Name: Richard

QUIZ 14 ♠

MATH 201

March 14, 2024

1. $\int_2^\infty \frac{1}{(x+1)^3}\,dx = \lim_{b\to\infty} \int_2^b \frac{1}{(x+1)^3}\,dx$

$$= \lim_{b\to\infty} \int_2^b (x+1)^{-3}\,dx \longleftarrow \begin{Bmatrix} u = x+1 \\ du = dx \end{Bmatrix}$$

$$= \lim_{b\to\infty} \left[\frac{(x+1)^{-2}}{-2}\right]_2^b$$

$$= \lim_{b\to\infty} \left[\frac{-1}{2(x+1)^2}\right]_2^b$$

$$= \lim_{b\to\infty} \left(\frac{-1}{2(b+1)^2} - \frac{-1}{2(2+1)^2}\right)$$

$$= 0 + \frac{1}{18} = \boxed{\frac{1}{18}}$$

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QUIZ 14 ♢

MATH 201
March 14, 2024

1. $\int_0^\infty \frac{e^x}{e^{2x}+1}\,dx = \lim_{b\to\infty} \int_0^b \frac{e^x}{(e^x)^2+1}\,dx$

$u = e^x$

$du = e^x dx$

$$= \lim_{b\to\infty} \int_{e^0}^{e^b} \frac{1}{u^2+1}\,du$$

$$= \lim_{b\to\infty} \left[\tan^{-1}(u)\right]_1^{e^b}$$

$$= \lim_{b\to\infty} \left[\tan^{-1}(e^b) - \tan^{-1}(1)\right]$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \boxed{\frac{\pi}{4}}$$

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QUIZ 14 ♣

MATH 201
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1. $\int_2^\infty \frac{\cos(\pi/x)}{x^2}\,dx = \lim_{b\to\infty}\int_2^b \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx$

$u = \frac{\pi}{x}$

$du = -\frac{\pi}{x^2}\,dx$

$\frac{-1}{\pi}\,du = \frac{1}{x^2}\,dx$

$$= \lim_{b\to\infty} \frac{-1}{\pi}\int_{\frac{\pi}{2}}^{\frac{\pi}{b}} \cos(u)\,du$$

$$= -\frac{1}{\pi}\lim_{b\to\infty}\Big[\sin(u)\Big]_{\frac{\pi}{2}}^{\frac{\pi}{b}}$$

$$= -\frac{1}{\pi}\lim_{b\to\infty}\left(\sin\left(\frac{\pi}{b}\right) - \sin\left(\frac{\pi}{2}\right)\right)$$

approaching 0!

$$= -\frac{1}{\pi}\left(\sin(0) - \sin(\pi/2)\right)$$

$$= -\frac{1}{\pi}(0-1) = \boxed{\frac{1}{\pi}}$$

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QUIZ 14 ♡

MATH 201
March 14, 2024

1. $\int_1^\infty \frac{dx}{x^2+x} = \lim_{b\to\infty} \int_1^b \frac{1}{x^2+x}\,dx$

$$= \lim_{b\to\infty} \int_1^b \frac{dx}{x(x+1)}$$

$$= \lim_{b\to\infty} \int_1^b \left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

Partial Fractions

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$1 = A(x+1) + Bx$$

$$x=0 \Rightarrow A=1$$

$$x=-1 \Rightarrow B=-1$$

$$= \lim_{b\to\infty} \Big[\ln|x| - \ln|x+1|\Big]_1^b$$

$$= \lim_{b\to\infty} \Big(\big(\ln|b| - \ln|b+1|\big) - \big(\ln|1| - \ln|1+1|\big)\Big)$$

$$= \lim_{b\to\infty} \left(\ln\left|\frac{b}{b+1}\right| - 0 - \ln|2|\right)$$

$$= \ln\left|\lim_{b\to\infty} \frac{b}{b+1}\right| - \ln|2|$$

$$= \ln|1| - \ln|2|$$

$$= 0 - \ln|2| = \boxed{\ln\left|\frac{1}{2}\right|}$$