Name: Richard

Quiz 5 ♠

MATH 201

February 1, 2024

1. Find the arc length of the curve $y = \frac{\sqrt{x}^3}{3}$ from $x = 0$ to $x = 60$.

$y = \frac{1}{3} x^{3/2}$

$$L = \int_0^{60} \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^{60} \sqrt{1 + \left(\frac{1}{2} x^{\frac{1}{2}}\right)^2}\, dx$$

$u = 1 + \frac{1}{4}x$

$\frac{du}{dx} = \frac{1}{4}$

$dx = 4\,du$

$$= \int_0^{60} \sqrt{1 + \frac{1}{4}x}\, dx$$

$$= \int_{1+\frac{1}{4}\cdot 0}^{1+\frac{1}{4}\cdot 60} \sqrt{u}\; 4\, du$$

$$= 4 \int_1^{16} u^{\frac{1}{2}}\, du = 4 \left[\frac{u^{3/2}}{3/2} \right]_1^{16}$$

$$= 4 \left[\frac{2\sqrt{u}^3}{3} \right]_1^{16} = 4 \left[\frac{2\sqrt{16}^3}{3} - \frac{2\sqrt{1}^3}{3} \right]$$

$$= 4\left(\frac{2 \cdot 4^3}{3} - \frac{2}{3} \right) = \boxed{168 \text{ units}}$$

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Name: Richard

QUIZ 5 ♢

MATH 201

February 1, 2024

1. Find the arc length of the curve $y = \dfrac{\sqrt{x^2+2}^3}{3}$ from $x = 0$ to $x = 1$.

$$y = \frac{1}{3}\left(x^2+2\right)^{\frac{3}{2}}$$

$$L = \int_0^1 \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_0^1 \sqrt{1 + \left(\frac{1}{3}\cdot\frac{3}{2}\left(x^2+2\right)^{1/2} 2x\right)^2}\, dx$$

$$= \int_0^1 \sqrt{1 + (x^2+2)\, x^2}\, dx$$

$$= \int_0^1 \sqrt{x^4 + 2x^2 + 1}\, dx$$

$$= \int_0^1 \sqrt{(x^2+1)^2}\, dx$$

$$= \int_0^1 (x^2+1)\, dx = \left[\frac{x^3}{3} + x\right]_0^1$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{0^3}{3} + 0\right)$$

$$= \boxed{\frac{4}{3} \text{ units}}$$

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QUIZ 5 ♣

MATH 201

February 1, 2024

1. Find the arc length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ from $x = 1$ to $x = 2$.

$$L = \int_1^2 \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \left(x^3 - \frac{1}{4x^3}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \left(x^3 - \frac{1}{4x^3}\right)\left(x^3 - \frac{1}{4x^3}\right)}\, dx$$

$$= \int_1^2 \sqrt{1 + x^6 - \frac{1}{4} - \frac{1}{4} + \frac{1}{16x^6}}\, dx$$

$$= \int_1^2 \sqrt{x^6 + \frac{1}{2} + \frac{1}{16x^6}}\, dx$$

$$= \int_1^2 \sqrt{\left(x^3 + \frac{1}{4x^3}\right)^2}\, dx = \int_1^2 \left(x^3 + \frac{1}{4x^3}\right) dx$$

$$= \left[\frac{x^4}{4} - \frac{1}{8x^2}\right]_1^2 = \left(\frac{2^4}{4} - \frac{1}{8 \cdot 2^2}\right) - \left(\frac{1^4}{4} - \frac{1}{8 \cdot 1^2}\right)$$

$$= 4 - \frac{1}{32} - \frac{1}{4} + \frac{1}{8} = \frac{128}{32} - \frac{1}{32} - \frac{8}{32} + \frac{4}{32}$$

$$= \boxed{\frac{123}{32} \text{ units}}$$

Name: Richard

QUIZ 5 ♡

MATH 201

February 1, 2024

1. Find the arc length of the curve $y = 3\ln(x) - \frac{x^2}{24}$ from $x = 1$ to $x = 6$.

$$L = \int_1^6 \sqrt{1 + (f'(x))^2}\, dx$$

$$= \int_1^6 \sqrt{1 + \left(\frac{3}{x} - \frac{x}{12}\right)^2}\, dx$$

$$= \int_1^6 \sqrt{1 + \left(\frac{3}{x} - \frac{x}{12}\right)\left(\frac{3}{x} - \frac{x}{12}\right)}\, dx$$

$$= \int_1^6 \sqrt{1 + \frac{9}{x^2} - \frac{1}{4} - \frac{1}{4} + \frac{x^2}{144}}\, dx$$

$$= \int_1^6 \sqrt{\frac{x^2}{144} + \frac{1}{2} + \frac{9}{x^2}}\, dx$$

$$= \int_1^6 \sqrt{\left(\frac{x}{12} + \frac{3}{x}\right)^2} dx = \int_1^6 \left(\frac{x}{12} + \frac{3}{x}\right) dx$$

$$= \left[\frac{x^2}{24} + 3\ln|x|\right]_1^6 = \left(\frac{6^2}{24} + 3\ln(6)\right) - \left(\frac{1^2}{24} + 3\ln(1)\right)$$

$$= \frac{36}{24} - \frac{1}{24} + 3\ln(6) = \boxed{\frac{35}{24} + 3\ln(6) \text{ units}}$$