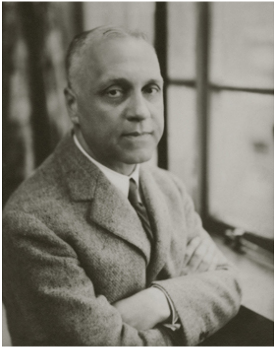
MATH 123
(Day 8)

Mathematical Architecture

Richard Hammack

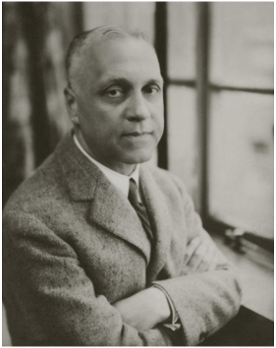
http://www.people.vcu.edu/∼rhammack/Math123/

Claude Bragdon



Claude F. Bragdon, 1866–1946

Claude Bragdon



Claude F. Bragdon, 1866–1946

Salisbury House in Franklin Square, Oswego, NY

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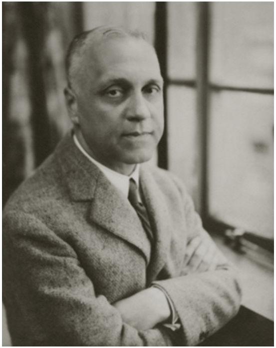
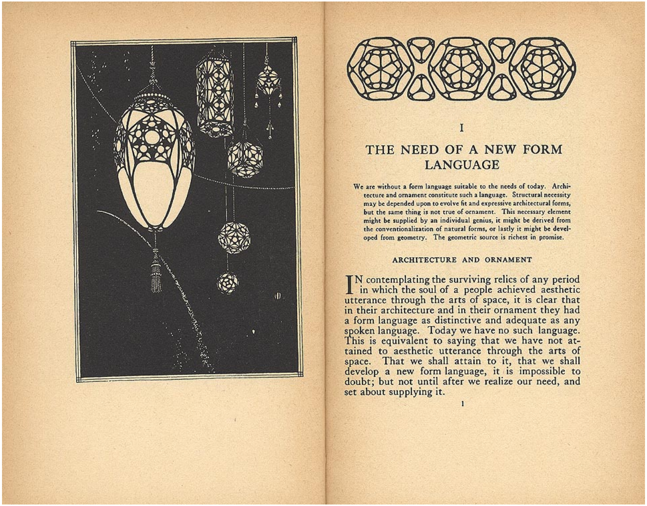


Claude F. Bragdon, 1866–1946

Salisbury House in Franklin Square, Oswego, NY

Books by Claude Bragdon

- A Primer of Higher Space, 1913
- Projective Ornament, 1915



I

THE NEED OF A NEW FORM LANGUAGE

We are without a form language suitable to the needs of today. Architecture and ornament constitute such a language. Structural necessity may be depended upon to evolve fit and expressive architectural forms, but the same thing is not true of ornament. This necessary element might be supplied by an individual genius, it might be derived from the conventionalization of natural forms, or lastly it might be developed from geometry. The geometric source is richest in promise.

ARCHITECTURE AND ORNAMENT

IN contemplating the surviving relics of any period in which the soul of a people achieved aesthetic utterance through the arts of space, it is clear that in their architecture and in their ornament they had a form language as distinctive and adequate as any spoken language. Today we have no such language. This is equivalent to saying that we have not attained to aesthetic utterance through the arts of space. That we shall attain to it, that we shall develop a new form language, it is impossible to doubt; but not until after we realize our need, and set about supplying it.

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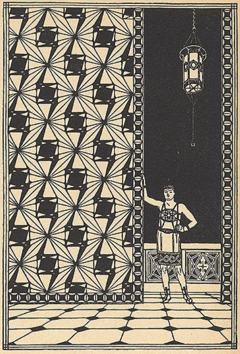
Claude Bragdon, page from *Projective Ornament*, 1915



II

ORNAMENT AND PSYCHOLOGY

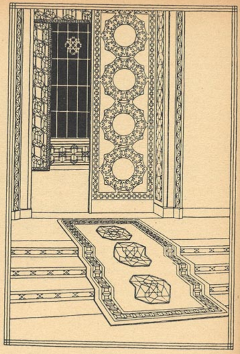
Ornament is the outgrowth of no practical necessity, but of a striving toward beauty. Our zeal for efficiency has resulted in a corresponding aesthetic infertility. Signs are not lacking that consciousness is now looking in a new direction—away from the contemplation of the facts of materiality towards the mysteries of the supersensuous life. This transfer of attention should give birth to a new aesthetic, expressive of the changing psychological mood. The new direction of consciousness is well suggested in the phrase, *The Fourth Dimension of Space*, and the decorative motifs of the new aesthetic may appropriately be sought in four-dimensional geometry.

THE ORNAMENTAL MODE AND THE PSYCHOLOGICAL MOOD

ARCHITECTURAL forms and features, such as the column, the lintel, the arch, the vault, are the outgrowth of structural necessity, but this is not true of ornament. Ornament develops not from the need and the power to build, but from the need and the power to beautify. Arising from a psychological impulse rather than from a physical necessity, it reflects the national and racial consciousness. To such a degree is this true that any mutilated and time-worn fragment out of the great past when art was a language can without difficulty be assigned its place and period. Granted a dependence of the ornamental mode upon the psychological mood, our first business is to discover what that mood may be.

A great change has come over the collective consciousness: we are turning from the accumula-

Claude Bragdon, page from *Projective Ornament*, 1915



IV

THREE REGULAR POLYHEDROIDS

The paradoxes of four-dimensional geometry are best understood by referring them to the corresponding truisms of plane and of solid geometry. This may profitably be done in the case of the pentahedroid, the tesseract, and the 16-hedroid, the four-fold figures of most use in Projective Ornament. In the plane representation of four-fold figures for decorative purposes certain conventions should be observed, conventions which, though they serve aesthetic ends, find justification in optical and physical laws.

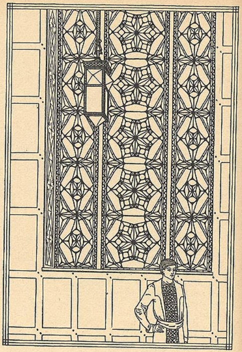
TWO-, THREE-, AND FOUR-FOLD FIGURES

THE most effective method for a novice to approach an understanding of any four-dimensional figure can be compared to the athletic exercise called the hop, skip and jump. In this the cumulative impetus given by the hop and the skip is concentrated and expended in the supreme effort of the jump. The jump into the fourth dimension is best prepared for, in any given case, by a preliminary hop in plane space, and a skip in solid space.

In the following cursory consideration of the three simplest regular polyhedroids of four-dimensional space let us apply this method. Even at the risk of wearisome reiteration let us resolve the paradoxes of hyperspace by referring them to the truisms of lower spaces.

A regular poly*gon*—a two-fold figure—consists of equal straight lines so joined as to enclose symmetrically a portion of *plane space*. A regular poly*hedron* a three-fold figure—consists of a number of equal

Claude Bragdon, page from *Projective Ornament*, 1915



V.

FOLDING DOWN

Regular polyhedroids of four-dimensional space may be unfolded in three-dimensional space, and these again unfolded in a space of two-dimensions; or, contrariwise, they may be built up by assembling the regular polyhedrons which compose them. In this way new and valuable decorative material is obtained.

ANOTHER METHOD OF REPRESENTING THE HIGHER IN THE LOWER

THE perspective method is not the only one whereby four-fold figures may be represented in three-dimensional and in two-dimensional space. Polyhedroids may be conceived of as cut apart along certain *planes*, and folded down into three-dimensional space in a manner analogous to that by which a cardboard box may be cut along certain of its *edges* and folded down into a plane. As the boundaries of a polyhedroid are polyhedrons, an unfolded polyhedroid will consist of a number of related polyhedrons. These can in turn be unfolded, and the aggregation of polygons—each a plane boundary of the solid boundary of a hypersolid—will represent a four-fold figure unfolded in a space of two dimensions.

An unfolded cube becomes a cruciform plane figure, made up of six squares, each one a boundary of the cube (A, Figure 15). Similarly, if we imagine a tesseract to be unfolded, its eight cubical cells will occupy three-dimensional space in the shape of a double-armed cross (B, Figure 15). In four-dimen-

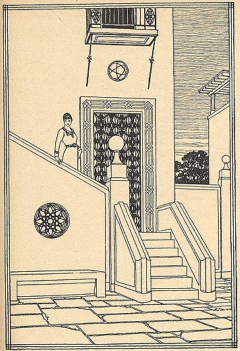
Claude Bragdon, page from *Projective Ornament*, 1915

tetrahedron and the cube—the pentahedroid and the tesseract—spread out in three-dimensional space. That is, they represent, in three-dimensional perspective, the symmetrical arrangement of the four boundaries of regular four-dimensional angles. In four-dimensional space the faces of those figures which lie adjacent to the common vertex are brought into coincidence, just as in three-dimensional space the *edges* of the triangles and squares adjacent to the common vertex are brought into coincidence, forming the summits of the tetrahedron and the cube.

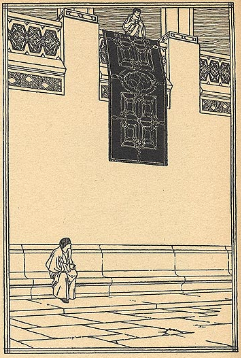
THE CONSTRUCTION OF THE 24-HEDROID

It is possible to build up any regular polyhedroid by putting together a set of polyhedrons. We take them in succession in such order that each is joined to those already taken by a set of polygons like the incomplete polyhedron.

Take the case of the four-fold icositetrahedroid or 24-hedroid. I, Figure 17, shows a summit with six octahedral boundaries arranged about it symmetrically in three-dimensional space. Conceive I to be transported into four-dimensional space, and the interstices between the adjacent triangular faces to be closed up by joining those faces two and two; the figure assumes a form whose projection is represented in J with dotted lines omitted. Adjust to this figure twelve other octahedrons in a symmetrical manner; three of these octahedrons are represented by the dotted lines of J. Again, close up the interstices between the adjacent faces; the outline of the figure assumes a form whose projection is represented in K.



Claude Bragdon, page from *Projective Ornament*, 1915



VI

MAGIC LINES IN MAGIC SQUARES

The numerical harmony inherent in magic squares finds graphic expression in the magic lines which may be traced in them. These lines, translated into ornament, yield patterns often of amazing richness and variety, beyond the power of the unaided aesthetic sense to compass. Magic lines have relations to spaces higher than a plane—they, too, are Projective Ornament.

THE HISTORY OF MAGIC SQUARES

ALMOST everyone knows what a magic square is. Briefly, it is a numerical acrostic, an arrangement of numbers in the form of a square, which, when added in vertical and horizontal rows and along the diagonals, yield the same sum. Magic squares are of Eastern and ancient origin. There is a magic square of 4 carved in Sanskrit characters on the gate of the fort at Gwalior, in India (Figure 20). Engraved on stone and metal, magic squares are worn at the present day in the East as talismans or amulets. They are known to have occupied the attention of Mediaeval philosophers, astrologers, and mystics. Albrecht Dürer introduced what is perhaps the most remarkable of all magic squares into his etching *Melancholia* (Figure 21). Today they find place in the puzzle departments of magazines. Their laws and formulas have engaged the serious attention of eminent mathematicians, and the discovery of so-called magical relations between numbers, not alone

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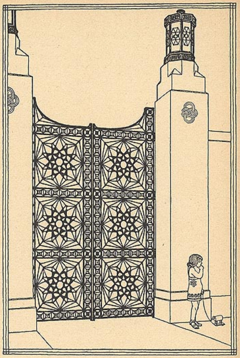


VII

A PHILOSOPHY OF ORNAMENT

The language of form is a symbolical expression of the world order. This order presents itself to individual consciousness most movingly and dramatically under the guise of fate and of free-will. For these two the straight line and the curve are graphic expressions. An ornamental mode should therefore embrace an intelligent and harmonious use of both. That Projective Ornament appears here so largely as a straight line system is because such a system is easier and more elementary than the other, and because this is an elementary treatise—merely a point of departure for an all-embracing art of the future, only to be developed by the coöperation of many minds.

THE WORLD ORDER AND THE WORD ORDER

PROJECTED solids and hypersolids, unfolded figures, magic lines in magic squares, these and similar translations of the truths of number into graphic form, are the words and syllables of the new ornamental mode. But we shall fail to develop a form language, eloquent and compelling, if we preoccupy ourselves solely with sources—the mere lexicography of ornament. There is a grammar and a rhetoric to be mastered as well. The words are not enough, there remains the problem of the word order.

Now the problem of the word order is the analogue of the problem of the world order. The sublime function of true art is to shadow forth the world order through any frail and fragmentary thing a man may make with his hands, so that the great thing can be sensed in the little, the permanent in

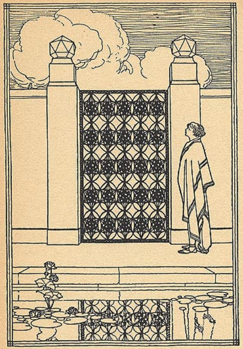
Claude Bragdon, page from *Projective Ornament*, 1915

SPACE AND TIME: THE FIELD AND THE FRAME

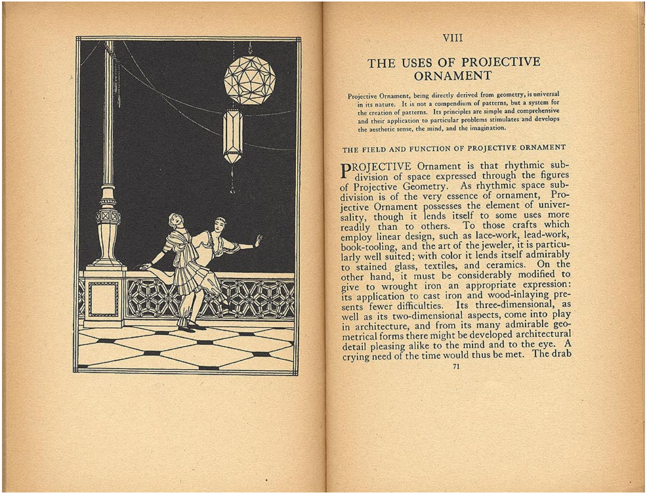
Now the characteristic of time is succession; in time alone one thing follows another in endless sequence. The unique characteristic of space is simultaneity, for in space alone everything exists at once. In classifying the arts, for example, music would go into the time box, for it is in time alone, being successive; architecture, on the other hand, would go into the space box. Yet because nothing is pure, so to speak, architecture has something of the element of succession, and music of simultaneousness. An arcade or a colonnade may be spoken of as successive; while a musical chord, consisting of several notes sounded together, is simultaneous.

The same thing holds true throughout nature. The time element and the space element everywhere appear, either explicitly or implicitly, the first as succession, the second as simultaneity.

In ornament we have the field and the frame, and the unfolding of living forms in space within some fixed time cycle may be thought of as symbolized by a foliated field and a geometrical frame or border. In the field, the units will be disposed with relation to points and radiating lines, implying the simultaneity of space, and in the border they will be arranged sequentially, implying the succession of time (Figure 31). Seeking greater interest, subtlety, and variety, we have, in the projected plane representations of symmetrical three-fold and four-fold solids, a frame rhythmically subdivided. These subdivisions of a frame may be taken to represent lesser time cycles within a greater, and the arabesque with which these spaces can be filled may be felt to



Claude Bragdon, page from *Projective Ornament*, 1915



VIII

THE USES OF PROJECTIVE ORNAMENT

Projective Ornament, being directly derived from geometry, is universal in its nature. It is not a compendium of patterns, but a system for the creation of patterns. Its principles are simple and comprehensive and their application to particular problems stimulates and develops the aesthetic sense, the mind, and the imagination.

THE FIELD AND FUNCTION OF PROJECTIVE ORNAMENT

PROJECTIVE Ornament is that rhythmic subdivision of space expressed through the figures of Projective Geometry. As rhythmic space subdivision is of the very essence of ornament, Projective Ornament possesses the element of universality, though it lends itself to some uses more readily than to others. To those crafts which employ linear design, such as lace-work, lead-work, book-tooling, and the art of the jeweler, it is particularly well suited; with color it lends itself admirably to stained glass, textiles, and ceramics. On the other hand, it must be considerably modified to give to wrought iron an appropriate expression: its application to cast iron and wood-inlaying presents fewer difficulties. Its three-dimensional, as well as its two-dimensional aspects, come into play in architecture, and from its many admirable geometrical forms there might be developed architectural detail pleasing alike to the mind and to the eye. A crying need of the time would thus be met. The drab

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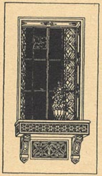
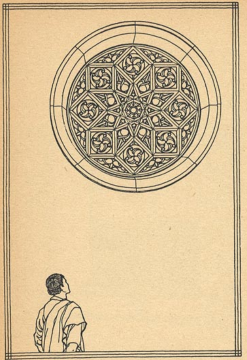
Claude Bragdon, page from *Projective Ornament*, 1915

field of hyperspace. But let him not seek to achieve results too easily and too quickly. In all his work he should follow an orderly sequence, quarrying his gold before refining it, and fashioning it to his uses only after it is refined: that is, he should endeavor to understand the figures before he draws them, and he should draw them as geometrical diagrams before he attempts to alter and combine them for decorative use. It is the author's experience that they will require very little alteration; that they are in themselves decorative. The filling in of certain spaces for the purpose of achieving *notan* (contrast) is all that is usually required. This done, the application of color is the next step in the process: first comes line, then light and dark, and lastly color values. Such is the method of the Japanese, those masters of decorative design.



THE ILLUSTRATIONS AND DIAGRAMS

The black-and-white designs interspersed throughout the text represent Projective Ornament removed only one degree from geometrical diagrams, yet they are seen to be highly decorative even in this form. At the pleasure of the designer they may be elongated, contracted, sheared, twisted, translated from straight lines



Claude Bragdon, page from *Projective Ornament*, 1915

Claude Bragdon

New York Central Railroad Station, Rochester

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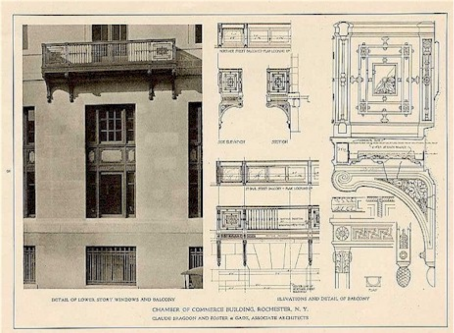
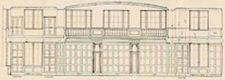
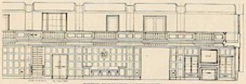


Claude Bragdon, Rochester Chamber of Commerce, 1917



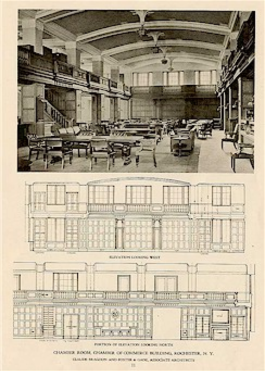
ELEVATION LOOKING WEST

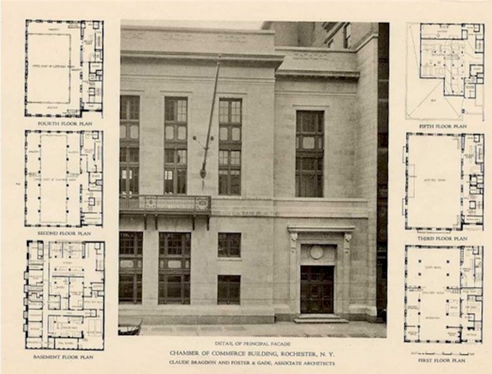


PORTION OF ELEVATION LOOKING NORTH

CHAMBER ROOM, CHAMBER OF COMMERCE BUILDING, ROCHESTER, N. Y.

CLAUDE BRAGDON AND FOOTE & GADE, ASSOCIATE ARCHITECTS





Claude Bragdon, Rochester Chamber of Commerce, 1917

Eero Saarinen
1910–1961

St. Louis Gateway Arch, 1964


Eero Saarinen
1910–1961

St. Louis Gateway Arch, 1964

Eero Saarinen
1910–1961

Dulles International Airport, 1958

Gherkin (London)

Design by Foster and Partners

London City Hall

Gherkin (London)

Design by Foster and Partners

London City Hall

Gherkin (London)

British Museum

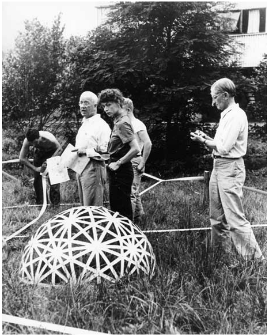
Design by Foster and Partners

Buckminster Fuller

1895–1983

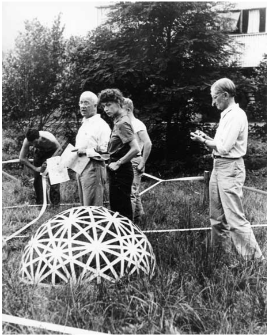
Buckminster Fuller

1895–1983



Buckminster Fuller

1895–1983



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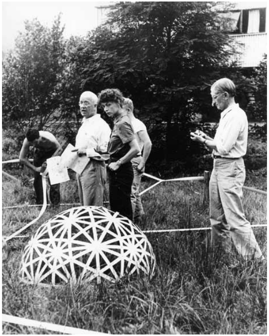

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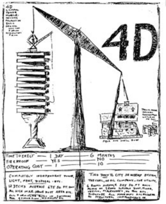

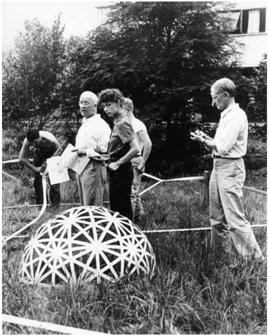

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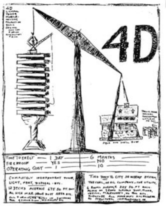

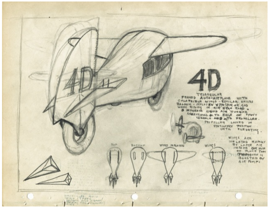

Ruben Margolin

Thanks for taking MATH 123!

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Next Time: Crit Day