

1. How many 4-digit numbers are even or divisible by 5?  
(Examples: 9764, 4475 and 4470 are even or divisible by 5, but 5421 is not.)

$A =$  (set of even  
4-digit #'s)

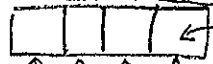
No 0 here



0, 2, 4, 6 or 8 here

$$|A| = 9 \cdot 10 \cdot 10 \cdot 5 = 4500$$

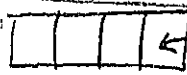
$B =$  (set of 4-digit #'s  
that are divisible by 5)



0 or 5 here

$$|B| = 9 \cdot 10 \cdot 10 \cdot 2 = 1800$$

$A \cap B =$  (set of 4-digit #'s  
that are even and  
divisible by 5)



only 0 here

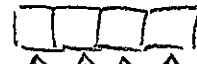
$$|A \cap B| = 9 \cdot 10 \cdot 10 \cdot 1 = 900$$

Answer:  $|A \cup B| = |A| + |B| - |A \cap B| = 4500 + 1800 - 900$   
 $= 4500 + 900$   
 $= \boxed{5400}$

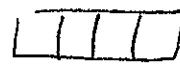
By inclusion-exclusion

1. A length-4 list is made from the letters A, B, C, D, E, with repetition allowed.  
How many such lists begin or end with a vowel?

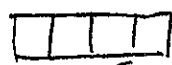
$A =$  (set of lists that  
begin with vowel)

$|A| =$    $= 2 \cdot 5 \cdot 5 \cdot 5 = 250$

$B =$  (set of lists that  
end with vowel)

$|B| =$    $= 5 \cdot 5 \cdot 5 \cdot 2 = 250$

$A \cap B =$  (set of lists that  
begin and end  
in a vowel)

$|A \cap B| =$    $= 2 \cdot 5 \cdot 5 \cdot 2 = 100$

Answer:  $|A \cup B| = |A| + |B| - |A \cap B| = 250 + 250 - 100$   
 $= \boxed{400}$

By inclusion-exclusion

1. A length-4 list is made from the letters A, B, C, D, E, with repetition allowed.  
How many such lists begin with a vowel or end with a consonant?

$$A = (\text{set of lists beginning with vowel}) \quad |A| = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 2 \cdot 5 \cdot 5 \cdot 5 = 250$$

$$B = (\text{set of lists ending with consonant}) \quad |B| = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 5 \cdot 5 \cdot 5 \cdot 3 = 375$$

$$A \cap B = (\text{lists beginning with vowel and ending with a consonant}) \quad |A \cap B| = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 2 \cdot 5 \cdot 5 \cdot 3 = 150$$

Answer:  $|A \cup B| = |A| + |B| - |A \cap B|$   
 $= 250 + 375 - 150 = \boxed{475}$

By Inclusion-  
exclusion

1. A length-4 list is made from the letters A, B, C, D, E, with repetition allowed.  
How many such lists begin with a vowel or have no repeated letters?  
(Examples: EDCC, EAAA, ABCD, DCAE, BCDE.)

$$A = (\text{set of lists beginning with vowel}) \quad |A| = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 2 \cdot 5 \cdot 5 \cdot 5 = 250$$

$$B = (\text{set of lists without repetition}) \quad |B| = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 5 \cdot 4 \cdot 3 \cdot 2 = 120$$

$$A \cap B = (\text{set of lists beginning with vowel and having no repetition}) \quad |A \cap B| = \begin{array}{|c|c|c|c|} \hline \square & \square & \square & \square \\ \hline \end{array} = 2 \cdot 4 \cdot 3 \cdot 2 = 48$$

Answer:  $|A \cup B| = |A| + |B| - |A \cap B|$   
 $= 250 + 120 - 48$

By  
inclusion-  
exclusion

$$= 370 - 48 = \boxed{322}$$