Name: Richard | QUIZ 20 ♠ | MATH 201
April 16, 2024

1. Use the ratio test, root test or alternating series test to determine whether the series converges.

$$\sum_{k=1}^{\infty} \frac{k^2}{4^k}$$

Ratio test $\lim_{k\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{k\to\infty} \left| \frac{\frac{(k+1)^2}{4^{k+1}}}{\frac{k^2}{4^k}} \right|$

$$= \lim_{k\to\infty} \frac{(k+1)^2}{4^{k+1}} \frac{4^k}{k^2}$$

$$= \lim_{k\to\infty} \frac{(k+1)^2}{k^2} \frac{4^k}{4^{k+1}} = \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 \frac{1}{4}$$

$$= \frac{1}{4} \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^2 = \frac{1}{4}\left(\lim_{k\to\infty} \frac{k+1}{k}\right)^2$$

$$= \frac{1}{4} \cdot 1^2 = \frac{1}{4}$$

Because $\frac{1}{4} < 1$, the ratio test says the series converges

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QUIZ 20 ◇

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1. Use the ratio test, root test or alternating series test to determine whether the series converges.
$\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{e^k}$

This is an alternating series, so let's apply the alternating series test.

Let $f(x) = \frac{x^2}{e^x}$, so $a_k = f(k)$.

Notice that $f'(x) = \frac{2x e^x - x^2 e^x}{(e^x)^2} = \frac{e^x x(1-x)}{(e^x)^2}$,

and this is <u>negative</u> on $(2, \infty)$. Therefore $f(x)$ decreases on $[2, \infty)$ and hence

$a_2 > a_3 > a_4 > a_5 > \cdots$

$\frac{\infty}{\infty}$

Next, note that $\lim_{k \to \infty} a_k = \lim_{k \to \infty} \frac{k^2}{e^k}$

$= \lim_{k \to \infty} \frac{2k}{e^k} = \lim_{k \to \infty} \frac{2}{e^k} = 0.$

$\frac{\infty}{\infty}$

Now we've shown ① $a_2 > a_3 > a_4 > a_5 > \cdots$ and ② $\lim_{k \to \infty} a_k = 0$. Therefore the <u>series converges</u> by the alternating series test.

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QUIZ 20 ♣

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1. Use the ratio test, root test or alternating series test to determine whether the series converges.

$$\sum_{k=1}^{\infty}(-1)^{k+1}ke^{-k}$$

Ratio Test $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to 1}\left|\frac{(-1)^{k+2}(k+1)e^{-(k+1)}}{(-1)^{k+1}ke^{-k}}\right|$

$$= \lim_{k\to\infty}\frac{(k+1)e^{-(k+1)}}{ke^{-k}}$$

$$= \lim_{k\to\infty}\frac{k+1}{k}\frac{1}{e^{-k+(k+1)}}$$

$$= \lim_{k\to\infty}\frac{k+1}{k}\frac{1}{e}$$

$$= \frac{1}{e}\lim_{k\to\infty}\frac{k+1}{k} = \frac{1}{e}\cdot 1 = \frac{1}{e} < 1$$

Because $\frac{1}{e} < 1$, the series converges by the ratio test.

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Quiz 20 ♡

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1. Use the ratio test, root test or alternating series test to determine whether the series converges.
$\sum_{k=1}^{\infty} \left(\frac{1}{k}\right)^k$

Root Test $\lim_{k\to\infty} \sqrt[k]{a_k} = \lim_{k\to\infty} \sqrt[k]{\left(\frac{1}{k}\right)^k}$

$= \lim_{k\to\infty} \frac{1}{k} = 0 < 1$

Because $0 < 1$, the root test says the series converges.