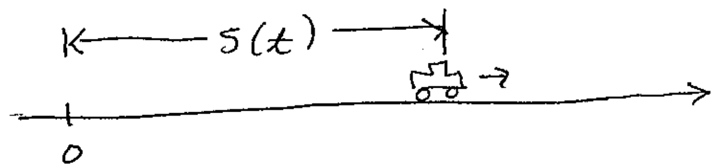
Chapter 6 Applications of Integration

§6.1 Velocity and Net Change

Motion Recall the following facts from Calculus I:

Suppose an object moves on a straight line (a number line).



Then at time t, its

- **position** is $s(t) = \text{-------} = \int v(t)\,dt$
- **velocity** is $v(t) = s'(t) = \text{-----} = \int a(t)\,dt$
- **acceleration** is $a(t) = v'(t) = s''(t)$
- **speed** is $|v(t)|$

But you'll have to find C!

Theorem 6.1

$$s(t) = s(0) + \int_0^t v(x)\,dx$$

$$v(t) = v(0) + \int_0^t a(x)\,dx$$

Theorem 6.2

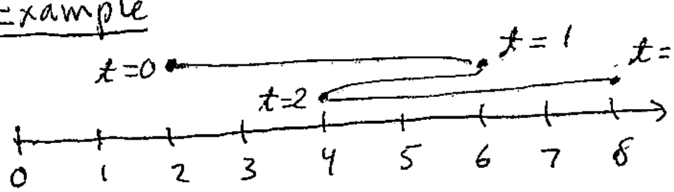
Observations

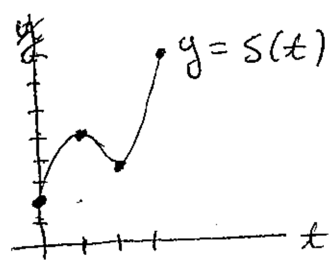
① $\displaystyle\int_a^b v(t)\,dt = \Big[s(t)\Big]_a^b = s(b) - s(a) = \int_a^b v(x)\,dx \Longrightarrow s(t) - s(0) = \int_0^t v(x)\,dx$

② $\displaystyle\int_a^b a(t)\,dt = \Big[v(t)\Big]_a^b = v(b) - v(a) = \int_a^b a(x)\,dx \Longrightarrow v(t) - v(0) = \int_0^t a(x)\,dx$

Displacement & Distance Traveled

Example



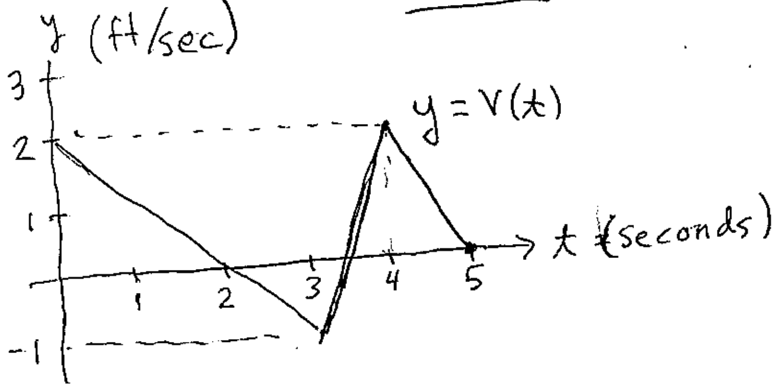


Displacement between $t=1$ and $t=3$ is $2 = s(3) - s(1) = \int_1^3 v(t)\,dt$

Distance traveled between $t=1$ & $t=3$ is $6 = \int_1^3 |v(t)|\,dt$

$$\boxed{\begin{array}{l}\text{Displacement between times } a \,\&\, b \text{ is } s(b) - s(a) = \int_a^b v(t)\,dt \\ \text{Distance traveled between } a \,\&\, b \text{ is } \int_a^b |v(t)|\,dt\end{array}}$$

Example 9 A velocity function is given below.
Known: $s(0) = 0$.

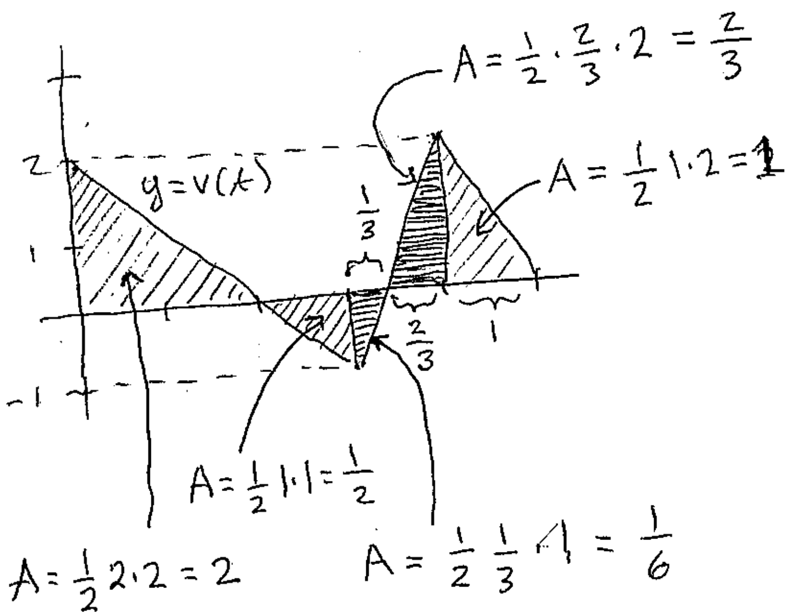


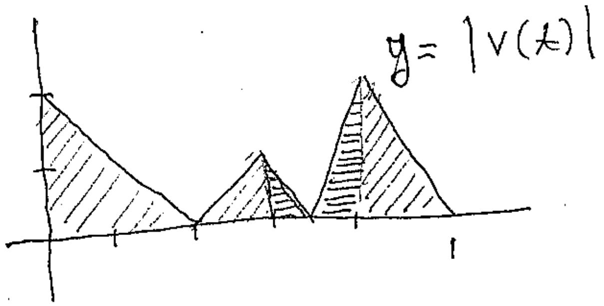
ⓐ Find displacement between $t=0$ and $t=5$

ⓑ Find distance traveled between $t=0$ & $t=5$.

ⓒ Position at $t=5$

Solutions





ⓐ Displacement =

$$s(5) - s(0) = \int_0^5 v(t)\,dt$$

$$= A_{up} - A_{down}$$

$$= 2 + \frac{2}{3} + 1 - \left(\frac{1}{2} + \frac{1}{6}\right)$$

$$= 3 + \frac{4}{6} - \left(\frac{3}{6} - \frac{1}{6}\right)$$

$$= \boxed{3 \text{ feet}}$$

ⓑ Distance traveled =

$$\int_0^5 |v(t)|\,dt = \text{Area}$$

$$= 2 + \frac{2}{3} + 1 + \frac{1}{2} + \frac{1}{6}$$

$$= \frac{12}{6} + \frac{4}{6} + \frac{6}{6} + \frac{3}{6} + \frac{1}{6}$$

$$= \frac{26}{6} = \boxed{\frac{13}{3} \text{ feet}}$$

ⓒ Position at $t=5$ is

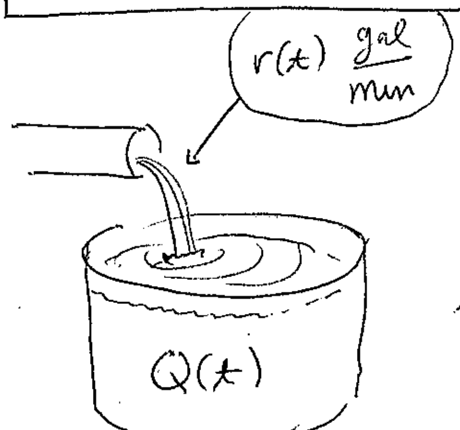
$$s(0) + \int_0^5 v(t)\,dt = 0 + 3 = \boxed{3}$$

Theorem 6.3

Suppose $Q(t)$ is some quantity that depends on time t. The net change in Q between times $t=a$ and $t=b$ is

$$Q(b) - Q(a) = \int_a^b Q'(t)\,dt.$$

and $Q(t) = Q(0) + \int_0^t Q'(x)\,dx$



Example

At time t, water is pouring into a tank at a rate of $r(t)$ gallons/min.

Amount of water added to tank between times $t=30$ and $t=60$ is

$$\int_{30}^{60} Q'(t)\,dt = \int_{30}^{60} r(t)\,dt$$