Name: Richard

QUIZ 18 ♡

MATH 211
April 18, 2023

1. This question concerns the statement $\forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^2 = a$

(a) Is this statement true or false?

For each $a \in \mathbb{R}$, there is a $b \in \mathbb{R}$ for which $b^2 = a$

False: If a were negative, Then There is no $b \in \mathbb{R}$ for which $b^2 = a$

(b) Form the negation of the statement, and simplify.

$$\neg(\forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^2 = a)$$
$$= \exists a \in \mathbb{R}, \neg(\exists b \in \mathbb{R}, b^2 = a)$$
$$= \exists a \in \mathbb{R}, \forall b \in \mathbb{R}, \neg(b^2 = a)$$
$$= \boxed{\exists a \in \mathbb{R}, \forall b \in \mathbb{R}, b^2 \neq a}$$

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1. This question concerns the statement $\forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^3 = a$

(a) Is this statement true or false?

For each $a \in \mathbb{R}$, there is a $b \in \mathbb{R}$ for which $b^3 = a$.

TRUE For any $a \in \mathbb{R}$, just let $b = \sqrt[3]{a}$.
Then $b^3 = \sqrt[3]{a}^3 = a$

(b) Form the negation of the statement, and simplify.

$$\neg(\forall a \in \mathbb{R}, \exists b \in \mathbb{R}, b^3 = a)$$
$$= \exists a \in \mathbb{R}, \neg(\exists b \in \mathbb{R}, b^3 = a)$$
$$= \exists a \in \mathbb{R}, \forall b \in \mathbb{R}, \neg(b^3 = a)$$
$$= \boxed{\exists a \in \mathbb{R}, \forall b \in \mathbb{R}, b^3 \neq a}$$