one A?

   (d) How many such lists are there that have no repetition and are in alphabetical order?

5. (10 points) How many 10-digit integers contain no 0's and exactly three 6's?

6. (10 points) In how many ways can you distribute 30 identical pieces of candy among 4 children?

7. (10 points)

(a) Here are the first several rows of Pascal's triangle. Write the next row.

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

(b) Use part (a) to find the coefficient of x^3y^3 in $(2x-y)^6$.
Please give the exact (i.e., worked out) value.

8. (10 points) Use the binomial theorem to show that
$3^n = 2^0\binom{n}{0} + 2^1\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + 2^4\binom{n}{4} + \cdots + 2^n\binom{n}{n}$.

9. (10 points) A department consists of 5 men and 7 women. From this department you select a committee with 3 men and 2 women. In how many ways can you do this?

10. (10 points) How many 4-digit positive integers are there that are even or contain no 0's?