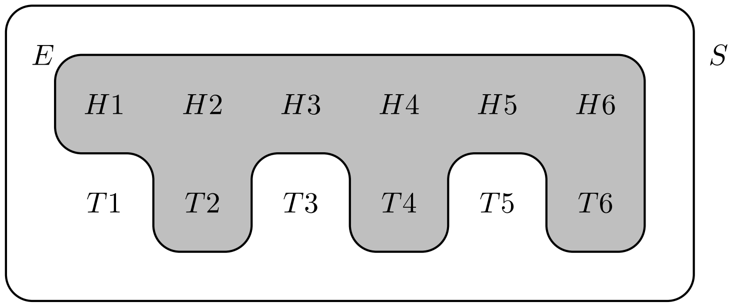
MATH 211 Test #2 ♣ October 31, 2019

Name:____________________ R. Hammack Score:______

Directions You must show your work to get full credit. This test is closed-book and closed-notes. No calculators or other electronic devices are allowed. Simplify your answers if it is easy to do so, but you may leave complex answers unsimplified. All you will need is something to write with.

1. (10 points) You toss a coin and then roll a 6-sided dice. Write out the sample space S for this experiment. Consider the event E : *The coin is heads or the dice is even.* Circle E in S. Find $p(E)$.



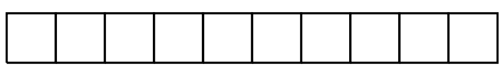
$$p(E) = \frac{|E|}{|S|} = \frac{9}{12} = \frac{3}{4} = \boxed{75\%}$$

2. (10 points) Toss a coin 10 times in a row.
What are the chances that exactly five of the tosses are tails?

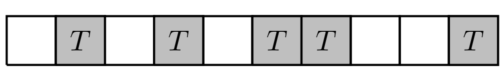
The sample space S is the set of all length-10 lists made from the symbols H and T, with repetition allowed. Thus $|S| = 2^{10}$.

Consider the event $E \subseteq S$ of exactly five tails.

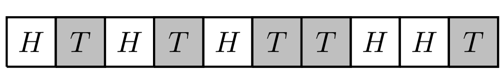
To make a list in E, start with 10 empty spots.



Then pick five out of the ten spots for T's. You can do this in $\binom{10}{5}$ ways.



Now fill the remaining spots with heads.



Thus $|E| = \binom{10}{5} = \frac{10!}{5!(10-5)!} = \frac{10!}{5!(5)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2} = 2 \cdot 3 \cdot 7 \cdot 6 = 252.$

Thus $p(E) = \frac{|E|}{|S|} = \frac{\binom{10}{5}}{2^{10}} = \frac{252}{1024} \approx \boxed{24.61\%}$

3. (10 points) A 7-card hand is dealt off a shuffled standard 52-card deck.
What is the probability that not all of the cards are red?

The sample space S is the set of all 7-element subsets of the deck of 52 cards, so $|S| = \binom{52}{7}$.

Let E be the event of all seven cards being red, so $|E| = \binom{26}{7}$.
(The number of ways to choose 7 out of 26 red cards.)

Now, $\overline{E}$ is the event of not all the seven cards being red. Thus our answer is

$$p(\overline{E}) = 1 - p(E) = 1 - \frac{|E|}{|S|} = 1 - \frac{\binom{26}{7}}{\binom{52}{7}} = 1 - \frac{\frac{P(26,7)}{7!}}{\frac{P(26,7)}{7!}} = 1 - \frac{P(26,7)}{P(52,7)} = 1 - \frac{55}{11186} \approx 99.5\%$$

4. (10 points) The top card and the bottom card of a shuffled 52-card deck are removed.
You win a dollar if the top card is black or the bottom card is a heart.
What are your chances of winning?

Consider the following events:
A : The top card is black
B : The bottom card is a heart

The answer will be $p(A \cup B)$.

$$\begin{aligned} p(A \cup B) &= p(A) + p(B) - p(A \cap B) \\ &= p(A) + p(B) - p(A) \cdot p(B|A) \\ &= \frac{26}{52} + \frac{13}{52} - \frac{26}{52} \cdot \frac{13}{51} \\ &\approx \mathbf{62.2549}\% \end{aligned}$$

5. (10 points) A box contains 7 red balls and 5 green balls. You reach in and remove two balls, one after the other. What is the probability that the two balls have the same color?

The sample space is $S = \{\, RR,\ RG,\ GR,\ GG \,\}$.
Thus the event of both balls having the same color is $E = \{RR, GG\}$.

Let A be the event of the first draw being R. So $\overline{A}$ be the event of the first draw being G.
Let B be the event of the second draw being R. So $\overline{B}$ be the event of the second draw being G.

$P(RR) = p(A \cap B) = p(A) \cdot p(B|A) = \frac{7}{12} \cdot \frac{6}{11}$.
$P(GG) = p(\overline{A} \cap \overline{B}) = p(\overline{A}) \cdot p(\overline{B}|\overline{A}) = \frac{5}{12} \cdot \frac{4}{11}$.

Then $p(E) = p\big(\{RR, GG\}\big) = p(RR) + p(GG) = \dfrac{7}{12} \cdot \dfrac{6}{11} + \dfrac{5}{12} \cdot \dfrac{4}{11} = \dfrac{7 \cdot 6 + 5 \cdot 4}{12 \cdot 11} = \dfrac{62}{132} \approx \boxed{\mathbf{46.969\%}}$

6. (10 points) Suppose $A, B \subseteq S$ are two events in the sample space S of some experiment.
Suppose $p(A) = 25\%,\ \ p(A|B) = 50\%$ and $p(B|A) = 40\%$.

(a) $p\left(\overline{A}\right) = 1 - p(A) = 1 - 0.25 = \boxed{\mathbf{75\%}}$.

(b) Are A and B independent or dependent?

Since $p(A) = 25\%$ and $p(A|B) = 50\%$, we see that $p(A) \neq p(A|B)$, so $\boxed{A \textbf{ and } B \textbf{ are dependent}}$

(c) $p(A \cap B) = p(A) \cdot p(B|A) = (0.25) \cdot (0.40) = 0.10 = \boxed{\mathbf{10\%}}$

(d) $p(B) =$

Combining the above information with $p(A|B) = \dfrac{p(A \cap B)}{p(B)}$ results in $0.5 = \dfrac{0.1}{p(B)}$.

From this, $p(B) = \frac{0.1}{0.5} = 0.2 = \boxed{\mathbf{20\%}}$.

(e) $p(A \cup B) = p(A) + p(B) - p(A \cap B) = 25\% + 20\% - 10\% = \boxed{\mathbf{35\%}}$.

7. (10 points) A coin is tossed three times in a row, and there more heads than tails. What is the probability that the first toss is a head?

The sample space can be considered to be $S = \{HHH, HHT, HTH, THH\}$.
The event of the first toss being a head is $E = \{HHH, HHT, HTH\} \subseteq S$.

Thus $p(E) = \frac{|E|}{|S|} = \frac{3}{4} = \boxed{\mathbf{75\%}}$.

8. (10 points) Give the output for the following chunk of pseudocode.

```
y := 5
for n := 1 to 5 do
|   output y
|   y := 10 + y
end
```

Output:	**5**	**15**	**25**	**35**	**45**

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9. (10 points) What does the following algorithm do?

Algorithm

Input: A list $X = (x_1, x_2, \ldots, x_n)$ of at least two integers
Output: ?
begin
  answer := Yes
  for $k := 1$ **to** $n - 1$
   do
    if $(x_k > x_{k+1})$ **then**
     answer := No
    end
  end
  output answer
end

It returns "Yes" if the list X is sorted from smallest to largest.
Otherwise it returns "No".

For example it would return "Yes" on input $(1, 2, 2, 3, 6, 8)$ or $(3, 3, 3, 3)$.
It returns "No" on input $(5, 6, 3, 5, 4)$.

10. (10 points) Write an algorithm whose input is a positive integer n and whose output is the first n terms of the sequence $6, 60, 600, 6000, \ldots$.

This is a geometric sequence whose first term is 6 and whose ratio is 10. (Multiply any term by 10 to get the next term.

Algorithm (computes first n terms of $6, 60, 600, 6000, \ldots$.)

Input: A positive integer n.
Output: First n terms of geometric series with first term 6 and ratio 10.
begin
  $a := 6$
  for $i = 1$ **to** n **do**
    output a ....(this is the ith term)
    $a := a \cdot 10$ .... (now multiply ith term by 10 to get the next term)
  end
end