Name: Richard

Quiz 13 ♠

MATH 201
March 12, 2024

1. $\int \frac{e^x}{e^{2x}+2e^x+17}\,dx = \int \frac{e^x}{(e^x)^2+2e^x+17}\,dx$

$$= \int \frac{du}{u^2+2u+17} \qquad \begin{aligned} u &= e^x \\ du &= e^x\,dx \end{aligned}$$

$$= \int \frac{du}{u^2+2u+1+16}$$

$$= \int \frac{du}{(u+1)^2+4^2}$$

$$= \int \frac{dw}{w^2+4^2} \qquad \begin{aligned} w &= u+1 \\ dw &= du \end{aligned}$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{w}{4}\right) + C$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{u+1}{4}\right) + C$$

$$= \boxed{\frac{1}{4}\tan^{-1}\left(\frac{e^x+1}{4}\right) + C}$$

Name: Richard

Quiz 13 ◊

MATH 201

March 12, 2024

1. $\displaystyle\int \frac{\sin(x)}{\cos^2(x)+\cos(x)}\,dx = -\int \frac{du}{u^2+u}$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$-du = \sin(x)\,dx$

$$= -\int \frac{du}{u(u+1)}$$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$$1 = A(u+1) + Bu$$

Put $u=0 \Longrightarrow A = 1$

Put $u=-1 \Longrightarrow B = -1$

$$= -\int \frac{1}{u} - \frac{1}{u+1}\,du$$

$$= \int \frac{1}{u+1} - \frac{1}{u}\,du$$

$$= \ln|u+1| - \ln|u| + C$$

$$= \ln\left|\frac{u+1}{u}\right| + C$$

$$= \boxed{\ln\left|\frac{\cos(x)+1}{\cos(x)}\right| + C}$$

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Quiz 13 ♣

MATH 201

March 12, 2024

1. $\displaystyle\int_0^{1/2} \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = \int_0^{1/2} \sin^{-1}(x)\frac{1}{\sqrt{1-x^2}}\,dx$

$$u = \sin^{-1}(x)$$

$$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \int_{\sin^{-1}(0)}^{\sin^{-1}(\frac{1}{2})} u\,du$$

$$= \int_0^{\frac{\pi}{6}} u\,du$$

$$= \left[\frac{u^2}{2}\right]_0^{\frac{\pi}{6}}$$

$$= \frac{(\pi/6)^2}{2} - \frac{0^2}{2}$$

$$= \boxed{\frac{\pi^2}{72}}$$

Name: Richard

QUIZ 13 ♡

MATH 201
March 12, 2024

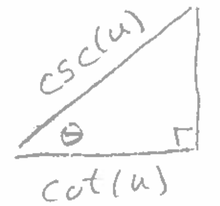
1. $\int e^x \cot^3(e^x)\,dx = \int \cot^3(e^x)\, e^x\,dx$

$u = e^x$

$du = e^x\,dx$

$$= \int \cot^3(u)\,du$$

$$= \int \cot(u)\cot^2(u)\,du$$



$$= \int \cot(u)\left(\csc^2(u) - 1\right)du$$

$$= \int \cot(u)\csc^2(u)\,du - \int \cot(u)\,du$$

$$= \int \cot(u)\csc^2(u)\,du - \ln|\sin(u)| + C$$

$w = \cot(u)$

$dw = -\csc^2(u)\,du$

$-dw = \csc^2(u)\,du$

$$= -\int w\,dw - \ln|\sin(u)| + C$$

$$= -\frac{w^2}{2} - \ln|\sin(u)| + C$$

$$= -\frac{\cot^2(u)}{2} - \ln|\sin(e^x)| + C$$

$$= \boxed{-\frac{\cot^2(e^x)}{2} - \ln|\sin(e^x)| + C}$$