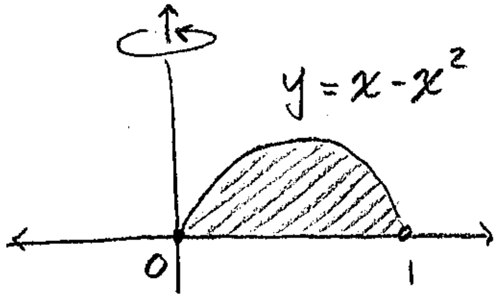
Name: Richard

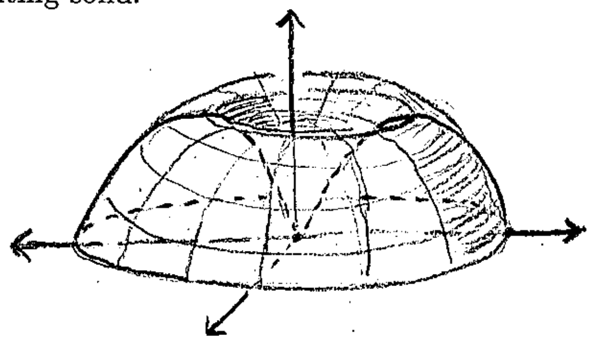
Quiz 4 ♠

MATH 201

January 30, 2024

1. The region between the graphs of $y = x - x^2$ and $y = 0$ is rotated around the y-axis. Use the shell method to find the volume of the resulting solid.





$$V = \int_0^1 2\pi x\,(x - x^2)\,dx$$

$$= 2\pi \int_0^1 x^2 - x^3\,dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

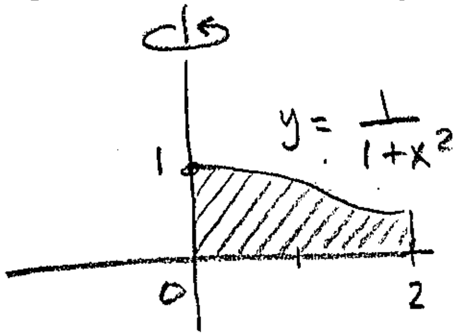
$$= 2\pi \left(\left(\frac{1^3}{3} - \frac{1^3}{4}\right) - \left(\frac{0^3}{3} - \frac{0^3}{4}\right) \right)$$

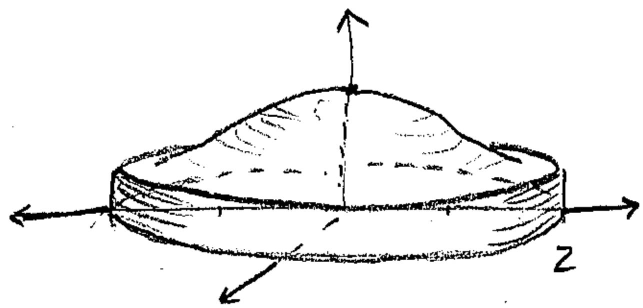
$$= 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = 2\pi\left(\frac{4}{12} - \frac{3}{12}\right)$$

$$= 2\pi \cdot \frac{1}{12} = \boxed{\frac{\pi}{6} \text{ cubic units}}$$

Name: Richard	Quiz 4 ♢	MATH 201 January 30, 2024

1. Consider the region bounded above by $y = \dfrac{1}{1+x^2}$, below by the x-axis, and for $0 \le x \le 2$. This region is rotated around the y-axis. Use the shell method to find the volume of the resulting solid.





$$V = \int_0^2 2\pi\, x \frac{1}{1+x^2}\, dx$$

$$u = 1 + x^2$$
$$\frac{du}{dx} = 2x$$
$$\frac{1}{2}du = x\,dx$$

$$= 2\pi \int_0^2 \frac{1}{1+x^2}\, x\, dx$$

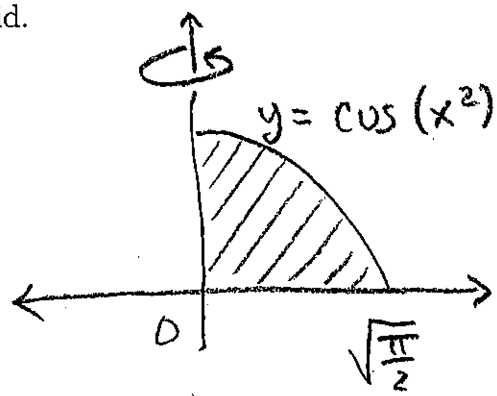
$$= 2\pi \int_{1+0^2}^{1+2^2} \frac{1}{u}\,\frac{1}{2}\, du$$

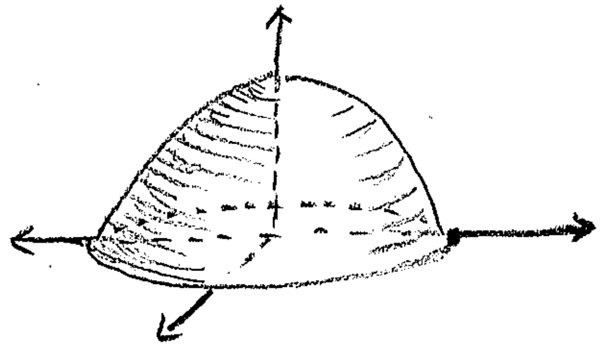
$$= \pi \int_1^5 \frac{1}{u}\, du = \pi \Big[\ln|u|\Big]_1^5$$

$$= \pi\big(\ln|5| - \ln|1|\big) = \boxed{\pi \ln(5) \text{ cubic units}}$$

Name: Richard | Quiz 4 ♣ | MATH 201
January 30, 2024

1. Consider the region bounded above by $y = \cos\left(x^2\right)$, below by the x-axis, and for $0 \leq x \leq \sqrt{\pi/2}$. This region is rotated around the y-axis. Use the shell method to find the volume of the resulting solid.





$$V = \int_0^{\sqrt{\frac{\pi}{2}}} 2\pi x \cos(x^2)\, dx$$

$$u = x^2$$
$$\frac{du}{dx} = 2x$$
$$\frac{1}{2} du = x\, dx$$

$$= 2\pi \int_0^{\sqrt{\pi/2}} \cos(x^2)\; x\, dx$$

$$= 2\pi \int_{0^2}^{\sqrt{\pi/2}^{\,2}} \cos(u)\, \frac{1}{2}\, du$$

$$= \pi \int_0^{\pi/2} \cos(u)\, du = \pi \Big[\sin(u)\Big]_0^{\pi/2}$$

$$= \pi\left(\sin\left(\tfrac{\pi}{2}\right) - \sin(0)\right) = \pi(1-0) = \boxed{\pi \text{ cubic units}}$$

Name: Richard

QUIZ 4 ♡

MATH 201
January 30, 2024

1. Consider the region bounded $y = \sqrt{x}$, $y = 0$ and $x = 4$. This region is rotated around the y-axis. Use the shell method to find the volume of the resulting solid.

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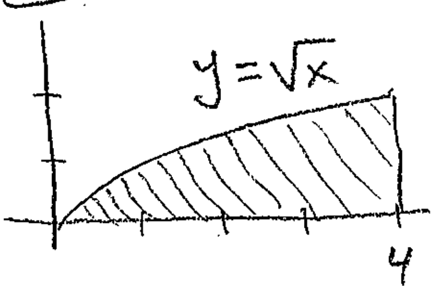

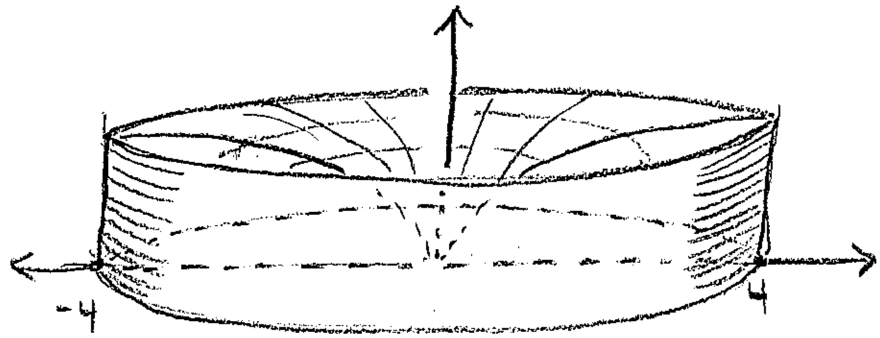

$$V = \int_0^4 2\pi\, x \sqrt{x}\, dx$$

$$= 2\pi \int_0^4 x^{\frac{3}{2}}\, dx$$

$$= 2\pi \left[\frac{x^{\frac{5}{2}}}{5/2} \right]_0^4 = 2\pi \left[\frac{2\sqrt{x}^5}{5} \right]_0^4$$

$$= 2\pi \left(\frac{2\sqrt{4}^5}{5} - \frac{2\sqrt{0}^5}{5} \right)$$

$$= \frac{2\pi\, 2 \cdot 2^5}{5} = \frac{2^7\pi}{5} = \boxed{\frac{128}{5}\pi \text{ cubic units}}$$