Name: ______________________________ QUIZ 19 ♠ MATH 201

April 11, 2024

1. Use the comparison test or the limit comparison tests to determine whether the series converges.

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k}+\sqrt{k}}$$

Notice that $\sqrt{k} \geq \sqrt[3]{k}$ for $k \geq 1$ and therefore

$$\frac{1}{\sqrt{k}+\sqrt{k}} \leq \frac{1}{\sqrt[3]{k}+\sqrt{k}}$$

left side has larger denominator

$$\frac{1}{2\sqrt{k}} \leq \frac{1}{\sqrt[3]{k}+\sqrt{k}}$$

$$\frac{1}{2k^{1/2}} \leq \frac{1}{\sqrt[3]{k}+\sqrt{k}}$$

Therefore $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k}+\sqrt{k}}$ diverges by comparison

with the divergent p-series $\sum_{k=1}^{\infty} \frac{1}{2} \cdot \frac{1}{k^{1/2}}$

1. Use the comparison test or the limit comparison tests to determine whether the series converges.
$\sum_{k=1}^{\infty} \frac{2+(-1)^k}{k^2}$

Notice that $2+(-1)^k$ equals either 1 or 3 depending on whether k is odd or even. Consequently this series has positive terms.

Moreover,

$$\frac{2+(-1)^k}{k^2} \leq \frac{2+1}{k^2} = \frac{3}{k^2}$$

Consequently $\sum_{k=1}^{\infty} \frac{2+(-1)^k}{k^2}$ converges

by comparison with the convergent p-series $\sum_{k=1}^{\infty} \frac{3}{k^2}$ $(p=2)$.

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1. Use the comparison test or the limit comparison tests to determine whether the series converges.
$$\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^3+4}$$

The terms of this series are positive and

$$\frac{\sqrt{k}}{k^3+4} \le \frac{\sqrt{k}}{k^3} \longleftarrow$$

because left side has larger denominator

$$\frac{\sqrt{k}}{k^3+4} \le \frac{k^{\frac{1}{2}}}{K^3} = \frac{1}{K^{3-1/2}} = \frac{1}{K^{5/2}}$$

Consequently $\sum_{K=1}^{\infty} \frac{\sqrt{K}}{K^3+4}$ converges by comparison with the convergent p-series $\sum_{k=1}^{\infty} \frac{1}{k^{5/2}}$

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1. Use the comparison test or the limit comparison tests to determine whether the series converges.
$\sum_{k=1}^{\infty} \frac{\cos^2(k)}{2^k}$

Note that $\cos^2(k) \leq 1$ and therefore

$$\frac{\cos^2(k)}{2^k} \leq \frac{1}{2^k}$$

and $\sum_{k=1}^{\infty} \frac{\cos^2(k)}{2^k}$ converges by comparison with the convergent geometric series $\sum_{k=1}^{\infty} \frac{1}{2^k}$ with ratio $\frac{1}{2} < 1$.