MATH 211 Test #1 October 4, 2016

Name:_____ R. Hammack Score:_____

Directions Unless noted otherwise, you must show and explain your work to get full credit.

---

1. (4 points) **Short answer.**

   (a) Express the set $\{3x + 1 : x \in \mathbb{N}\}$ by listing its elements between braces.

   (b) Write $\left\{\ldots \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8 \ldots\right\}$ in set-builder notation.

2. (10 points) **Short answer.** In this problem $A = \{1, 2, 3\}$ and $B = \{1, 4\}$.

   (a) $A \cap B =$

   (b) $(A - B) \times B =$

   (c) $\mathscr{P}(B) =$

   (d) $\mathscr{P}(B) - \mathscr{P}(A) =$

   (e) How many subsets of cardinality 4 does $A \times B$ have?

3. (6 points) **Short answer.** Suppose $A = \{1, 3, 4, 6, 9\}$ and $B = \{4, 5, 6, 8, 9\}$ are two sets in a universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

   (a) $\overline{A \cap B} =$

   (b) $\overline{A} \cup \overline{B} =$

4. (20 points) Five cards are dealt off of a standard 52-card deck and lined up in a row.

   (a) How many different lineups are possible?

   (b) How many of these lineups have at least one red card?

   (c) How many lineups have exactly one red card?

   (d) How many lineups are either all black or all hearts?

5. (10 points) How many 8-digit binary strings end in 1 or have exactly four 1's?

6. (10 points) This problem concerns length-6 lists made from the symbols A, B, C, D, E, F, without repetition. How many such lists are there in which the D occurs before the A?

7. (10 points) How many permutations of the letters A, B, C, D, E, F, G are there in which the letters ABC appear consecutively, in alphabetical order?

8. (10 points) In how many ways can you place 20 identical balls into 5 different boxes?

9. (10 points)

   (a) Write out Pascal's triangle to Row 5.

   (b) Use part (a) as an aid in expanding $(2x - y)^5$.

10. (10 points)

   (a) State the binomial theorem.

   (b) Use the binomial theorem to show that $2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \cdots + \binom{n}{n}$