Name: Richard

Quiz 19 ♣

MATH 211
April 20, 2023

1. Suppose $a, b, c \in \mathbb{Z}$. Use direct proof to prove: If $a \mid b$ and $b \mid c$, then $a \mid c$.

Proof. Suppose $a|b$ and $b|c$.
Because $a|b$, we know $b = ad$ for some $d \in \mathbb{Z}$.
Because $b|c$, we know $c = be$ for some $e \in \mathbb{Z}$.
From the above, $c = be = ade = a(de)$.
So $c = a(de)$, where $de \in \mathbb{Z}$.
Therefore $a|c$, by definition of divides. ■

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1. Suppose $a, b \in \mathbb{Z}$. Use direct proof to prove: If $a \mid b$, then $a^2 \mid b^2$.

Proof. Suppose $a|b$.
By definition od divides, $b = ac$ for some $c \in \mathbb{Z}$.
Consequently $b^2 = (ac)^2 = a^2c^2$, or $\boxed{b^2 = a^2c^2}$
So $b^2 = a^2 \cdot d$, where $d = c^2 \in \mathbb{Z}$.
Therefore $a^2|d^2$, by definition of divides. ■