Section 6.7 Physical Applications

Force: when you push a car, you are exerting force on it. The force causes the car to accelerate

Force = (mass)(acceleration) $= kg\ \text{meters}/\text{sec}^2$ (e.g.)

Units of Force

Metric Newton (N) 1 newton of force causes 1 kg to accelerate 1 m/s²

English Pound (lb) 1 pound of force causes 1 slug to accelerate 1 ft/s

$1N \rightarrow \boxed{1kg} \longrightarrow 1\ m/s^2$

$1lb \rightarrow \boxed{1\ slug} \longrightarrow 1\ ft/s^2$

Ex Acceleration due to gravity is $9.8\ m/s^2$ or $32\ ft/s^2$

Force exerted on 10 kg is $(10\ kg)(9.8\ m/s^2) = 98\ N$

Force exerted on 2 slugs is $(2\ slug)(32\ ft/s^2) = 64\ lb$

Work

Work = (force)(distance) = mad

Units of work

Metric Joule (J) = work done by exerting 1 N over 1 meter.

English foot-pound (ft-lb) = work done by exerting 1 lb over 1 foot.

Ex A constant force of 4 newtons moves an object 25 m along a line. Work done is $4 \cdot 25 = 100\ J$.

Ex Gravity moves a 2 kg object 10 meters.

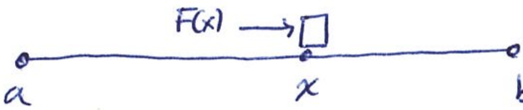
$W = F \cdot d = (2 \cdot 9.8)(10) = 196\ J$.

Ex How much work is done lifting a 2 kg object 10 m?

Answer $= W = F \cdot d = (F)(10)$ and that depends on force exerted. If only enough force is used to overcome gravity, then answer is $(2)(9.8)(10) = 196\ J$

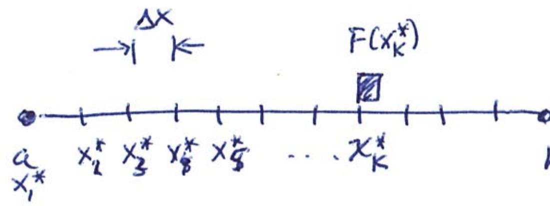
In most realistic situations, the force exerted on a moving object is variable. For example, if you move a car, you start off using more force than you finish with. How can you compute work in such a situation?

<u>Problem</u>



A variable force moves an object from a to b.
At point x, The force is $F(x)$. How much work is done?

<u>Solution</u>:

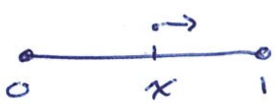


$$W \approx \sum_{K=1}^{n} F(x_i^*)\,\Delta x$$

$$W = \lim_{n\to\infty} \sum_{K=1}^{n} F(x_i^*)\,\Delta x = \int_a^b F(x)\,dx$$

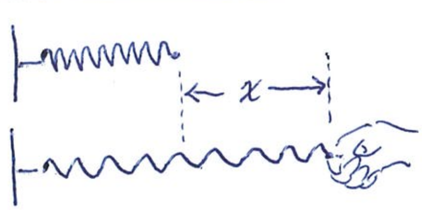
<u>Conclusion</u> Suppose a force causes an object to move from a to b on x-axis, and $F(x)$ = force exerted at point x. Then the total work done is $\int_a^b F(x)\,dx$.

<u>Ex</u> x^2 N of force is exerted at point x



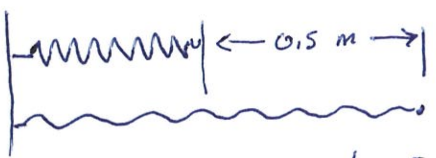
Work done is $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$ J

<u>Hookes' Law</u>



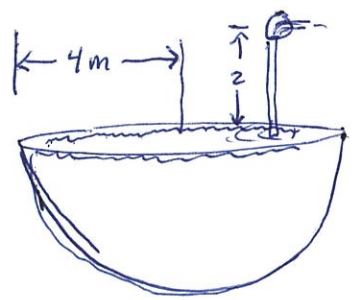
If a spring is pulled x units beyond its natural length, it pulls back with a force of
$F(x) = kx$
where k is a constant (depending on spring)

Ex A spring has constant $k=2$ (units are in Newtons)



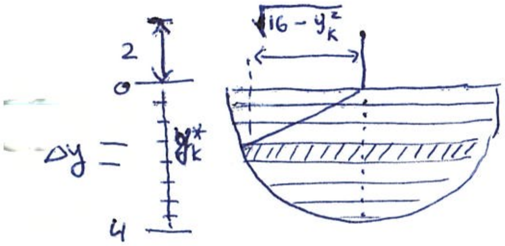
How much work is done pulling it 0.5 m beyond its' natural length? $W = \int_a^b F(x)\,dx = \int_0^{1/2} 2x\,dx = \left[x^2\right]_0^{1/2} = \frac{1}{4}$ J

Ex



Hemispherical tank is filled with water. How much work must be done to pump all the H_2O to a height of 2m?

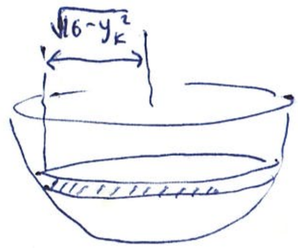
Relevant fact Density of H_2O: 1000 kg per cubic meter.



The idea is to think of removing the H_2O in layers. Lower levels have less H_2O, but you must pump it higher.

Volume of layer #k: $\pi \sqrt{16-y_k^2}^{\,2}\,\Delta y$

Mass of layer #k $1000\pi \sqrt{16-y_k^2}^{\,2}\,\Delta y$



Work done in lifting layer k:

$$W_k = Fd = mad = 1000\pi(16-y_k^2)\Delta y\, 9.8(2+y_k)$$

$$= 9800\pi(16-y_k^2)(2+y_k)\Delta y$$

$$= 9800\pi(32+16y_k-2y_k^2-y_k^3)\Delta y$$

Total work done: $W \approx \sum_{k=1}^{n} 9800\pi(32+16y_k-2y_k^2-y_k^3)\Delta y$

To get exact value, let $n\to\infty$, so $W = \int_0^4 9800\pi(32+16y-2y^2-y^3)\,dy$

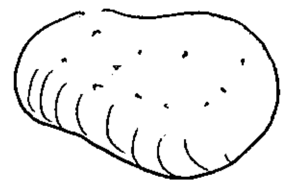
$= \dots = \frac{4390400\pi}{3}$ J $\approx 4{,}597{,}616.13$ J

Density and Mass

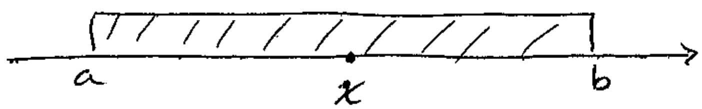
Density of a material: Measured in mass per volume, e.g $\begin{cases} Kg/m^3 \\ g/cm^3 \end{cases}$

If density is uniform: Mass = density · volume.

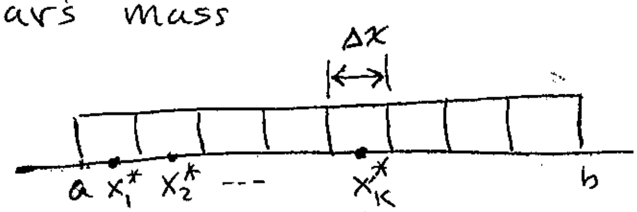
In practice, density may vary from point to point. How can we compute mass?



We will look at a simplified version of this question, namely the density of a wire or bar. (1D instead of 3D)



Suppose a bar runs between a & b on the x-axis. Say the density at x is $\rho(x)$ gram/cm. Find the bar's mass



$$\Delta x = \frac{b-a}{n}$$

$$\text{Mass} \approx \sum_{k=1}^{n} \rho(x_k^*)\Delta x$$

$$\text{Mass} = \lim_{n\to\infty} \sum_{k=1}^{n} \rho(x_k^*)\Delta x = \int_a^b \rho(x)\,dx$$

Ex A bar from 0 to π has density $\rho(x) = 1+\sin(x)$ g/cm at point x. Find the bar's mass.

$$m = \int_0^{\pi} (1+\sin(x))\,dx = \Big[x - \cos(x)\Big]_0^{\pi} = (\pi - \cos(\pi)) - (0 - \cos(0))$$

$$= \pi + 2 \text{ g.}$$

You can skip the material on force and pressure.