MATH 211	FINAL EXAM	December 13, 2016

Name: ____________________ R. Hammack Score: ________

Directions: Answer each question in the space provided. To get full credit you must show all of your work, unless instructed otherwise. Use of calculators is **not** allowed on this test.

1. (10 points) Write each set by listing its elements between braces.

   (a) $\{m \in \mathbb{N} \,:\, 3|m\} =$

   (b) $\big\{x \in \mathbb{R} \,:\, x^2 - 2x = 0\big\} =$

   (c) $\mathscr{P}(\{1,2\}) =$

   (d) $\big\{1,2\big\} \times \mathscr{P}(\{1,2\}) =$

   (e) $\big\{1,2\big\} \cap \mathscr{P}(\{1,2\}) =$

2. (6 points)

   (a) Suppose the following statement is **false:** $(P \wedge \sim Q) \Rightarrow (R \Rightarrow S)$
   Is there enough information given to determine the truth values of P, Q, R and S? If so, what are they?

   (b) Write a sentence that is the negation of the following sentence:
   There exists a real number a for which $a + x = x$ for every real number x.

   (c) Decide if the following statement true or false. Briefly justify answer. $\forall\, n \in \mathbb{N}, \exists\, X \in \mathscr{P}(\mathbb{N}), |X| = n - 1$

3. (6 points) Write a truth table for $(P \Rightarrow Q) \Leftrightarrow (P \vee Q)$.

4. (6 points) A 5-card poker hand is called a *flush* if all cards are the same suit. How many different flushes are there?

5. (6 points) Consider 4-card hands dealt off of a standard 52-card deck. How many hands are there for which all 4 cards are of the same suit or all 4 cards are red?

6. (6 points) In how many ways can you place 20 identical balls into five different boxes?

7. (6 points) Suppose $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$.
Prove: If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then $ac \equiv bd \pmod{n}$.

8. (6 points) Prove: If $n \in \mathbb{Z}$, then $4 \mid n^2$ or $4 \mid (n^2 + 3)$.

9. (6 points) Prove: If $n \in \mathbb{Z}$, then $4 \nmid (n^2 - 3)$.

10. (6 points) Suppose $a, b \in \mathbb{Z}$. Prove ab is odd if and only if both a and b are odd.

11. (6 points) Prove or disprove: If a relation R on a set A is both transitive and symmetric, then it is also reflexive.

The questions on this page involve the function $f : \mathbb{R} \times \mathbb{N} \to \mathbb{R} \times \mathbb{N}$ defined as $f((x, y)) = (3xy, y)$

12. (6 points) Is f is injective?

13. (6 points) Is f is surjective?

14. (6 points) Does the inverse function f^{-1} exist? If so, find it.

15. (6 points) Use mathematical induction to prove $2^1 + 2^2 + 2^3 + 2^4 + \cdots + 2^n = 2^{n+1} - 2$ for every $n \in \mathbb{N}$.

16. (6 points) Prove: If $a, b \in \mathbb{Z}$, then $a^2 - 4b - 3 \neq 0$. [Contradiction may be easiest.]