Name: Richard

Quiz 18 ♠

MATH 201
April 9, 2024

1. Determine whether the series converges or diverges. Explain your answer.

(a) $\sum_{k=1}^{\infty}\left(\frac{5}{2^k}+\frac{5}{k^2}\right)$ $= \sum_{k=1}^{\infty}\frac{5}{2^k} + \sum_{k=1}^{\infty}\frac{5}{k^2} = 5\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^k + 5\sum_{k=1}^{\infty}\frac{1}{k^2}$

Geometric series with $r=\frac{1}{2}<1$. Converges

p-series with $p=2>1$. converges

As the sum of two convergent series, the series

$$\boxed{\sum_{k=1}^{\infty}\left(\frac{5}{2^k}+\frac{5}{2^k}\right) \text{ converges}}$$

(b) $\sum_{k=1}^{\infty}\frac{k^2+1}{2k^2-k}$

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty}\frac{k^2+1}{2k^2-k} = \lim_{k\to\infty}\frac{k^2+1}{2k^2-k}\,\frac{\frac{1}{k^2}}{\frac{1}{k^2}}$$

$$= \lim_{k\to\infty}\frac{1+\frac{1}{k^2}}{2-\frac{1}{k}} = \frac{1+0}{2+0} = \frac{1}{2} \neq 0.$$

The $\boxed{\text{series diverges}}$ by the divergence test

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1. Determine whether the series converges or diverges. Explain your answer.

(a) $\displaystyle\sum_{k=1}^{\infty}\sqrt{\frac{2}{k}} = \sum_{k=1}^{\infty}\frac{\sqrt{2}}{\sqrt{k}} = \sum_{k=1}^{\infty}\frac{\sqrt{2}}{k^{1/2}} = \sqrt{2}\sum_{k=1}^{\infty}\frac{1}{k^{1/2}}$

p-series with $p=\frac{1}{2}<1$ Diverges

Divergent p-series.

(b) $\displaystyle\sum_{k=1}^{\infty}\frac{\sqrt{k}+1}{k^2} = \sum_{k=1}^{\infty}\left(\frac{\sqrt{k}}{k^2}+\frac{1}{k^2}\right) = \sum_{k=1}^{\infty}\left(\frac{k^{\frac{1}{2}}}{k^2}+\frac{1}{k^2}\right)$

$$= \sum_{k=1}^{\infty}\left(\frac{1}{k^{3/2}}+\frac{1}{k^2}\right)$$

$$= \sum_{k=1}^{\infty}\frac{1}{k^{3/2}} + \sum_{k=1}^{\infty}\frac{1}{k^2}$$

p-series $p=3/2>1$ Converges!

p-series $p=2>1$ converges

As the sum of two convergent p-series, this series <u>converges</u>.

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Quiz 18 ♣

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1. Determine whether the series converges or diverges. Explain your answer.

(a) $\sum_{k=1}^{\infty} \sqrt{\frac{2k}{k+1}}^{3}$

Let's try the divergence test.

$$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \sqrt{\frac{2k}{k+1}}^{3} = \sqrt{\lim_{k\to\infty} \frac{2k}{k+1}}^{3}$$

$$= \sqrt{\lim_{k\to\infty} \frac{2}{1+0}}^{3} = \sqrt{2}^{3} = 2\sqrt{2} \neq 0$$

Therefore the series <u>diverges</u> by the divergence test.

(b) $\sum_{k=1}^{\infty} ke^{-k^2}$

Let's try the integral test. $a_k = ke^{-k^2}$ is <u>positive</u> and $D_x\left[xe^{-x^2}\right] = e^{-x^2} + xe^{-x^2}(-2x)$ $= e^{-x^2}(1-2x)$ and this is <u>negative</u> on $[1,\infty)$ so xe^{-x^2} <u>decreases</u>. Thus the integral test applies. $\int_1^{\infty} xe^{-x^2}dx = -\frac{1}{2}\int_1^{\infty} e^{-x^2}(-2x)\,dx =$

$$-\frac{1}{2}\lim_{b\to\infty}\left[e^{-x^2}\right]_1^b = -\frac{1}{2}\lim_{b\to\infty}\left[\frac{1}{e^{b^2}} - \frac{1}{e^{1^2}}\right] = -\frac{1}{2}\left(0-\frac{1}{e}\right)$$

$= \frac{1}{2e}$. Because the integral converges, the <u>series converges</u> by the integral test.

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QUIZ 18 ♡

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1. Determine whether the series converges or diverges. Explain your answer.

(a) $\sum_{k=1}^{\infty} k^{1/k}$

Let's try the divergence test.

$$\lim_{k\to\infty} k^{1/k} = \lim_{k\to\infty} e^{\ln(k^{1/k})} = \lim_{k\to\infty} e^{\frac{1}{k}\ln(k)}$$

form ∞^0

$$= e^{\lim_{k\to\infty} \frac{\ln(k)}{k}}$$

form $\frac{\infty}{\infty}$

$$= e^{\lim_{k\to\infty} \frac{\frac{1}{k}}{1}} = e^0 = 1 \neq 0$$

Therefore the series diverges by the divergence test.

(b) $\sum_{k=3}^{\infty} \frac{\ln(k)}{k}$

$$D_x\left[\frac{\ln(k)}{k}\right] = \frac{\frac{1}{k}k - \ln(k)\cdot 1}{k^2} = \frac{1-\ln(k)}{k^2}$$

negative for $k > e$!

So $\frac{\ln(k)}{k}$ is positive and decreasing on $[3,\infty)$

So the integral test applies.

$$\int_3^{\infty} \frac{\ln(x)}{x}\,dx = \int_3^{\infty} \ln(x)\,\frac{1}{x}\,dx$$

$u = \ln(x)$
$du = \frac{1}{x}\,dx$

$$= \int_{\ln(3)}^{\infty} u\,du = \lim_{b\to\infty}\left[\frac{u^2}{2}\right]_{\ln(3)}^{b} = \lim_{b\to\infty}\left(\frac{b^2}{2} - \frac{\ln^2(3)}{2}\right)$$

$$= \boxed{\infty}$$

Therefore series diverges (by integral test).