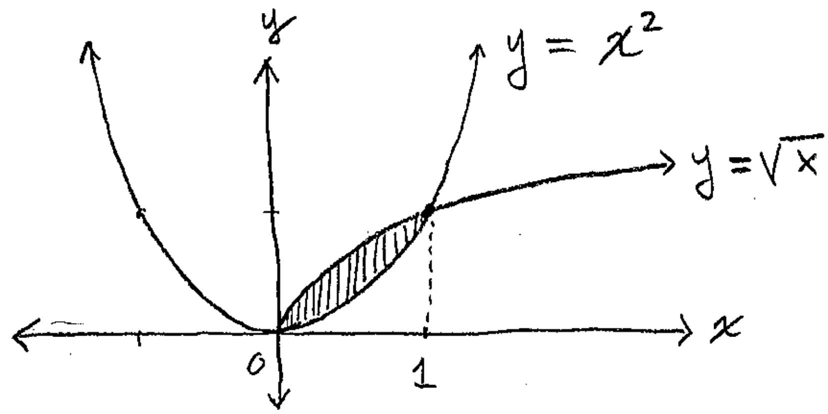
Name: Richard

QUIZ 2 ♠

MATH 201
January 23, 2024

1. Find the area of the region bounded by $y = x^2$ and $y = \sqrt{x}$. (Sketch the curves first!)

The two curves intersect at (0,0) and (1,1)



$$A = \int_0^1 (\sqrt{x} - x^2)\,dx = \int_0^1 (x^{\frac{1}{2}} - x^2)\,dx$$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1 = \left[\frac{2\sqrt{x}^3}{3} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{2\sqrt{1}^3}{3} - \frac{1^3}{3}\right) - \left(\frac{2\sqrt{0}^3}{3} - \frac{0^3}{3}\right)$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) - (0 - 0)$$

$$= \boxed{\frac{1}{3} \text{ square unit}}$$

Name: Richard QUIZ 2 ◇ MATH 201
January 23, 2024

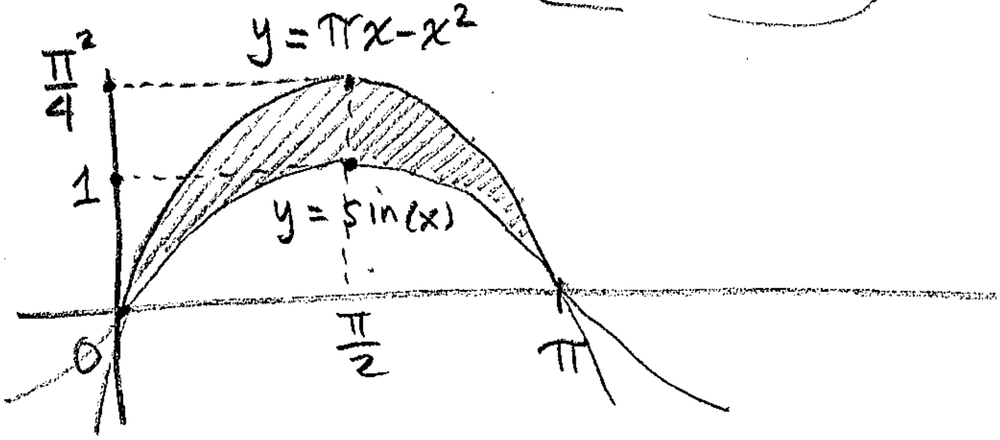
1. Find the area of the region bounded by $y = \pi x - x^2$ and $y = \sin(x)$. (Sketch the curves first!)

$y = \pi x - x^2$ ← parabola, opens down.

To find x-intercepts: $\pi x - x^2 = 0$

$x(\pi - x) = 0$

$x = 0$ $x = \pi$ ← x-intercepts



$$A = \int_0^{\pi} \left((\pi x - x^2) - \sin(x) \right) dx$$

$$= \int_0^{\pi} \left(\pi x - x^2 - \sin(x) \right) dx$$

$$= \left[\pi \frac{x^2}{2} - \frac{x^3}{3} + \cos(x) \right]_0^{\pi}$$

$$= \left(\frac{\pi \cdot \pi^2}{2} - \frac{\pi^3}{3} + \cos(\pi) \right) - \left(\frac{\pi \cdot 0^2}{2} - \frac{0^3}{3} + \cos(0) \right)$$

$$= \frac{\pi^3}{2} - \frac{\pi^3}{3} - 1 - 1 = \boxed{\frac{\pi^3}{6} - 2 \text{ square units}}$$

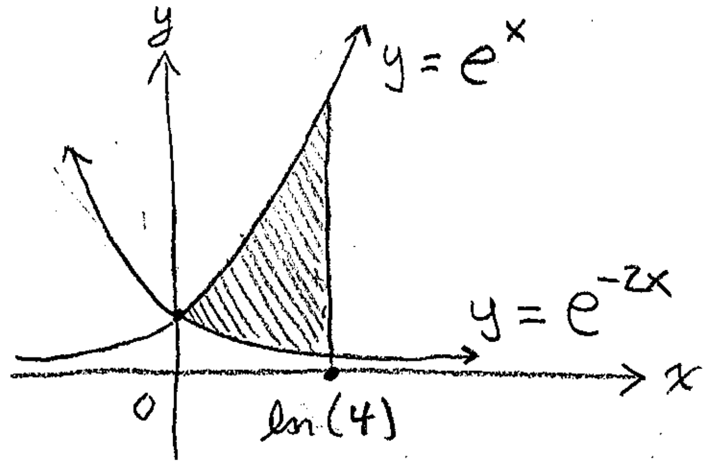
Name: Richard

Quiz 2 ♣

MATH 201

January 23, 2024

1. Find the area of the region bounded by $y = e^x$, $y = e^{-2x}$ and $x = \ln(4)$. (Sketch the curves first!)



$$A = \int_0^{\ln(4)} \left(e^x - e^{-2x} \right) dx$$

$$= \left[e^x - \left(-\tfrac{1}{2} e^{-2x} \right) \right]_0^{\ln(4)}$$

$$= \left[e^x + \tfrac{1}{2} e^{-2x} \right]_0^{\ln(4)}$$

$$= \left(e^{\ln(4)} + \frac{1}{2} e^{-2\ln(4)} \right) - \left(e^0 + \tfrac{1}{2} e^{-2\cdot 0} \right)$$

$$= 4 + \frac{1}{2} e^{\ln(4^{-2})} - 1 - \frac{1}{2}$$

$$= 4 + \frac{1}{2} \cdot \frac{1}{16} - 1 - \frac{1}{2} = \frac{128}{32} + \frac{1}{32} - \frac{32}{32} - \frac{16}{32}$$

$$= \boxed{\frac{81}{32} \text{ square units}}$$

Name: Richard

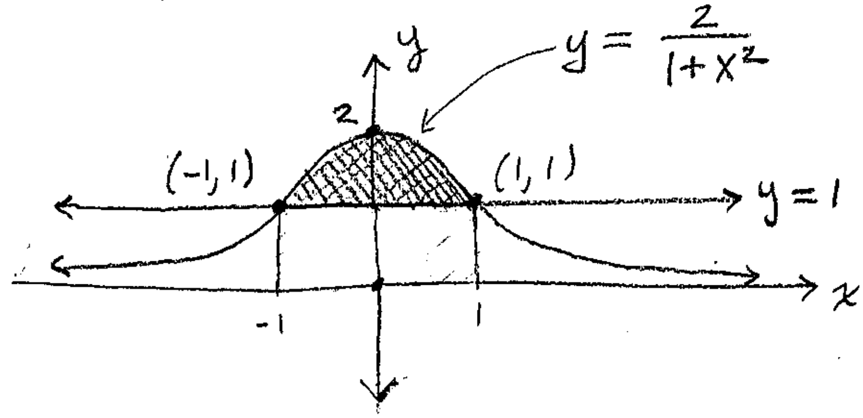
Quiz 2 ♡

MATH 201

January 23, 2024

1. Find the area of the region bounded by $y = \dfrac{2}{1+x^2}$ and $y = 1$. (Sketch the curves first!)

Note that the curve $y = \frac{2}{1+x^2}$ has a y-intercept of $y = \frac{2}{1+0^2} = 2$.



Also as $\lim\limits_{x\to\infty} \frac{1}{1+x^2} = 0$, it has a horizontal asymptote of $y = 0$. (See sketch above)

To find where $y = \frac{2}{1+x^2}$ and $y = 1$ intersect, solve $\frac{2}{1+x^2} = 1 \Rightarrow 2 = 1 + x^2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\therefore$ the intersection points are $(-1, 1)$ and $(1, 1)$. see the sketch above.

$$A = \int_{-1}^{1} \left(\frac{2}{1+x^2} - 1 \right) dx = \Big[2\tan^{-1}(x) - x \Big]_{-1}^{1}$$

$$= \left(2\tan^{-1}(1) - 1\right) - \left(2\tan^{-1}(-1) - (-1)\right)$$

$$= \left(2\frac{\pi}{4} - 1\right) - \left(-2\frac{\pi}{4} + 1\right) = \frac{\pi}{2} + \frac{\pi}{2} - 2$$

$$= \boxed{\pi - 2 \text{ square units}}$$