Name: Richard

Quiz 20 ♡

MATH 211
April 25, 2023

1. Suppose $n \in \mathbb{Z}$. Use contrapositive to prove: If $3 \nmid n^2$, then $3 \nmid n$.

Proposition If $3 \nmid n^2$, then $3 \nmid n$.

Proof (Contrapositive) Suppose $3 \nmid n$ is not true.
Then $3 \mid n$, which means $n = 3c$ for some $c \in \mathbb{Z}$.
Consequently $n^2 = (3c)^2 = 9c^2 = 3 \cdot 3c^2$.
Thus $n^2 = 3d$ for $d = 3c^2 \in \mathbb{Z}$, so $3 \mid n^2$.
Hence $3 \nmid n^2$ is not true. ■

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Quiz 20 ♢

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1. Suppose $a \in \mathbb{Z}$. Use contrapositive to prove: If a^2 is not divisible by 4, then a is odd.

Proposition If a^2 is not divisible by 4, then a is odd

Proof (Contrapositive). Suppose it is not true that a is odd.
Then a is even, so $a = 2c$ for some $c \in \mathbb{Z}$.
Hence $a^2 = (2c)^2 = 4c^2$.
So $a^2 = 4d$ for $d = c^2 \in \mathbb{Z}$.
Consequently $4 \mid a^2$, so it is not true that a^2 is not divisible by 4. ■