§11.2 Power Series

Definitions

- Power series centered at a:

$$\sum_{k=0}^{\infty} c_k (x-a)^k = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

- Power series centered at 0

$$\sum_{k=0}^{\infty} c_k x^k = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

Examples

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k}(x-1)^k = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \cdots$$

Taylor series for $\ln(x)$ centered at 1

$$\sum_{k=0}^{\infty} \frac{1}{k!} x^k = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$$

Maclaurin series for e^x

$$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots$$

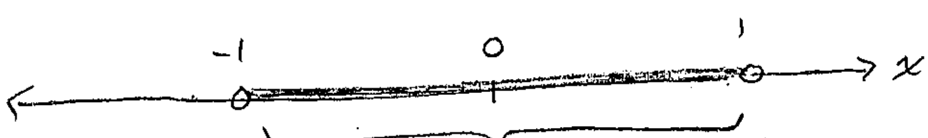
Geometric series, $a=1$ $r=x$

Maclaurin series for $\frac{1}{1-x}$

When you plug in a value for x in a power series you get a particular infinite series. The series may converge for some x and diverge for others.

Ex $\displaystyle\sum_{k=0}^{\infty} x^k$

$$\begin{cases} x=2: & 1+2+2^2+2^3+2^4+\cdots \quad \underline{\text{diverges}} \\ x=\frac{1}{2}: & 1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\cdots \quad \underline{\text{converges}} \end{cases}$$



Interval of convergence for Σx^k. Series converges for values of x in this interval.

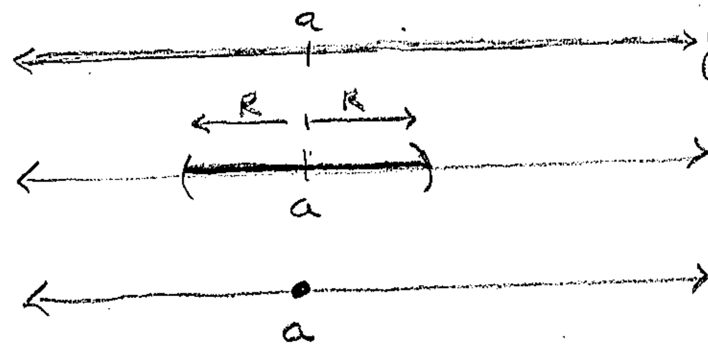
Note $\sum c_k(x-a)^k$ always converges for $x=a$.
Possibly converges for other values of x too!

Fact For any power series $\sum c_k(x-a)^k$, the interval of convergence will be one of the following

Ⓐ $(-\infty, \infty)$

Ⓑ $(a-R, a+R)$

Ⓒ Only $x=a$



Radius of convergence is ∞

Radius of convergence is R

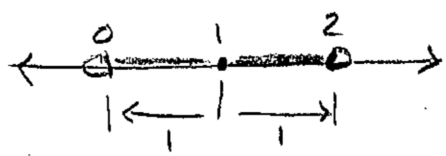
Radius of convergence is 0.

Example Taylor series for $\ln(x)$ centered at 1 is $\sum \frac{(-1)^{k-1}}{k}(x-1)^k$.
Find its interval of convergence.

Ratio Test: $\lim\limits_{k\to\infty}\left|\dfrac{\frac{(-1)^k}{k+1}(x-1)^{k+1}}{\frac{(-1)^{k-1}}{k}(x-1)^k}\right| = \lim\limits_{k\to\infty}\left|\frac{k}{k+1}(x-1)\right| = |x-1|.$

Series converges for $|x-1|<1 \Rightarrow -1 < x-1 < 1$
$\Rightarrow 0 < x < 2$

Radius of convergence: $R=1$.



What about $x=0$? $\sum \frac{(-1)^{k-1}}{k}(0-1)^k = \sum \frac{1}{k} = 1+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}$
Harmonic series – diverges.

What about $x=2$? $\sum \frac{(-1)^{k-1}}{k}(2-1)^k = \sum \frac{(-1)^{k-1}}{k} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+$
Alternating harmonic series – converges!

True interval of convergence $(0, 2]$

Unanswered question Does series converge to $\ln(x)$ on this interval? Stay tuned.