

$$1. \int x e^{5x} dx = x \frac{e^{5x}}{5} - \int \frac{e^{5x}}{5} dx = \boxed{\frac{x e^{5x}}{5} - \frac{e^{5x}}{25} + C}$$

$$\begin{aligned} u &= x & dv &= e^{5x} dx \\ du &= dx & v &= \frac{e^{5x}}{5} \end{aligned}$$

$$\text{Check: } \frac{d}{dx} \left[\frac{x e^{5x}}{5} - \frac{e^{5x}}{25} + C \right] =$$

$$\begin{aligned} & \frac{1}{5} (1 e^{5x} + x 5 e^{5x}) - \frac{1}{25} e^{5x} \cdot 5 + 0 \\ &= \frac{e^{5x}}{5} + x e^{5x} - \frac{e^{5x}}{5} = \underline{x e^{5x}} \quad \checkmark \end{aligned}$$

$$1. \int x \cos(3x) dx = x \frac{1}{3} \sin(3x) - \int \frac{1}{3} \sin(3x) dx$$

$$\begin{aligned} u &= x & dv &= \cos(3x) dx \\ du &= dx & v &= \frac{1}{3} \sin(3x) \end{aligned}$$

$$= \boxed{\frac{x \sin(3x)}{3} + \frac{\cos(3x)}{9} + C}$$

Check:

$$\frac{d}{dx} \left[\frac{1}{3} x \sin(3x) + \frac{1}{9} \cos(3x) + C \right]$$

$$= \frac{1}{3} (1 \cdot \sin(3x) + x \cos(3x) \cdot 3) - \frac{1}{9} \sin(3x) \cdot 3 + 0$$

$$= \frac{\sin(3x)}{3} + x \cos(3x) - \frac{\sin(3x)}{3} = \underline{x \cos(3x)} \quad \checkmark$$

$$1. \int x^2 \ln(x) dx = \ln(x) \frac{x^3}{3} - \int \frac{x^3}{3} \frac{1}{x} dx$$

$$\left. \begin{aligned} u &= \ln(x) & dv &= x^2 dx \\ du &= \frac{1}{x} dx & v &= \frac{x^3}{3} \end{aligned} \right\}$$

$$= \frac{\ln(x) x^3}{3} - \frac{1}{3} \int x^2 dx$$

$$= \boxed{\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C}$$

$$\text{Check: } \frac{d}{dx} \left[\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C \right] =$$

$$\frac{1}{3} \left(3x^2 \ln(x) + x^3 \frac{1}{x} \right) - \frac{x^2}{3} + 0 = x^2 \ln(x) - \frac{1}{3} x^2 + \frac{x^2}{3} = \underline{x^2 \ln(x)}$$

$$1. \int x \sec^2(x) dx = x \tan(x) - \int \tan(x) dx$$

$$\left. \begin{aligned} u &= x & dv &= \sec^2(x) dx \\ du &= dx & v &= \tan(x) \end{aligned} \right\}$$

$$= \boxed{x \tan(x) - \ln|\sec(x)| + C}$$

$$\text{Check: } \frac{d}{dx} \left[x \tan(x) - \ln|\sec(x)| + C \right]$$

$$= 1 \tan(x) + x \sec^2(x) - \frac{\sec(x) \tan(x)}{\sec(x)} + 0$$

$$= \tan(x) + x \sec^2(x) - \tan(x) = \underline{x \sec^2(x)}$$