MATH 123
(Day 3)

Fourth-Dimensional Platonic Solids

Richard Hammack

http://www.people.vcu.edu/∼rhammack/Math123/

Building a hypercube

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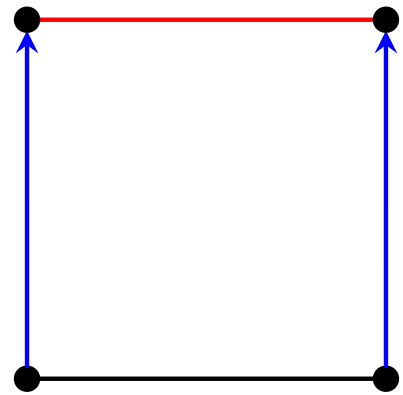

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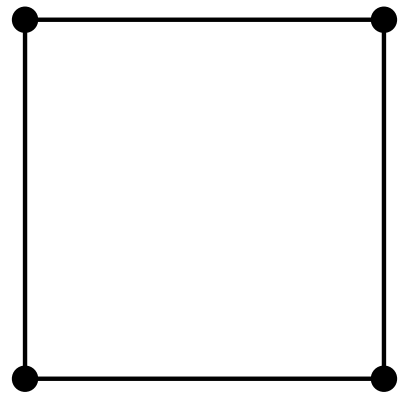

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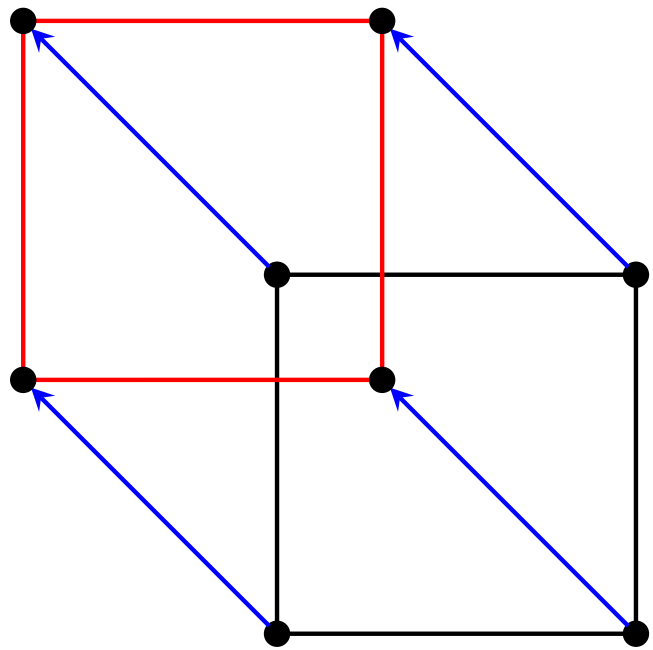

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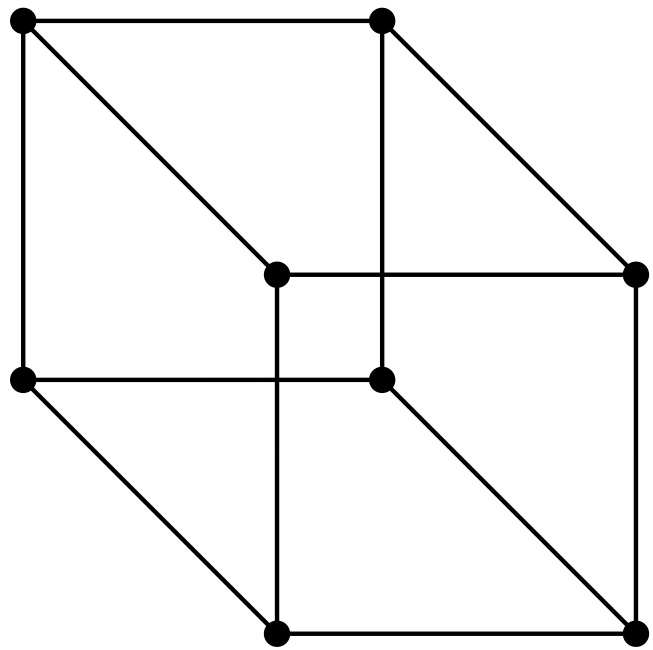

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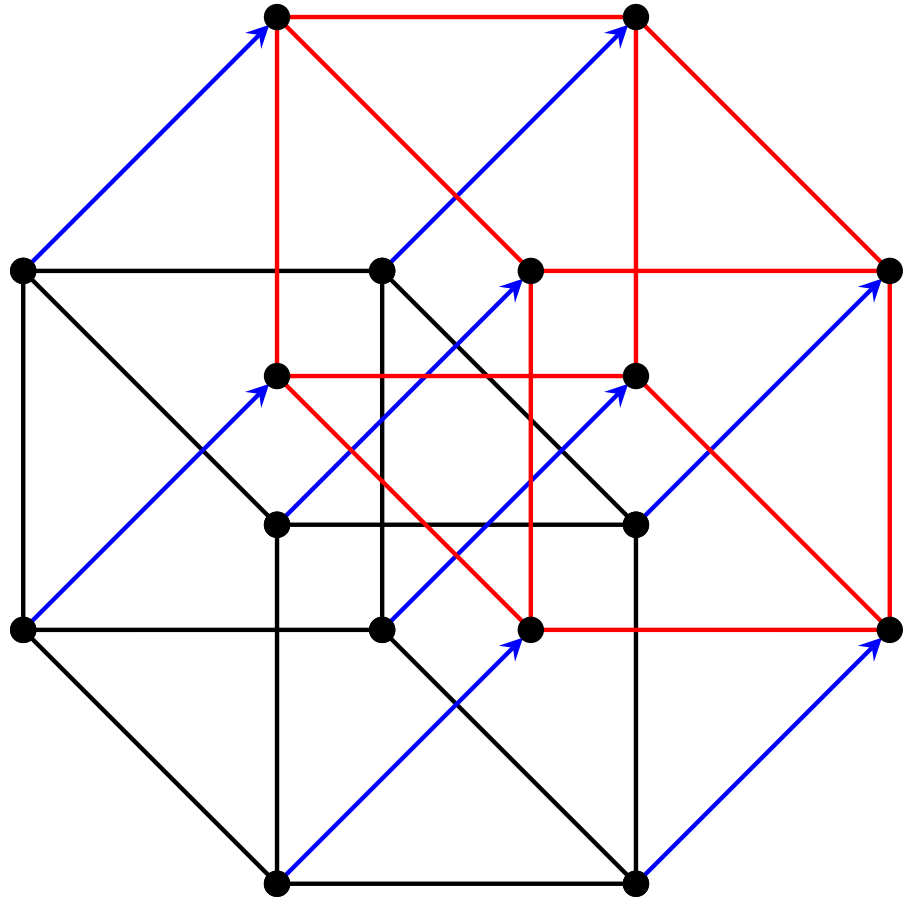

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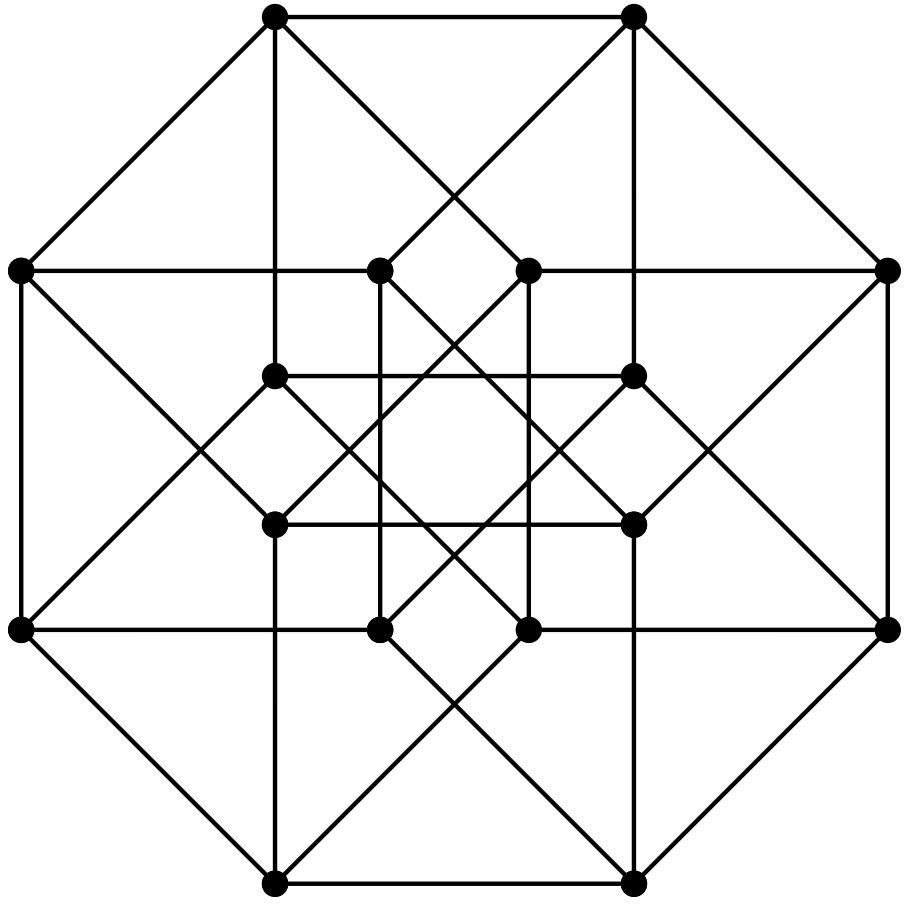

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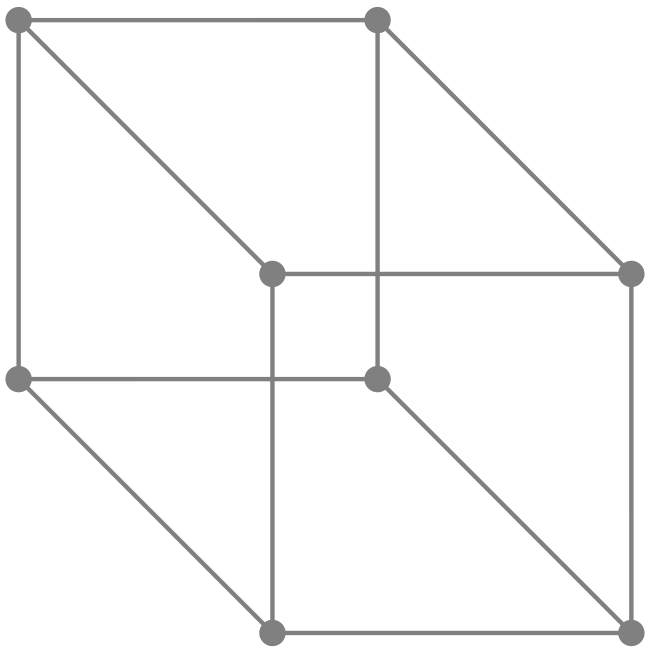

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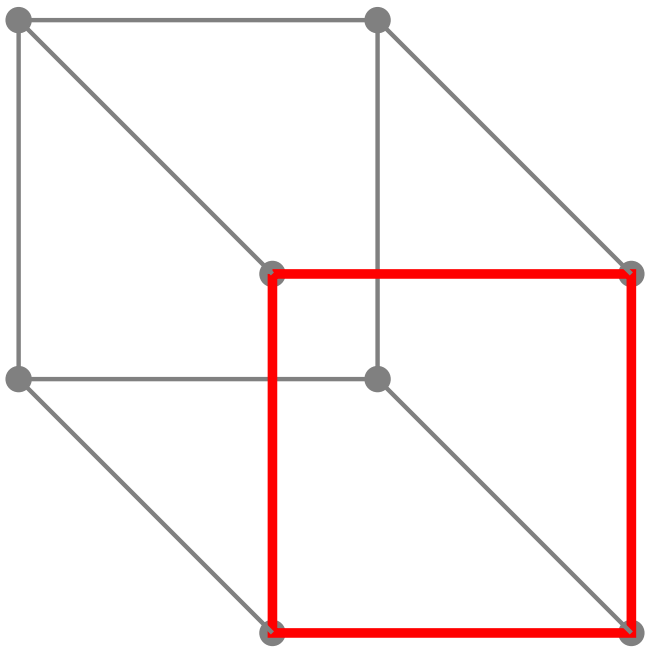

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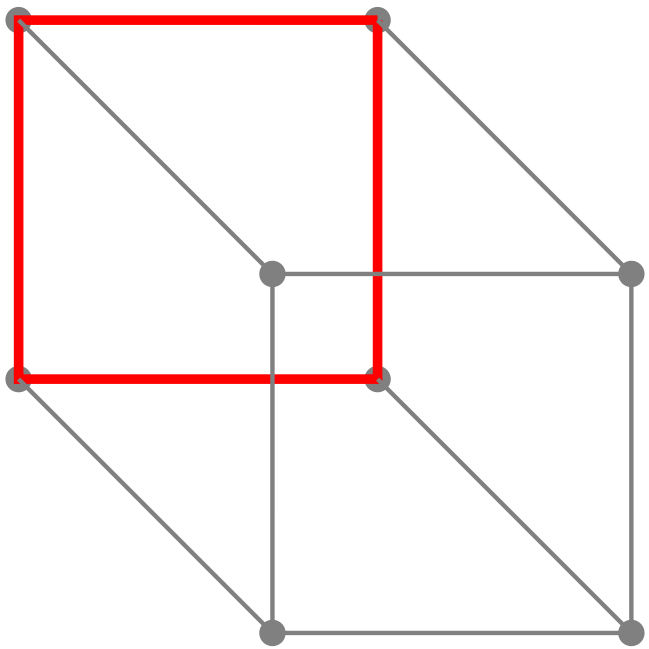

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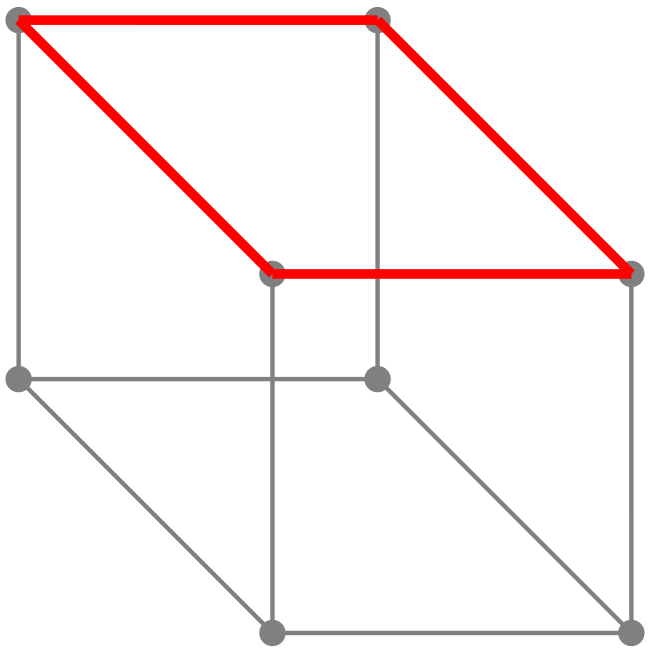

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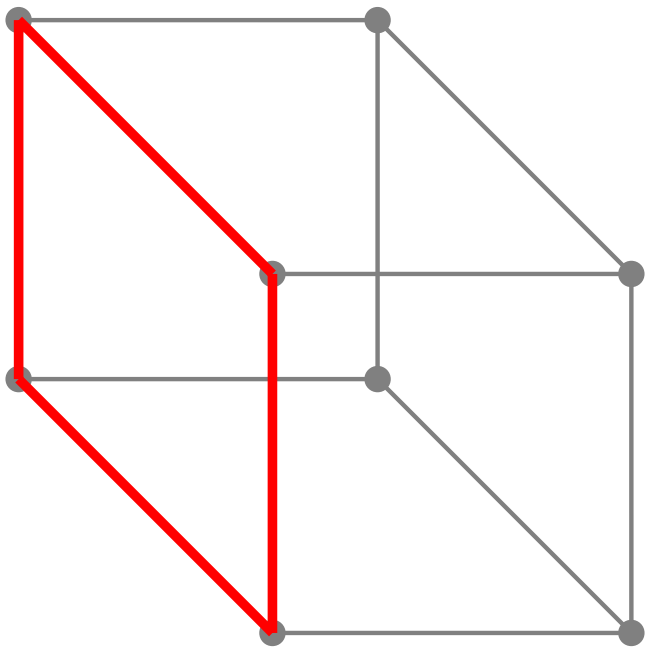

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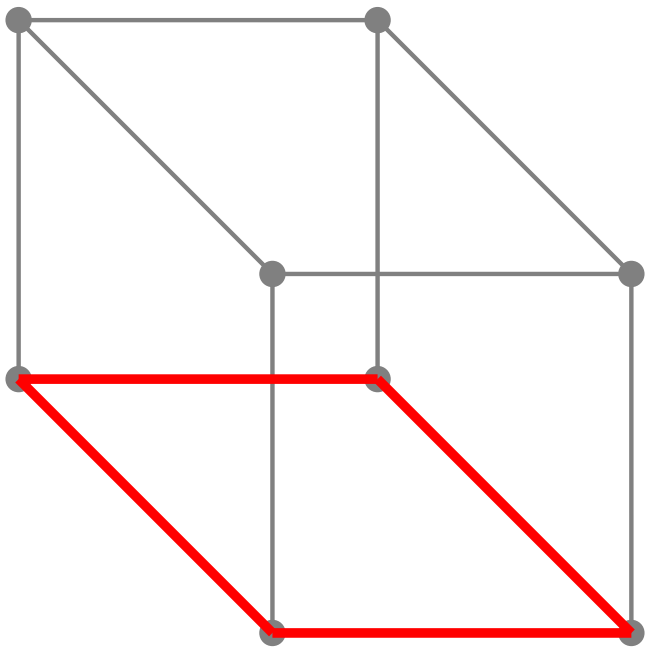

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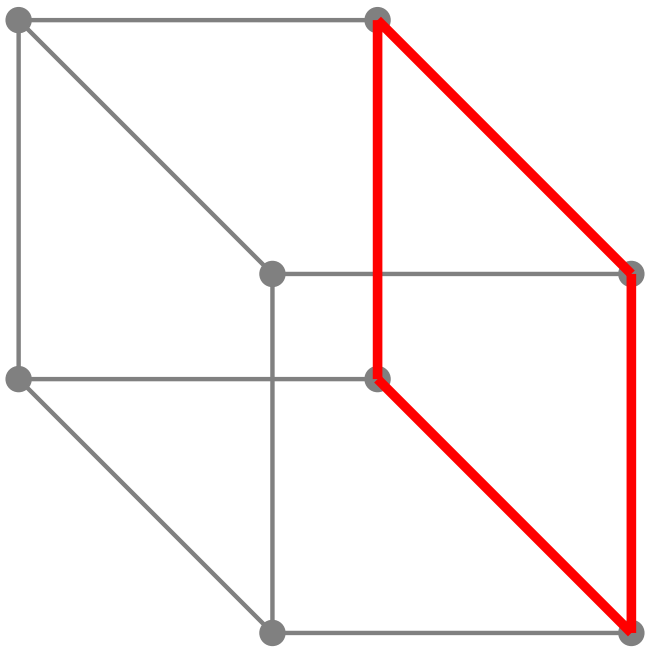

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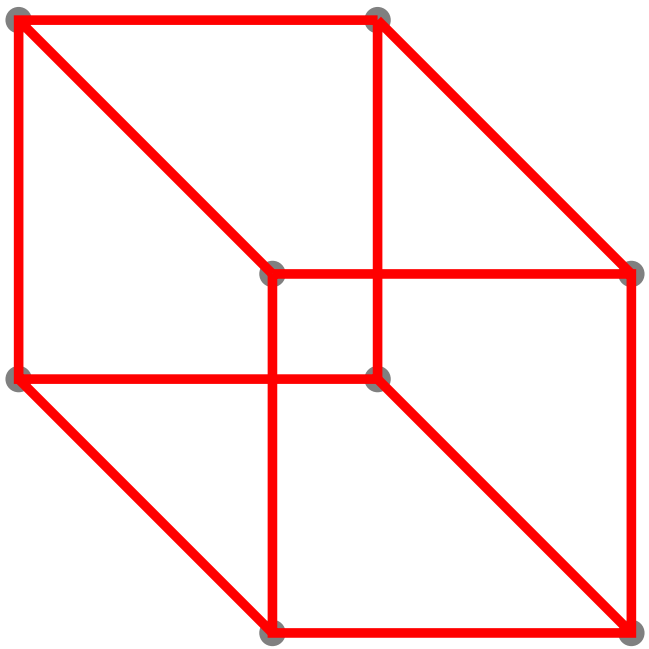

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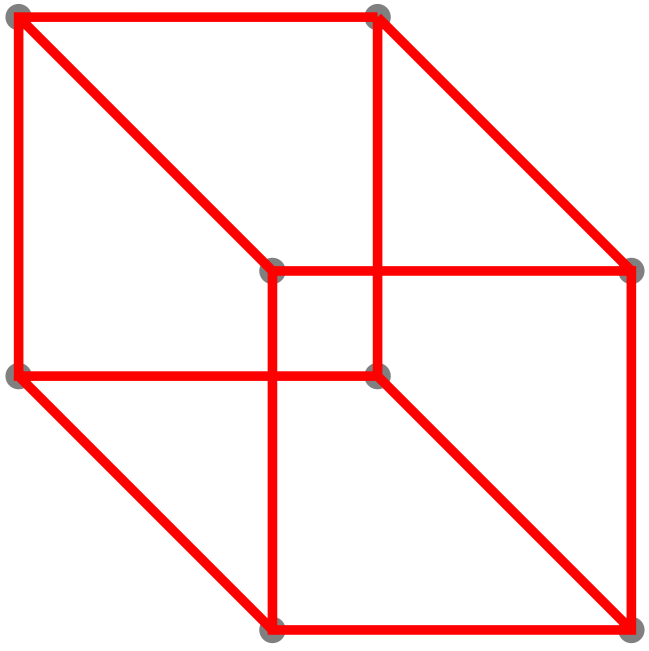

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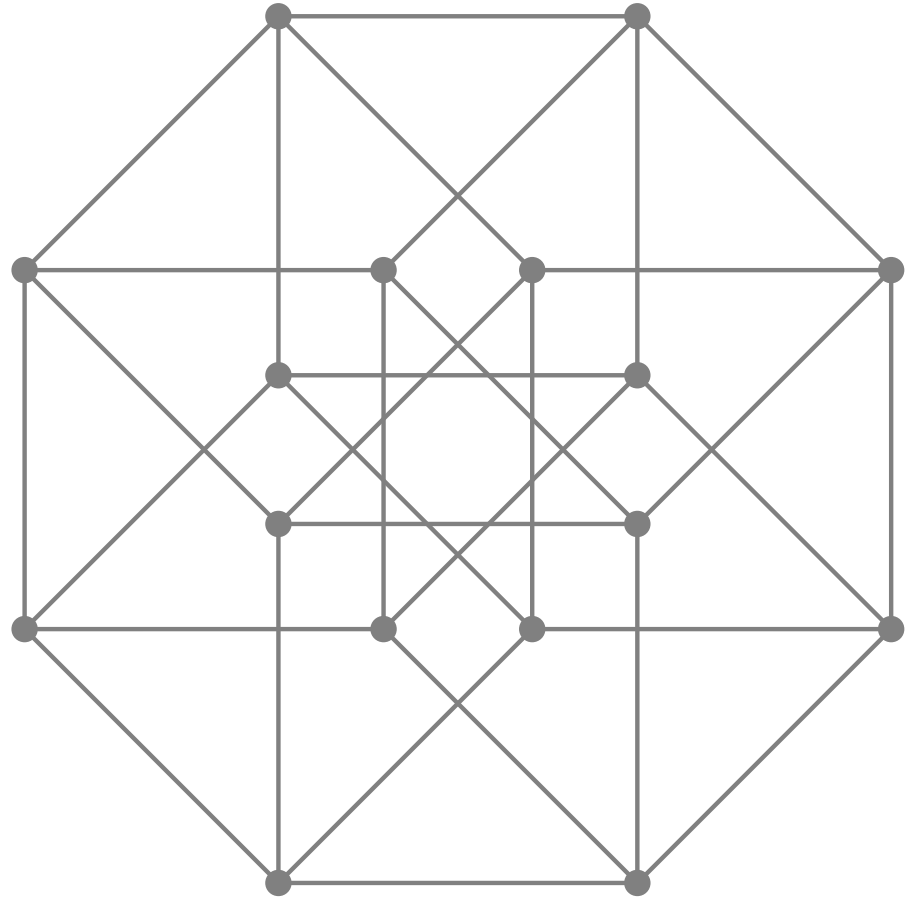

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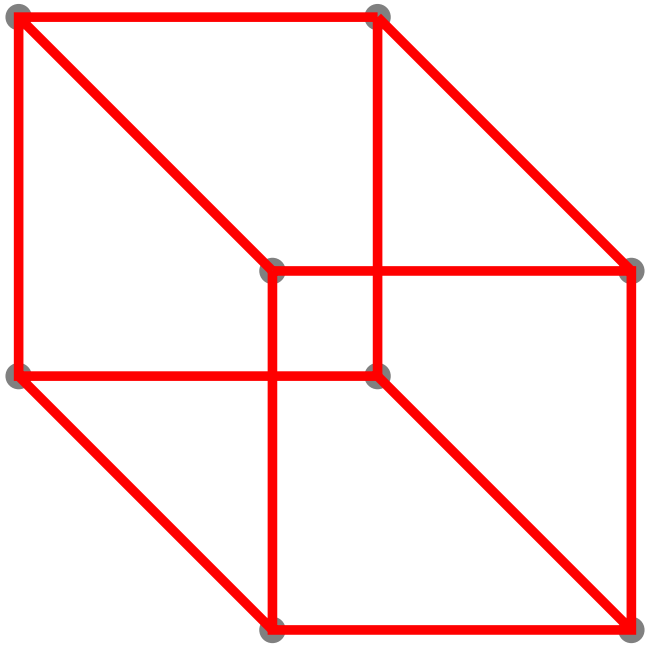

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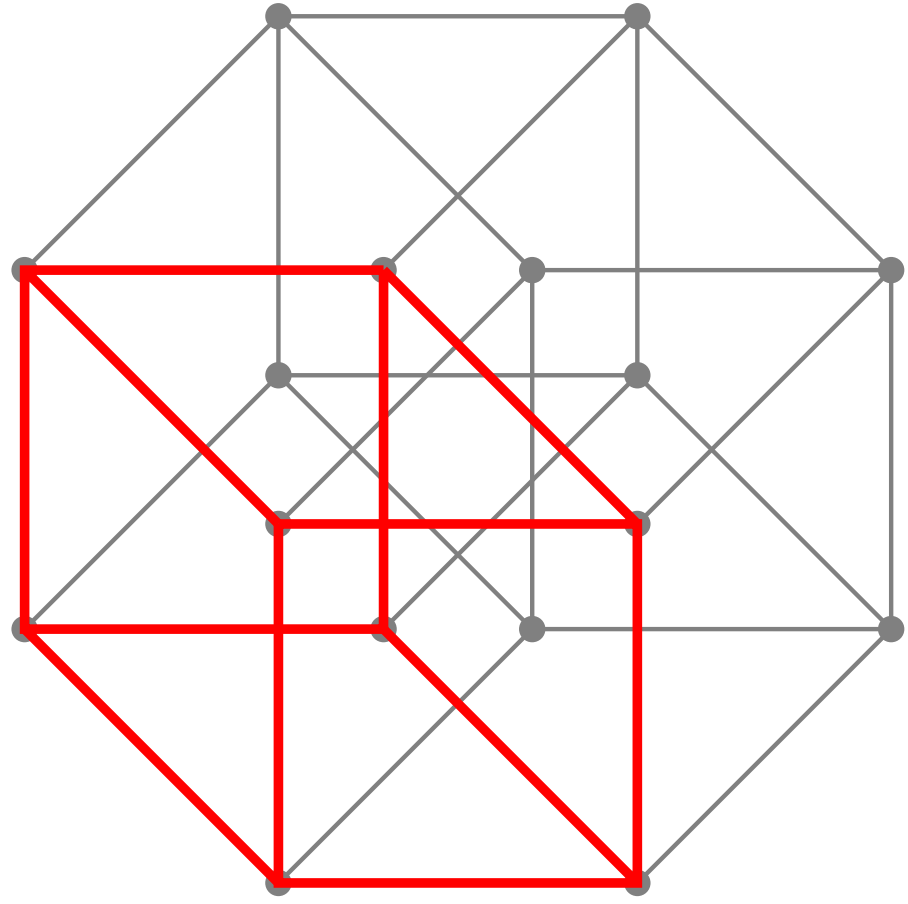

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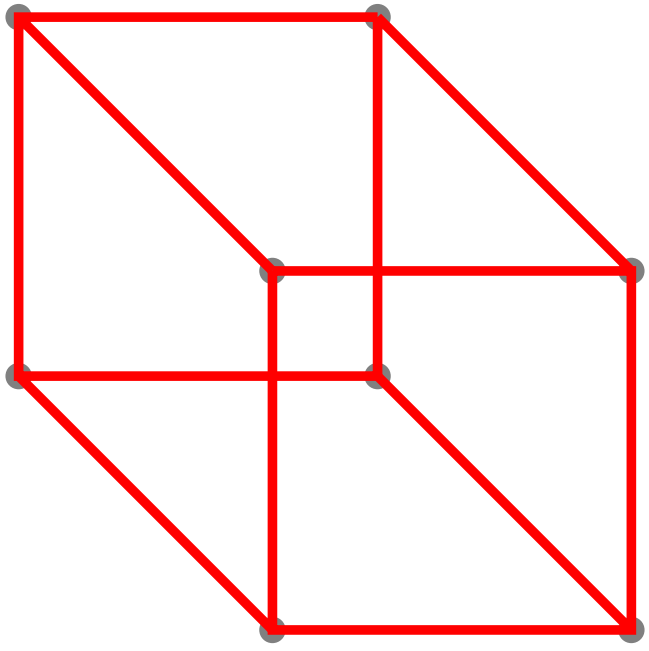

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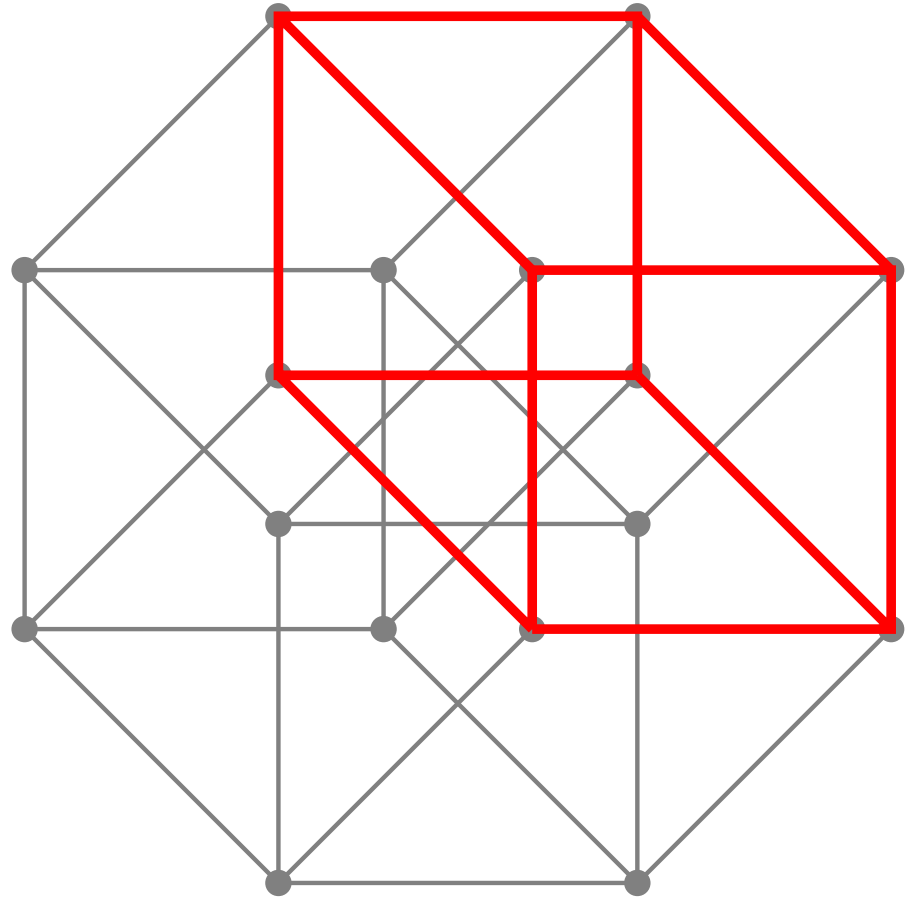

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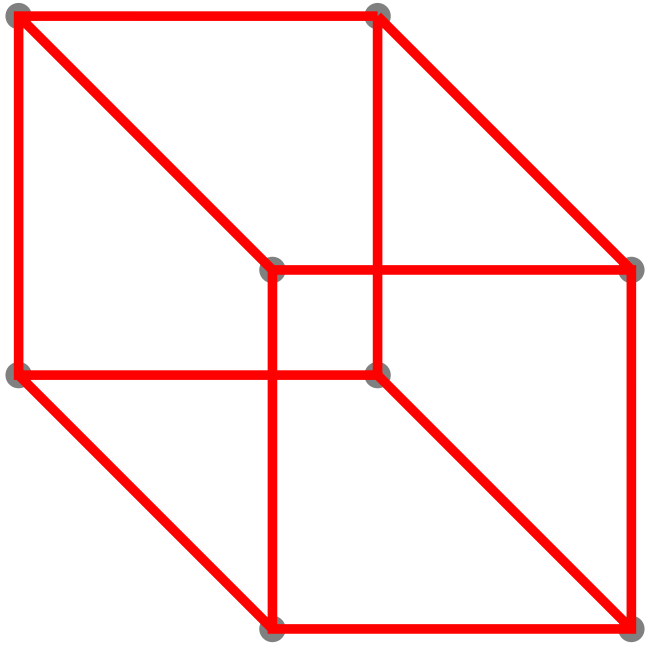

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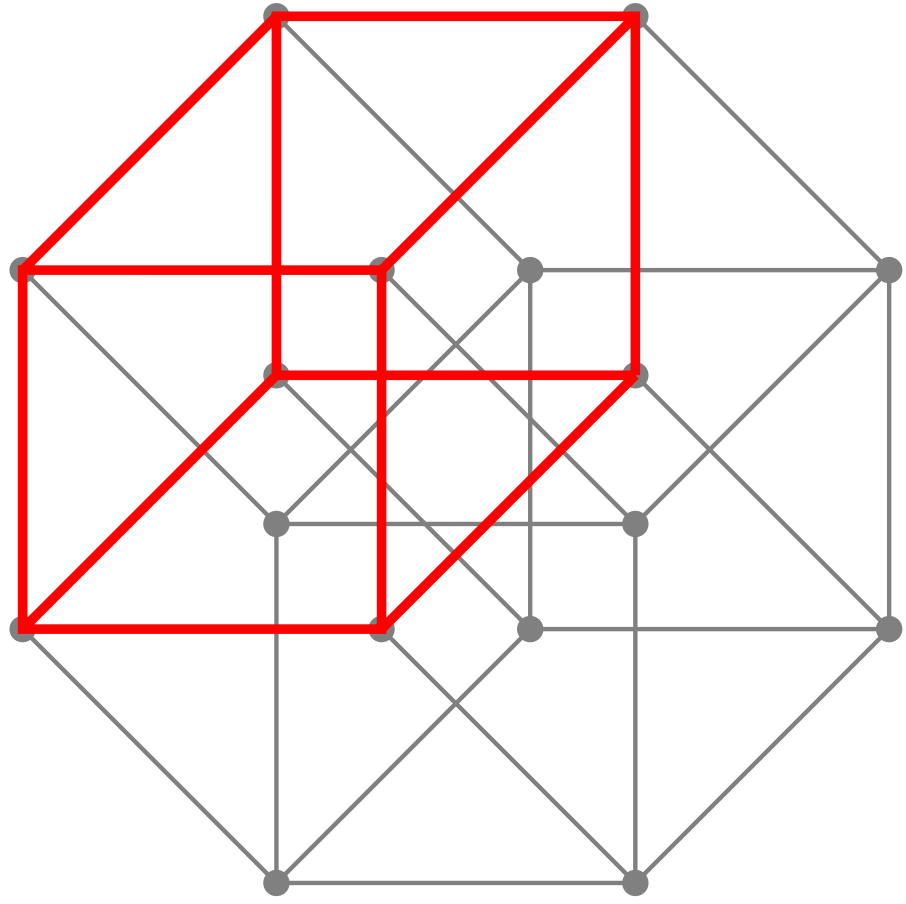

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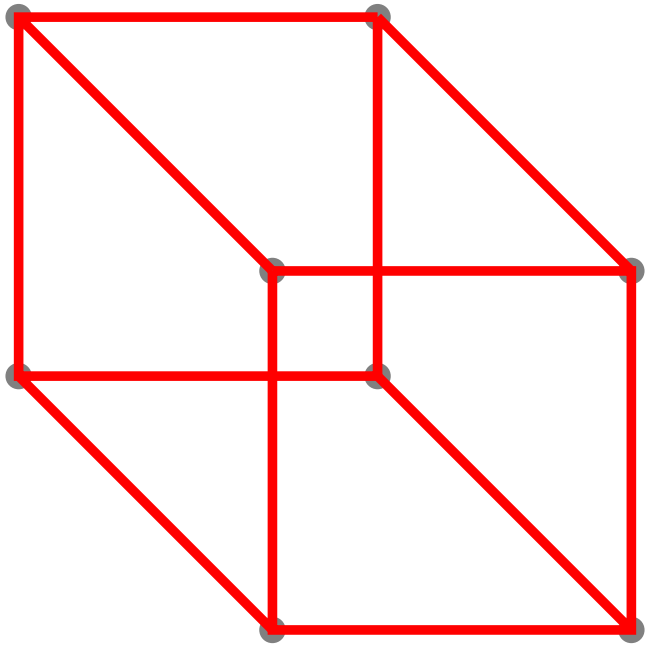

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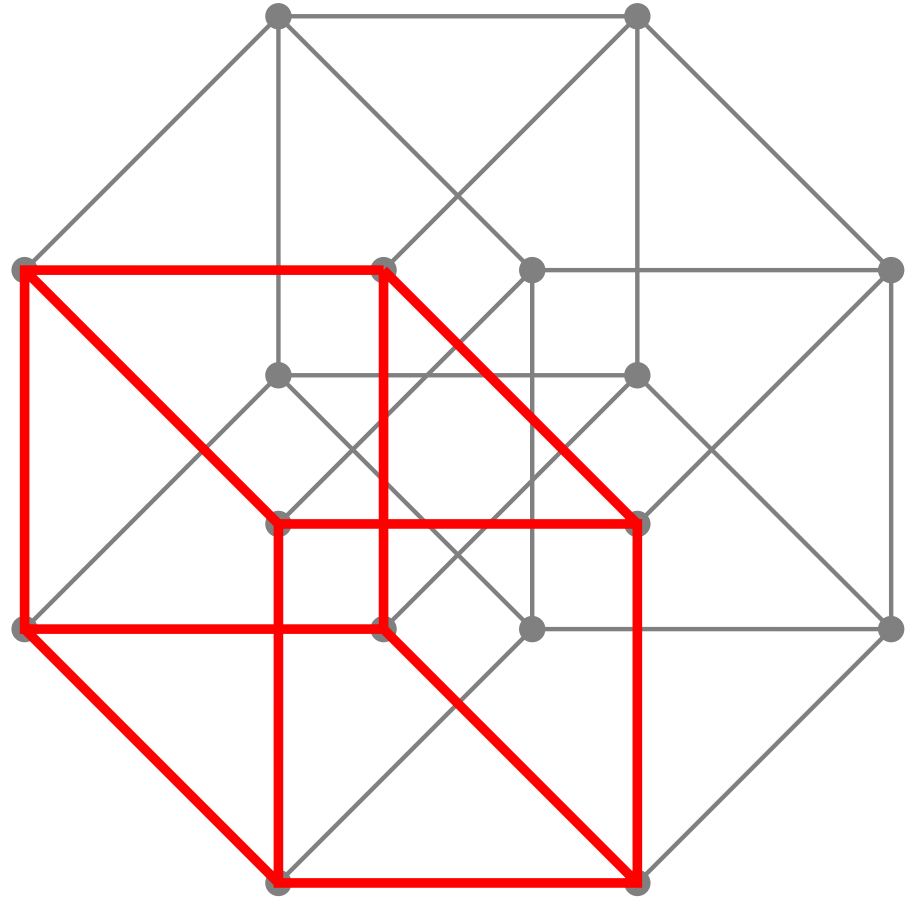

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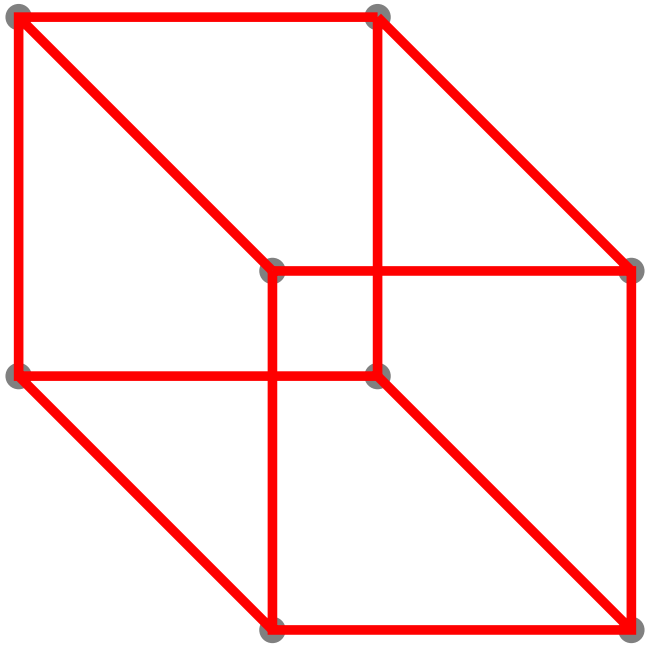

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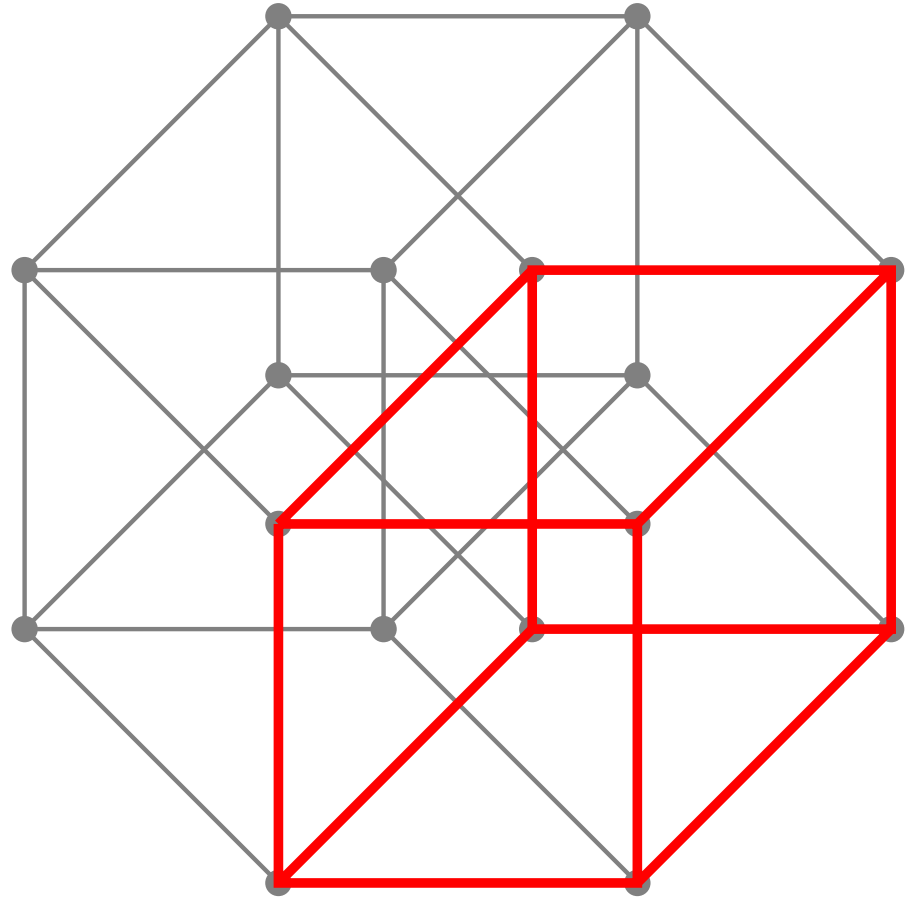

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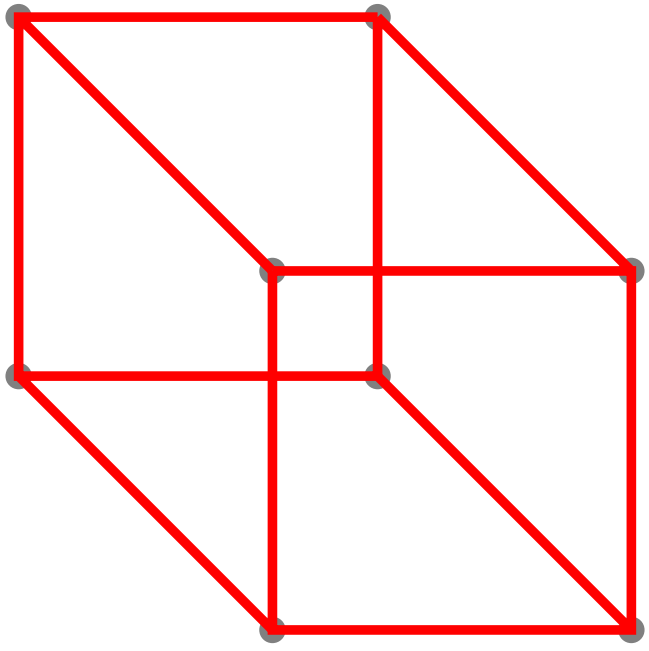

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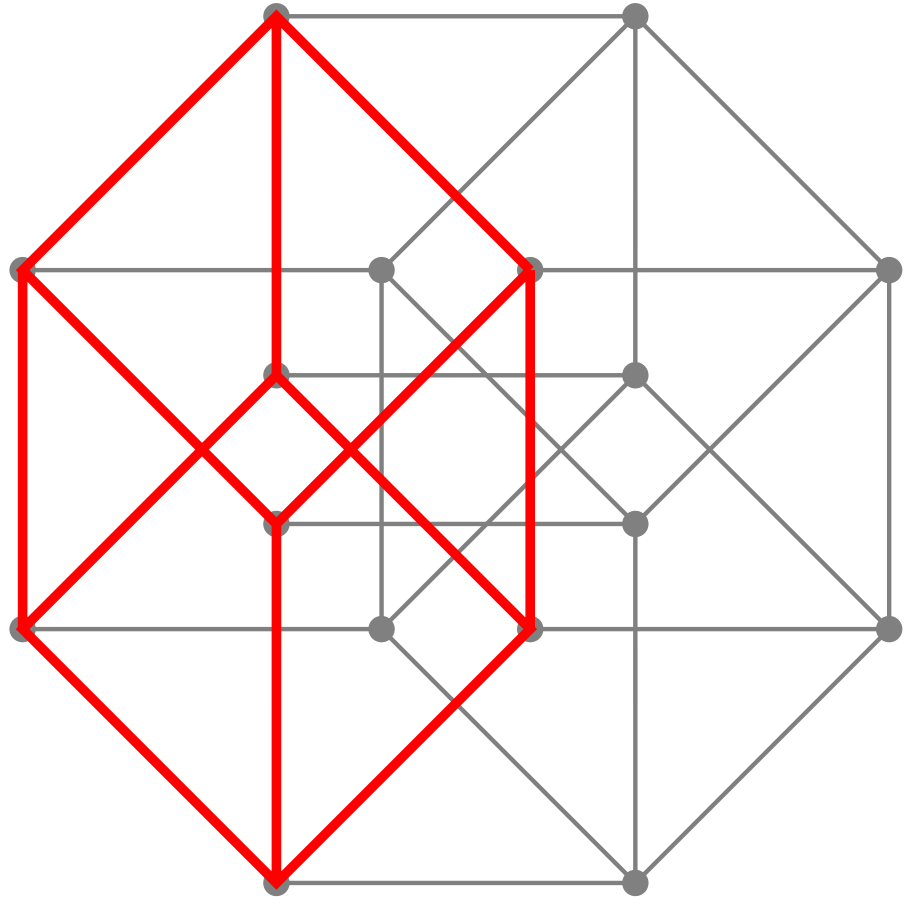

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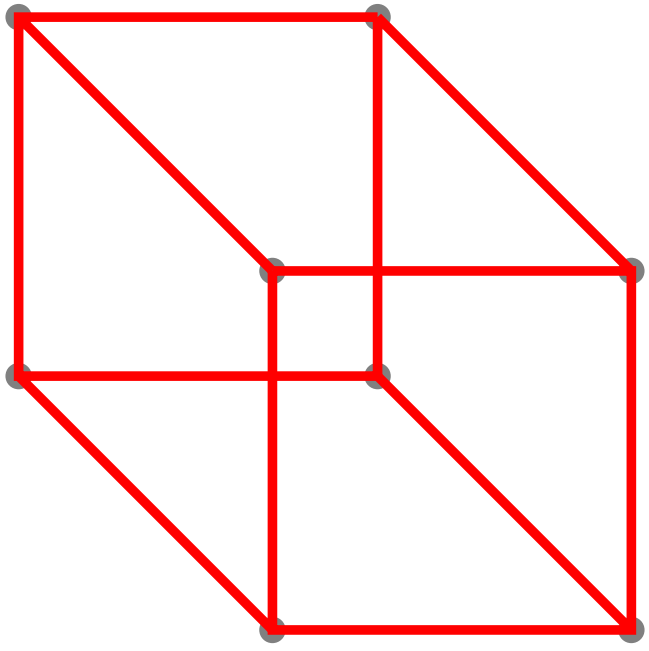

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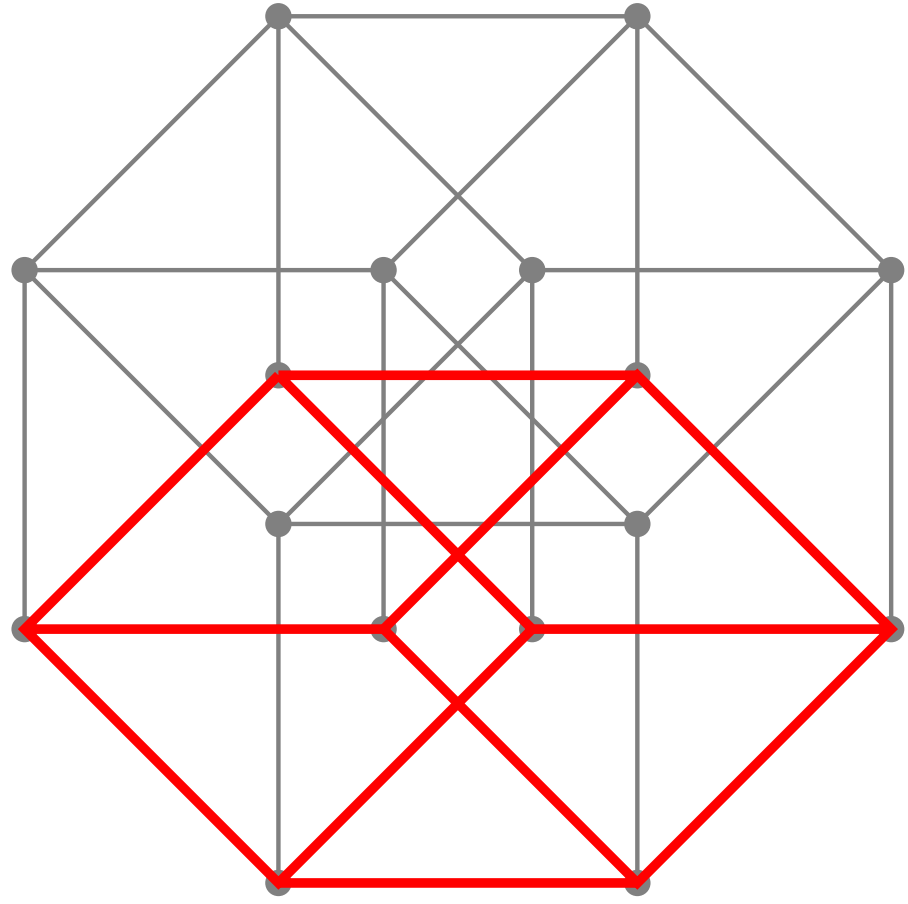

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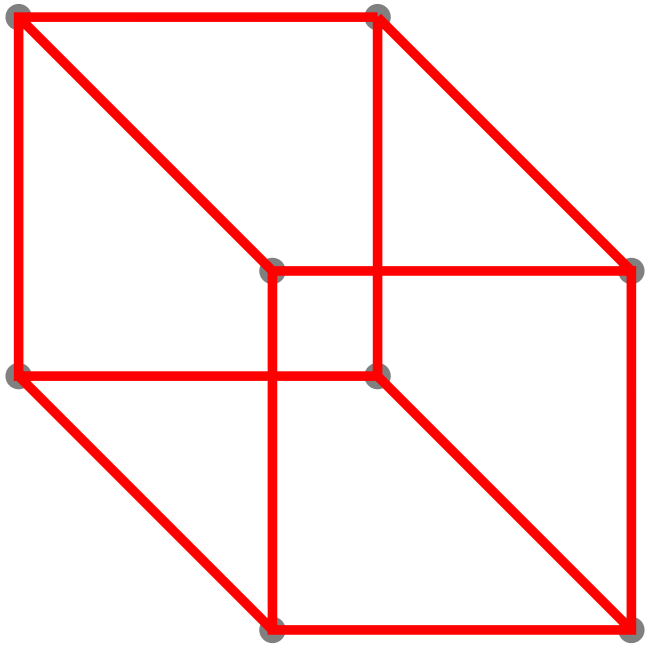

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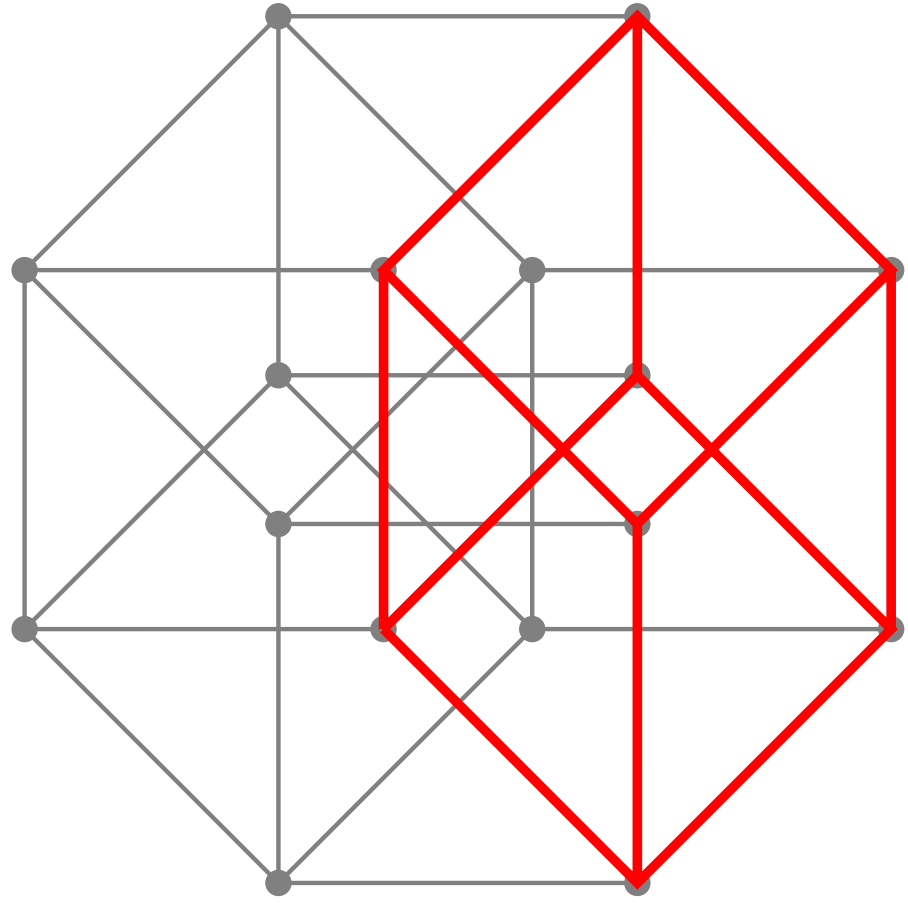

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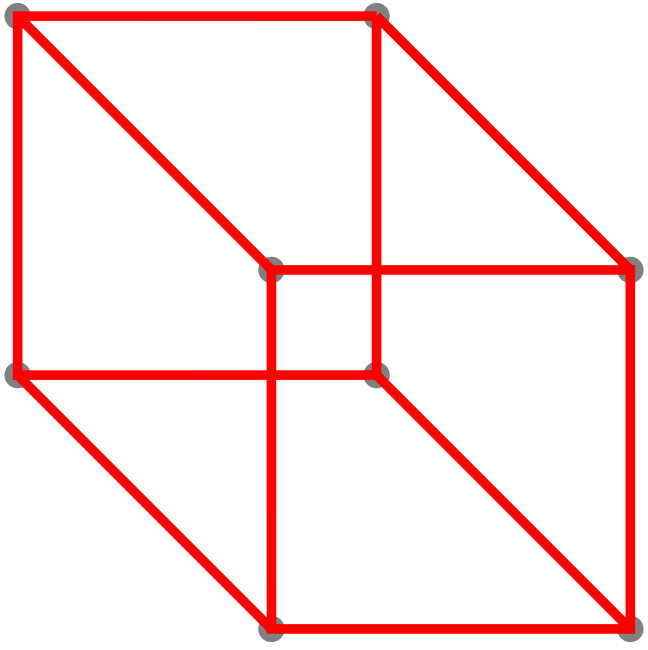

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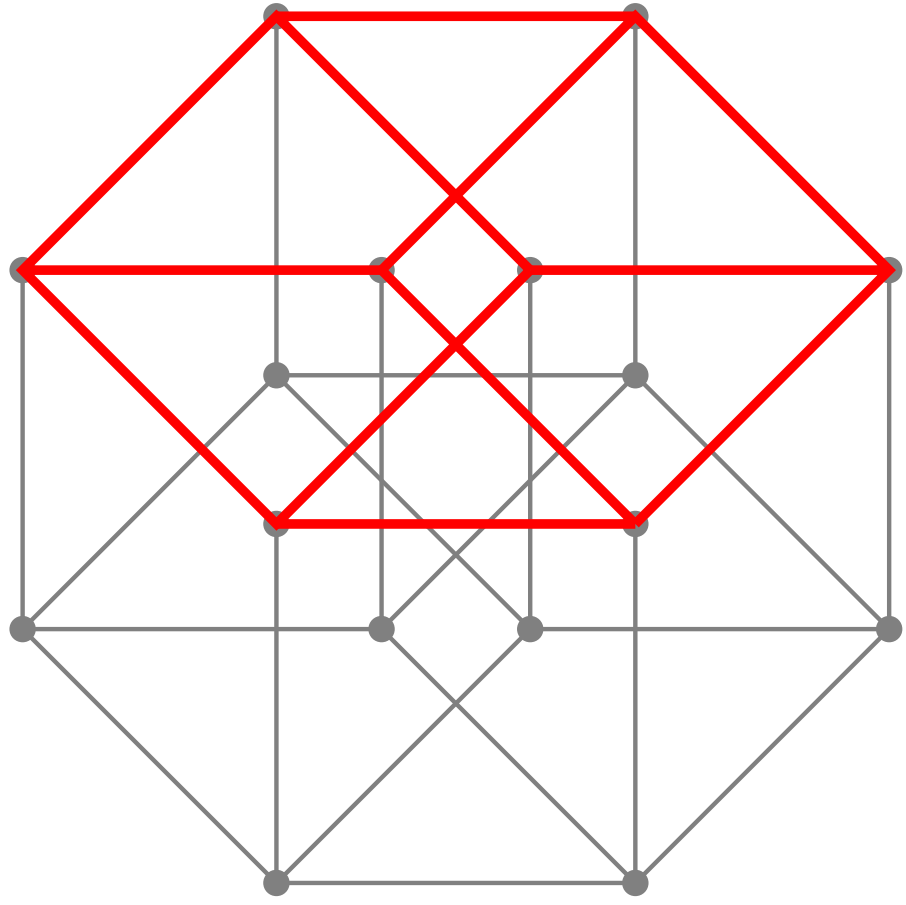

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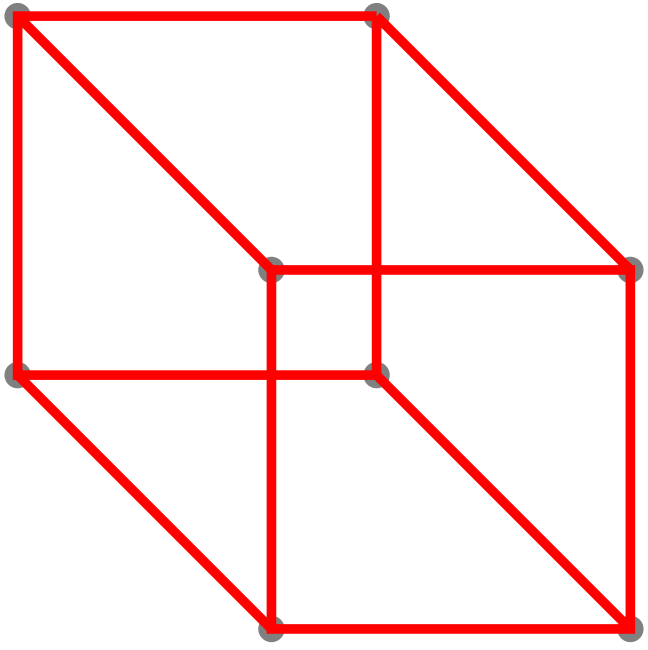

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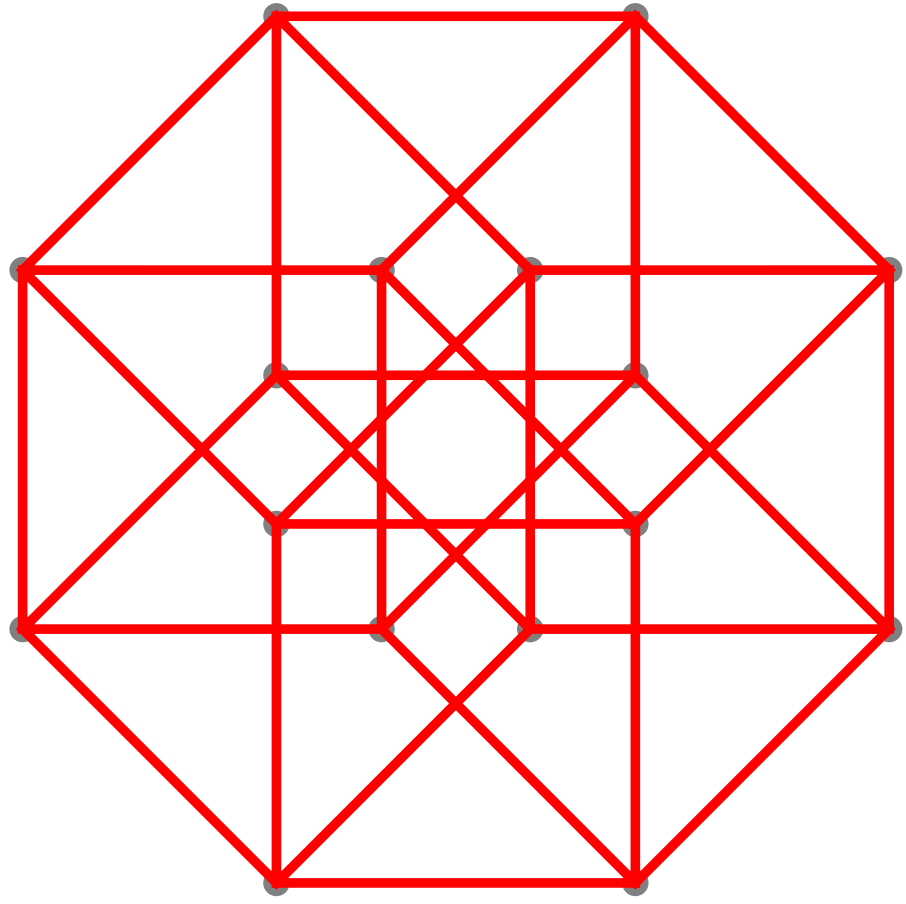

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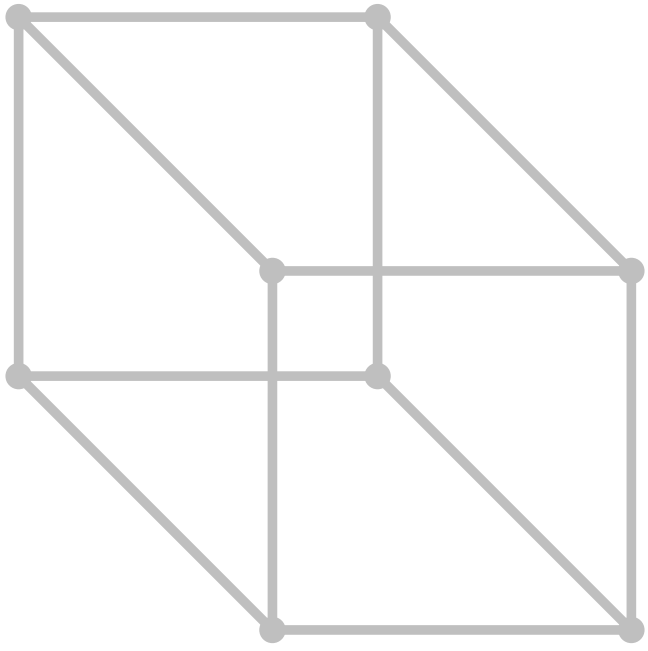

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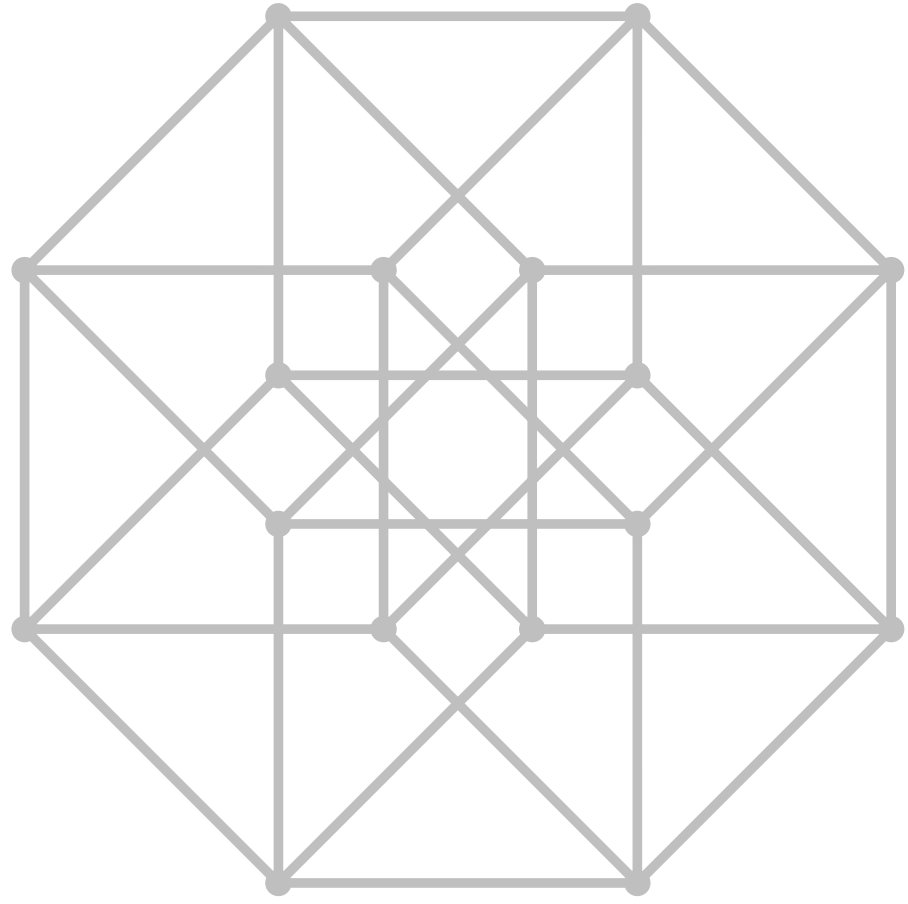

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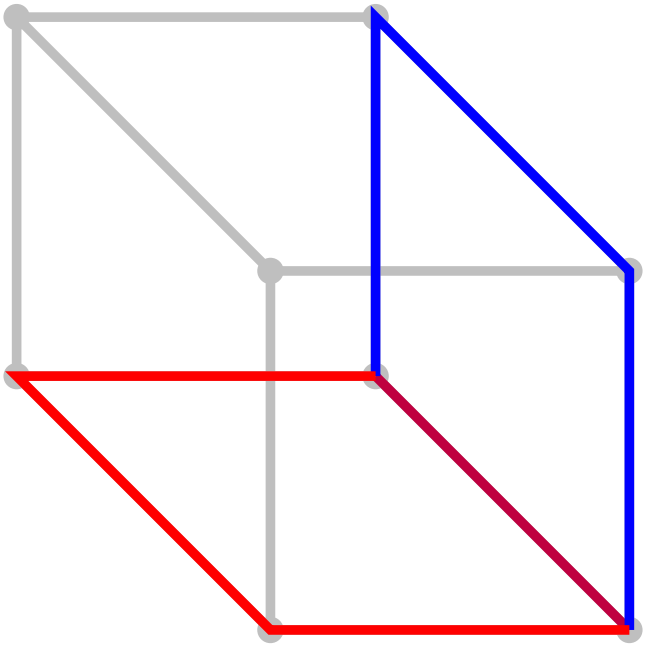

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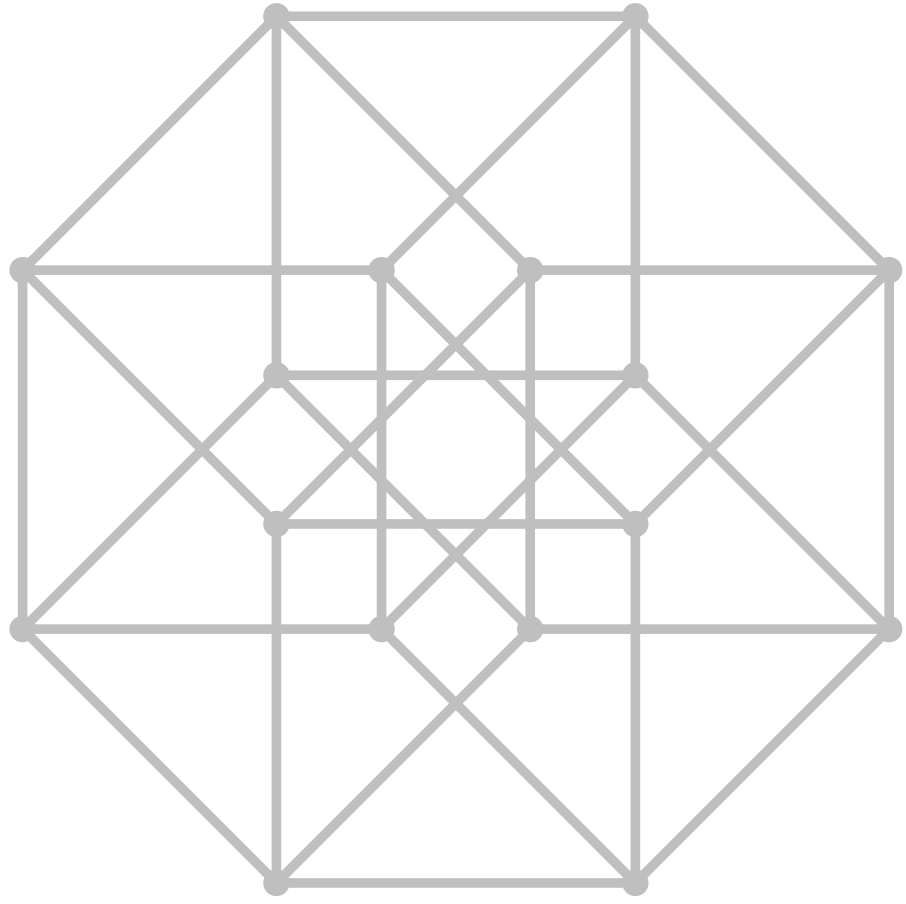

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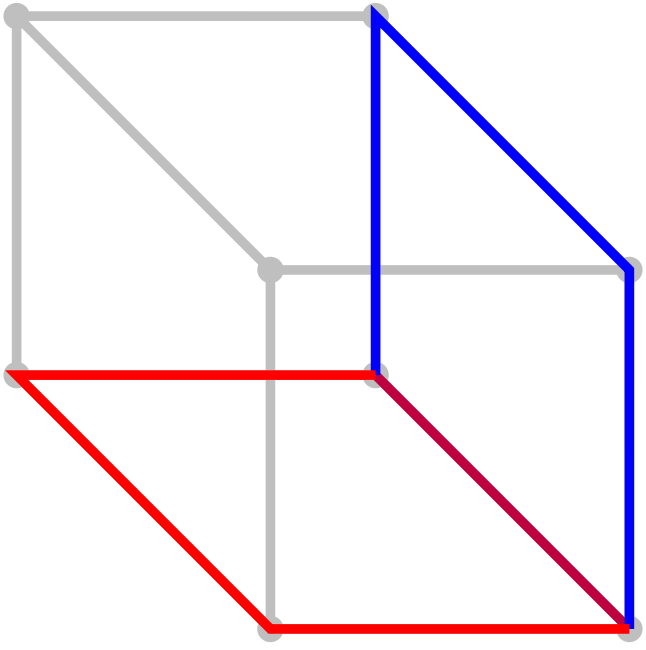

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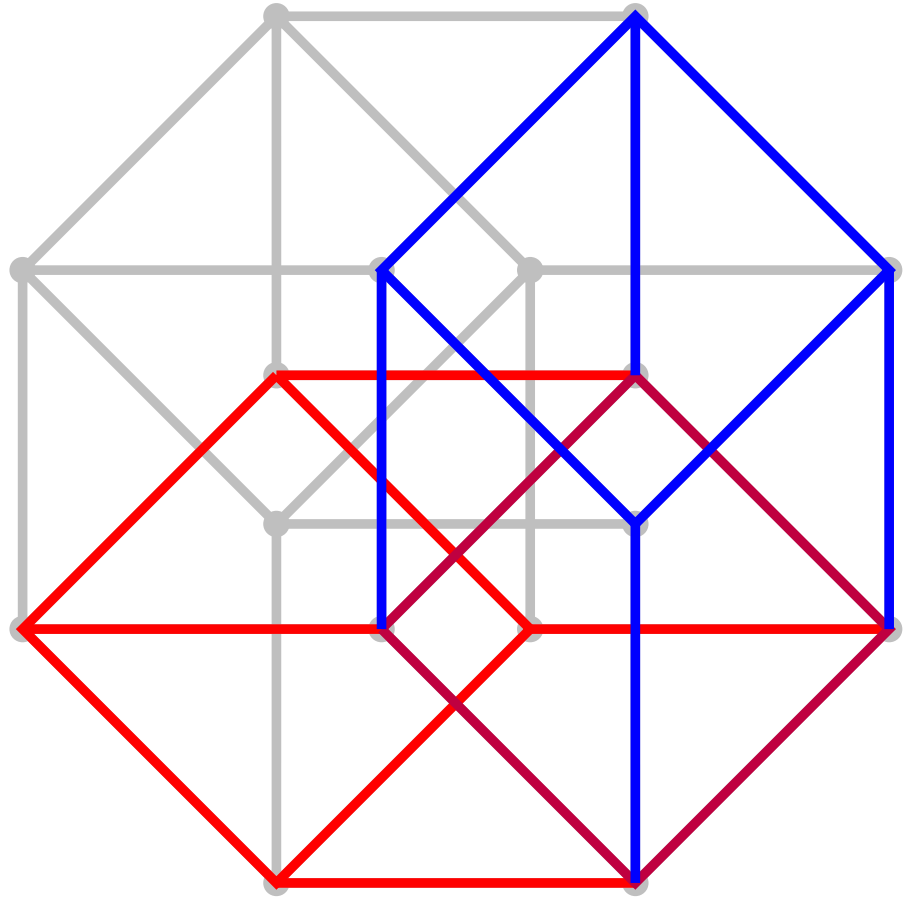

Hypercube

Today's Topic: *Other kinds of 4-D shapes*

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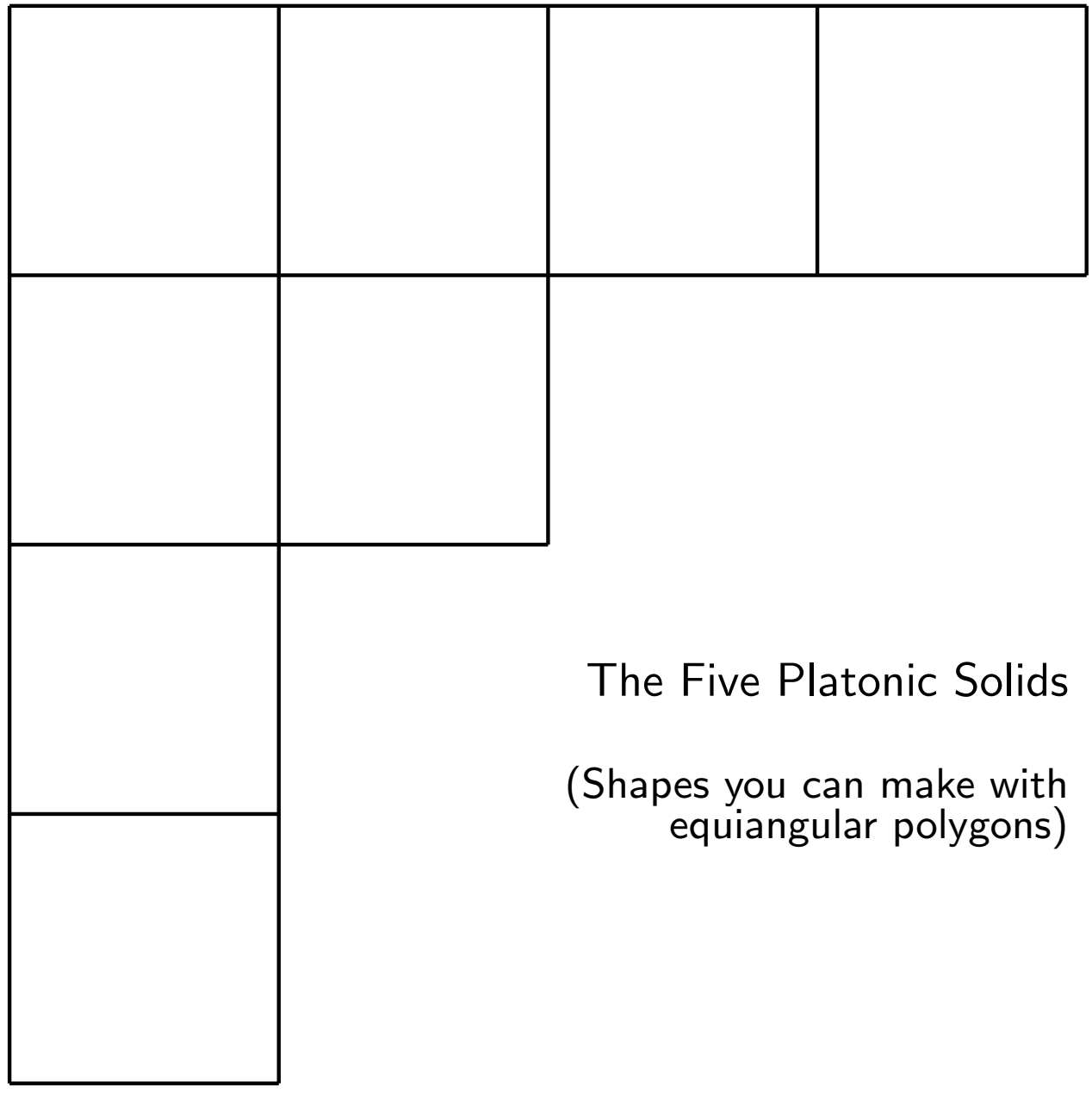
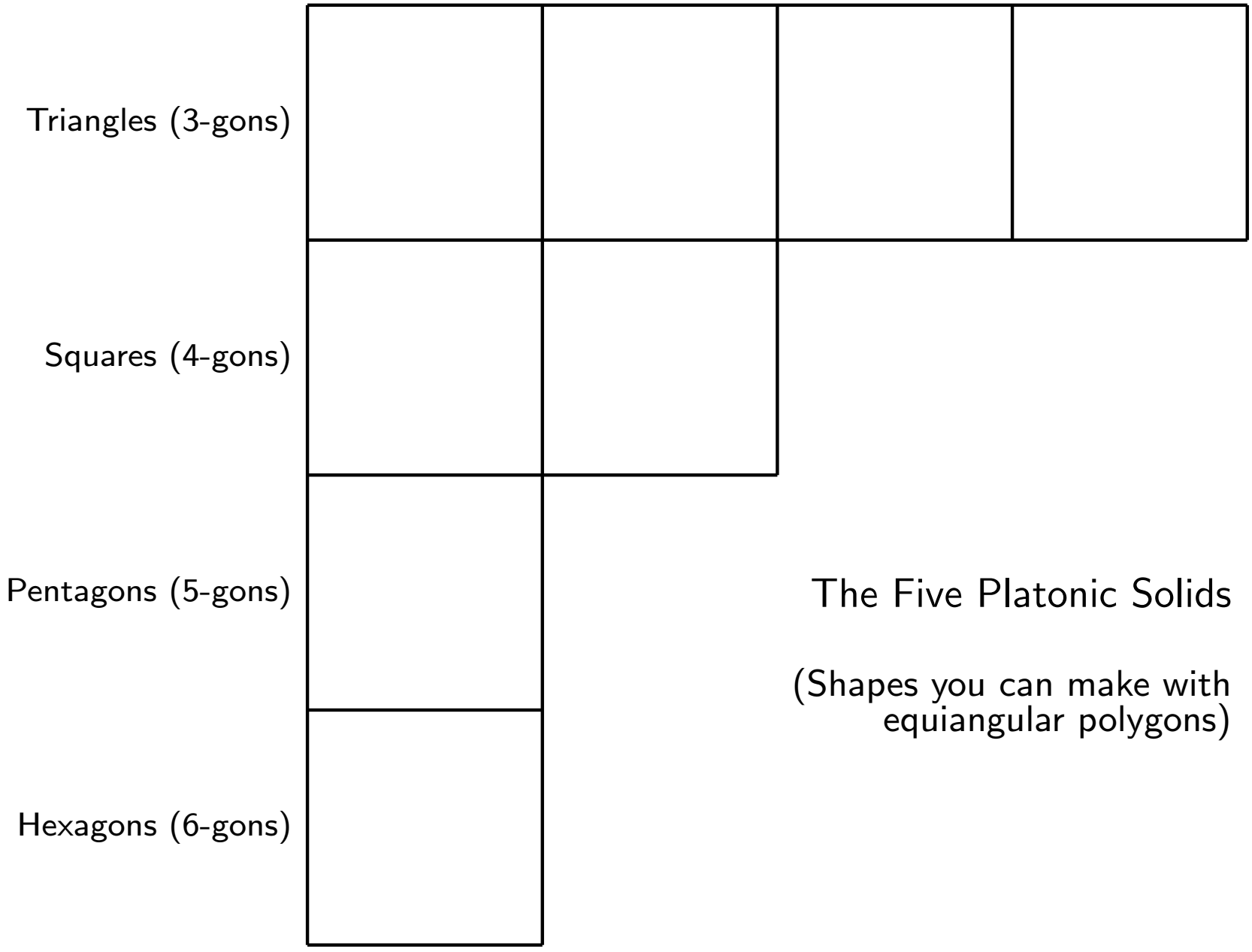
First, back to 3-D...



The Five Platonic Solids

(Shapes you can make with
equiangular polygons)



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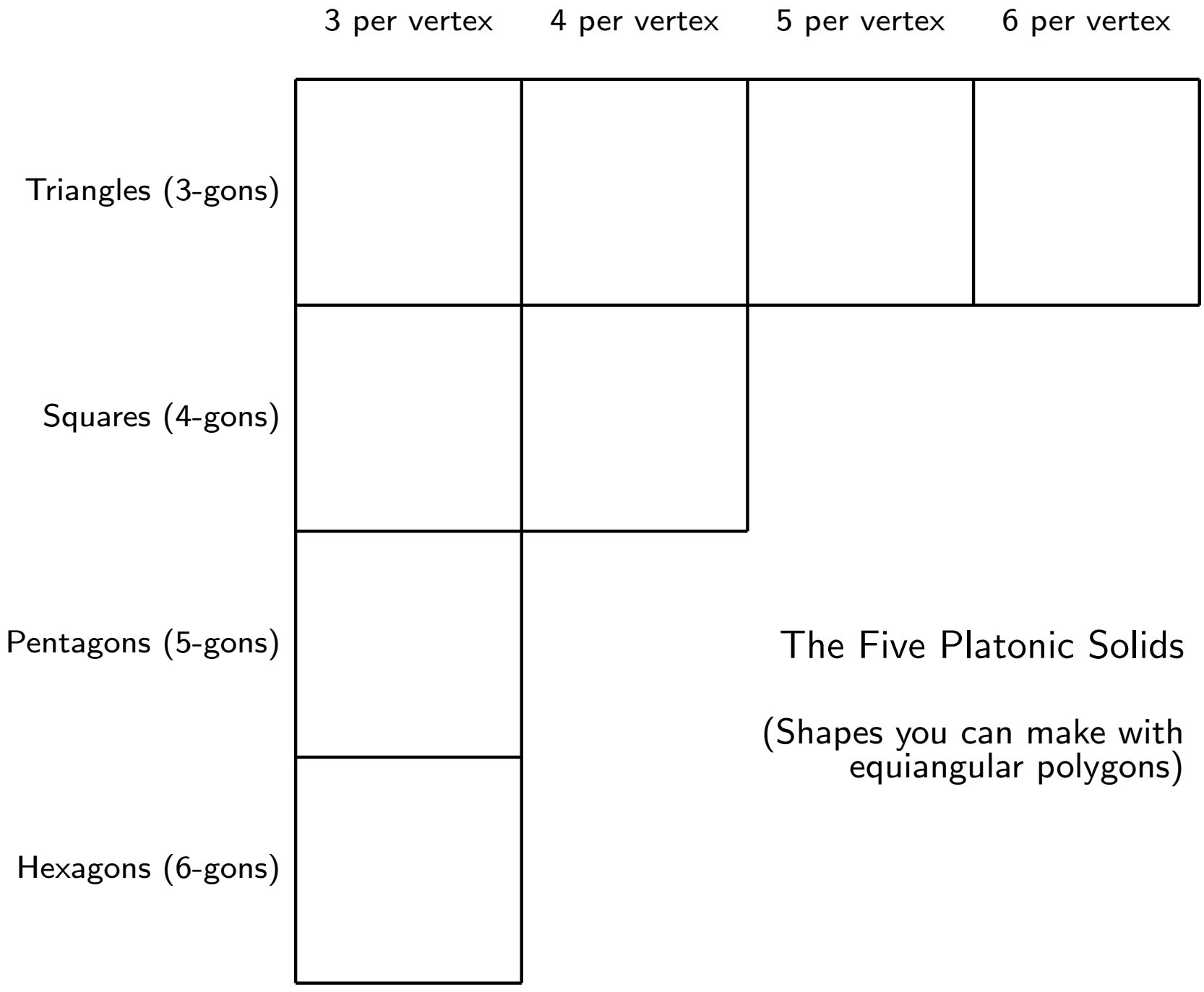
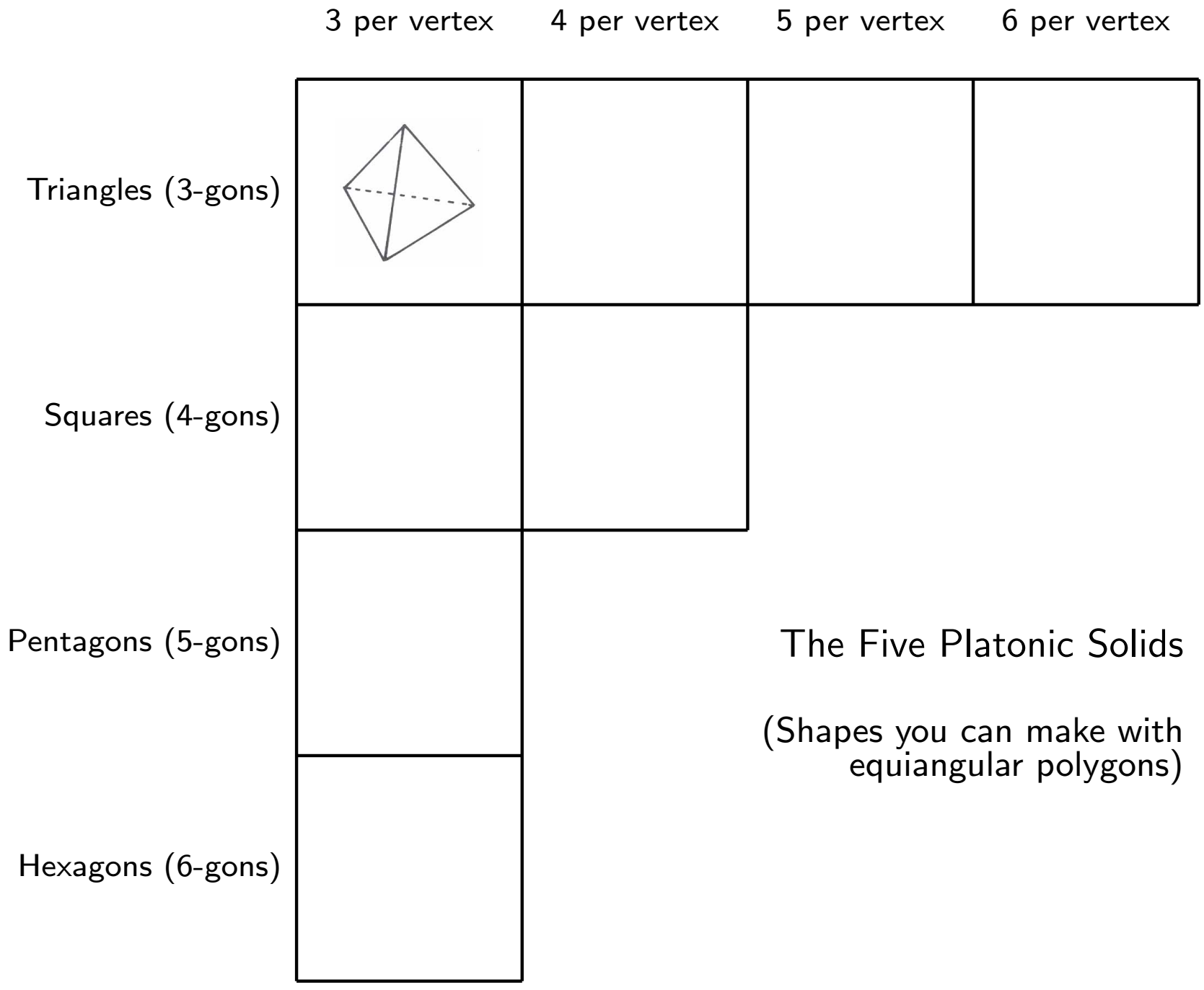

3 per vertex
4 per vertex
5 per vertex
6 per vertex
Triangles (3-gons)
Squares (4-gons)
Pentagons (5-gons)
Hexagons (6-gons)
The Five Platonic Solids
(Shapes you can make with equiangular polygons)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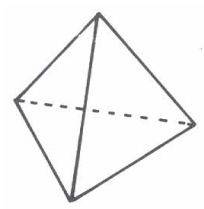
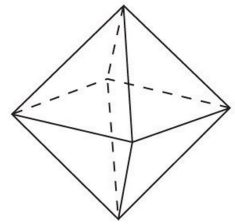
	3 per vertex	4 per vertex	5 per vertex	6 per vertex
Triangles (3-gons)				
Squares (4-gons)				
Pentagons (5-gons)				
Hexagons (6-gons)				

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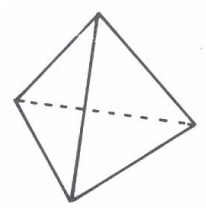
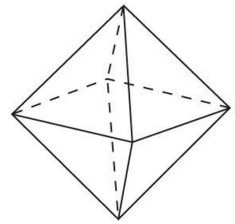
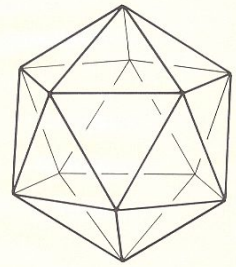
(Shapes you can make with equiangular polygons)

	3 per vertex	4 per vertex	5 per vertex	6 per vertex
Triangles (3-gons)				
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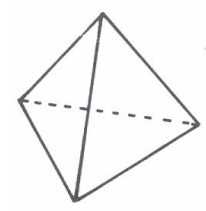
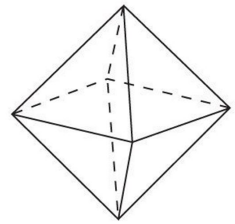
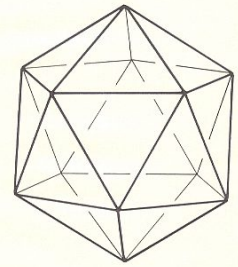
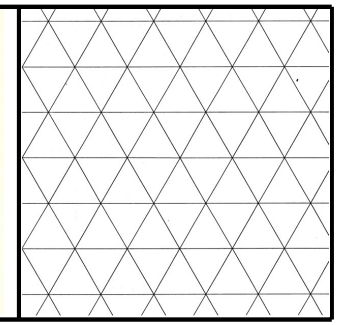
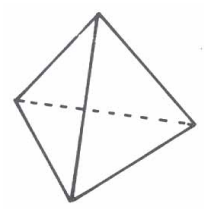
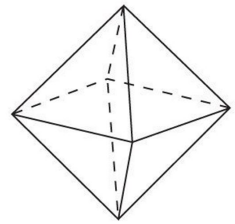
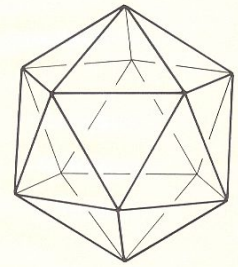
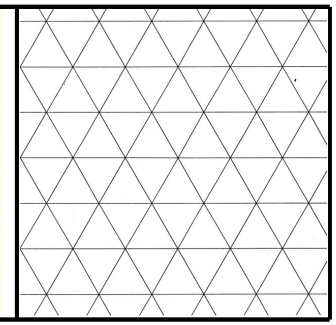
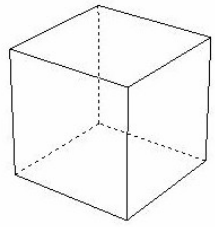
(Shapes you can make with equiangular polygons)

	3 per vertex	4 per vertex	5 per vertex	6 per vertex
Triangles (3-gons)				
Squares (4-gons)				
Pentagons (5-gons)				
Hexagons (6-gons)				

The Five Platonic Solids

(Shapes you can make with equiangular polygons)

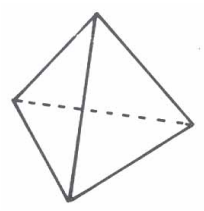
	3 per vertex	4 per vertex	5 per vertex	6 per vertex
Triangles (3-gons)				
Squares (4-gons)				
Pentagons (5-gons)				
Hexagons (6-gons)				

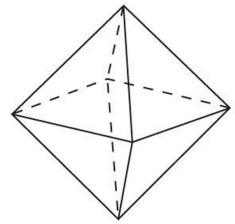
The Five Platonic Solids

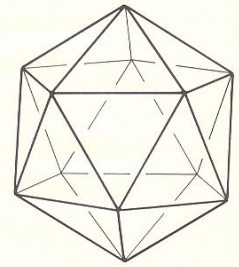
(Shapes you can make with equiangular polygons)

3 per vertex | 4 per vertex | 5 per vertex | 6 per vertex

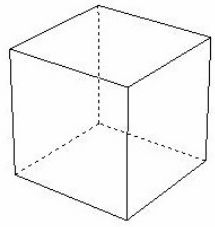
Triangles (3-gons)







Squares (4-gons)

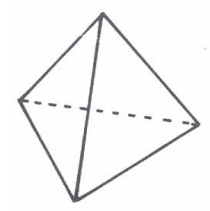
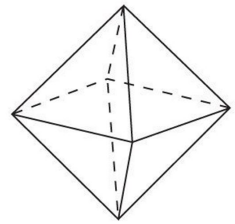
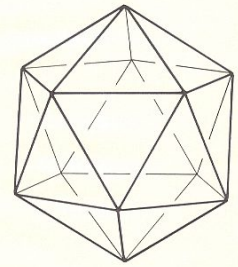
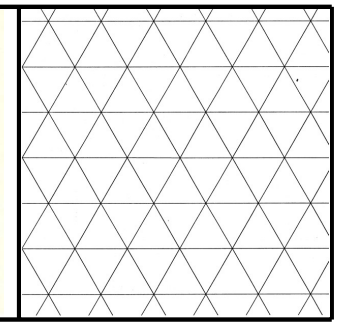
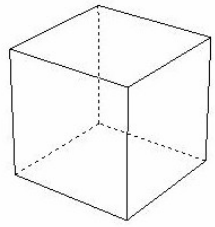
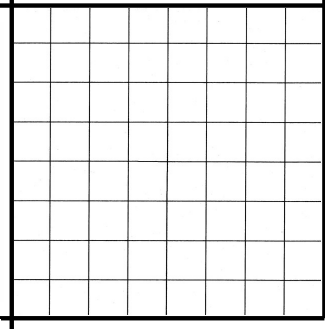
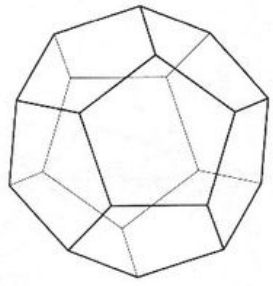


Pentagons (5-gons)

Hexagons (6-gons)

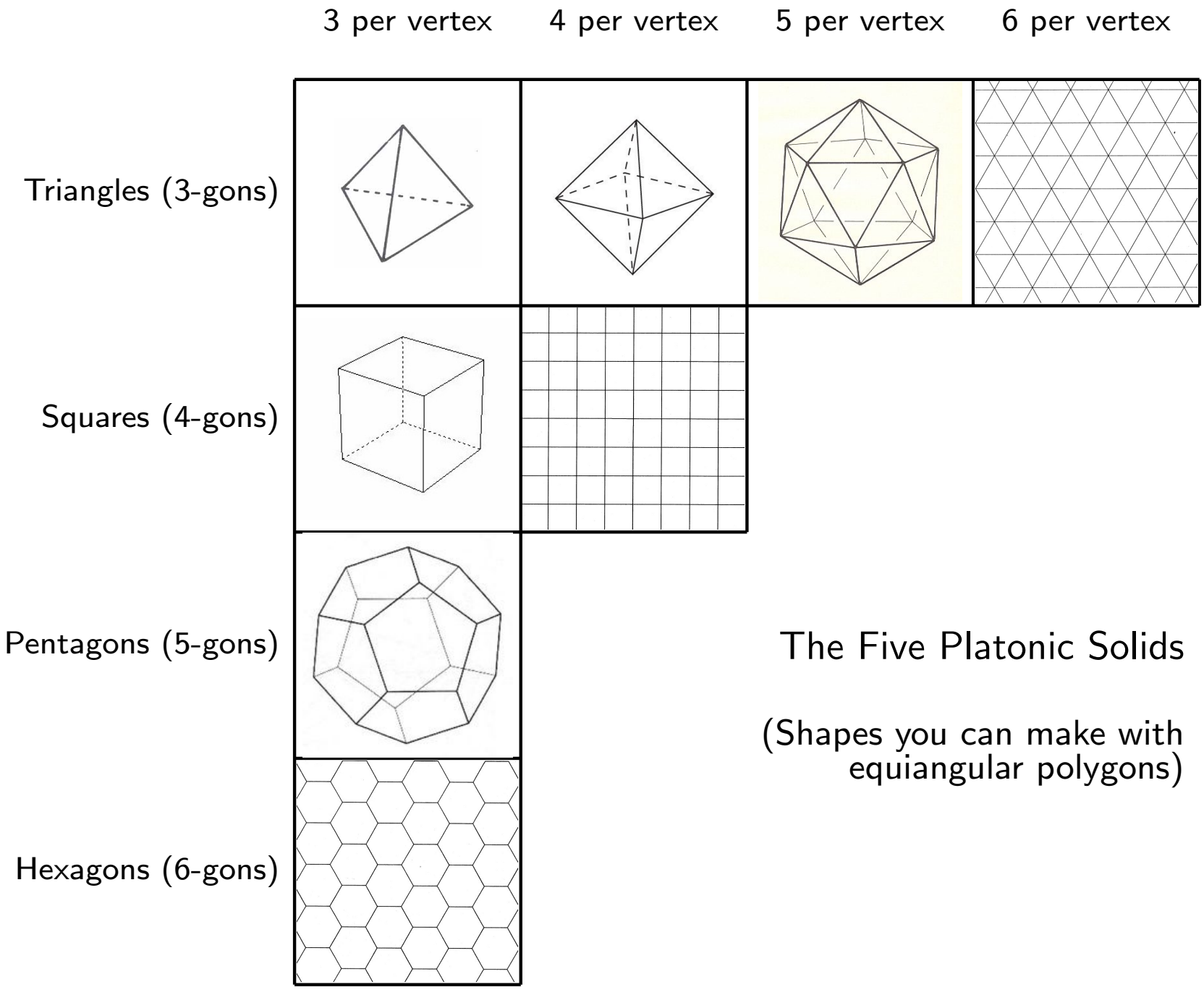
The Five Platonic Solids

(Shapes you can make with equiangular polygons)

	3 per vertex	4 per vertex	5 per vertex	6 per vertex
Triangles (3-gons)				
Squares (4-gons)				
Pentagons (5-gons)				
Hexagons (6-gons)				

The Five Platonic Solids

(Shapes you can make with equiangular polygons)


3 per vertex
4 per vertex
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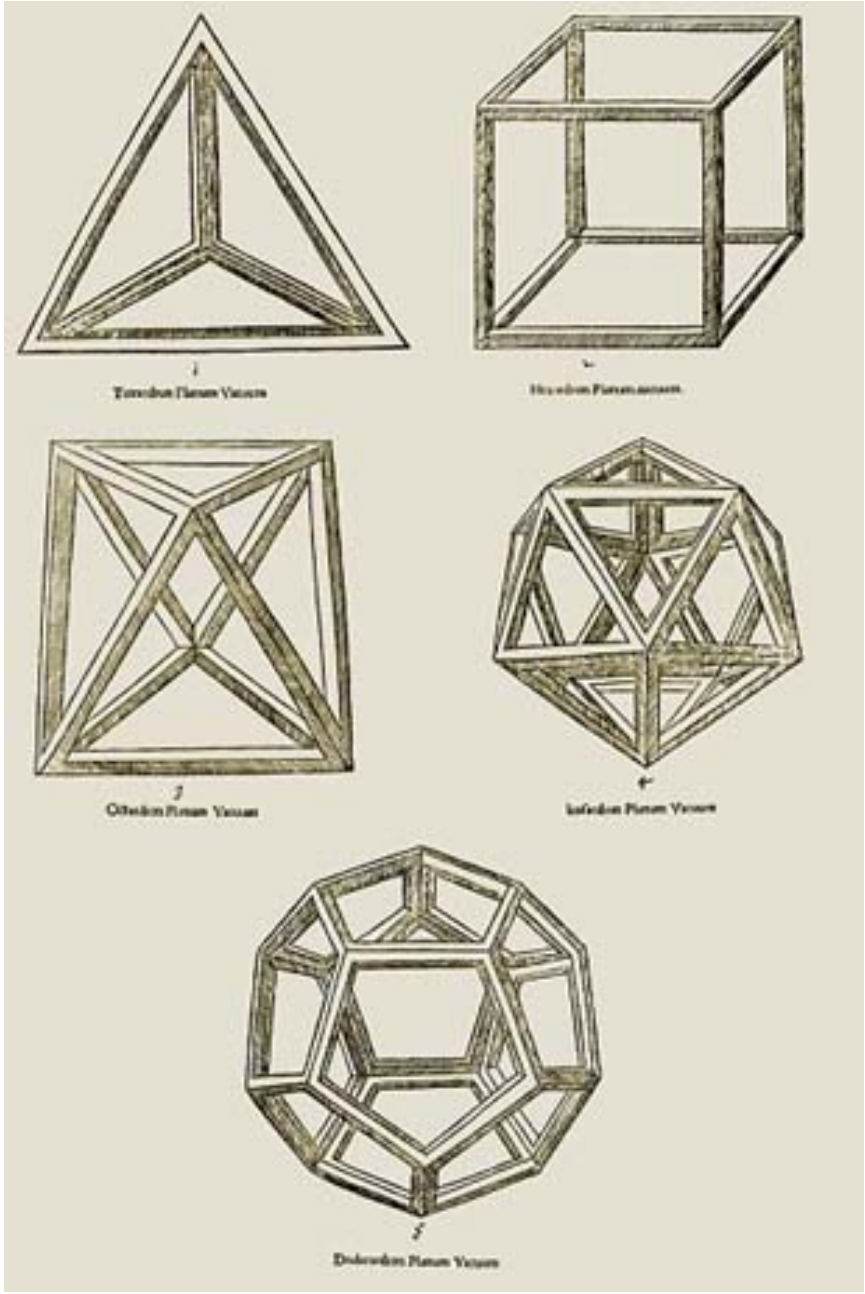
Triangles (3-gons)
Squares (4-gons)
Pentagons (5-gons)
Hexagons (6-gons)
The Five Platonic Solids
(Shapes you can make with equiangular polygons)

The Five Platonic Solids
Drawn by Leonardo da Vinci
for Luca Pacioli's book
The Divine Proportion, 1509



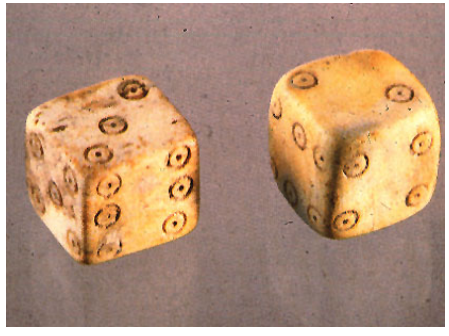
Neolithic stone polyhedrons, 2000 B.C.

Neolithic stone polyhedrons, 2000 B.C.

Roman examples.

Neolithic stone polyhedrons, 2000 B.C.

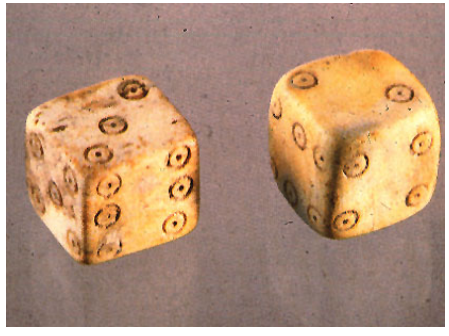
Roman examples.



200 A.D.

Neolithic stone polyhedrons, 2000 B.C.

Roman examples.



200 A.D.

Pompeii, 100 A.D.

Neolithic stone polyhedrons, 2000 B.C.

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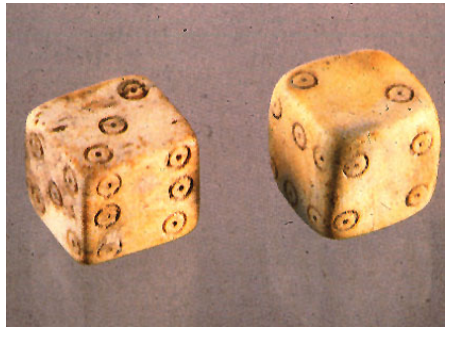
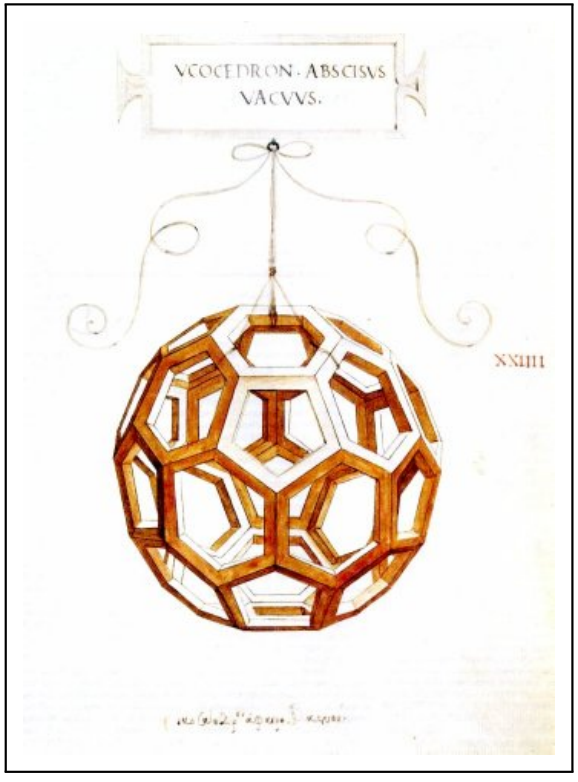


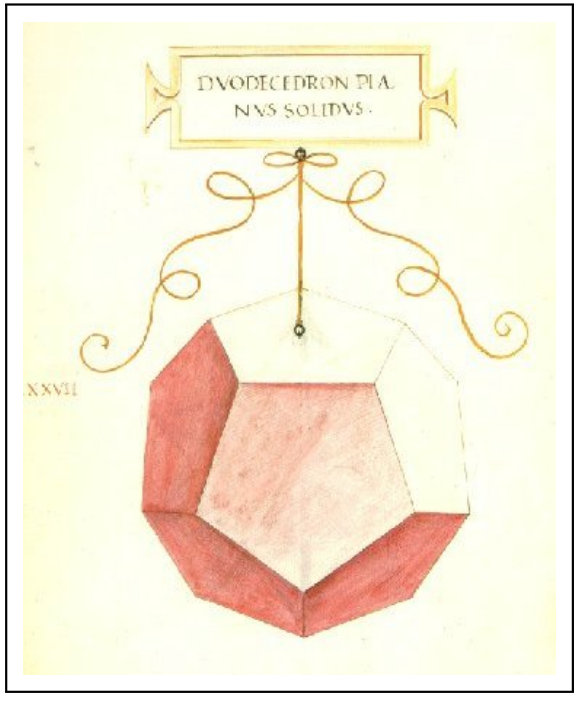
200 A.D.

Pompeii, 100 A.D.

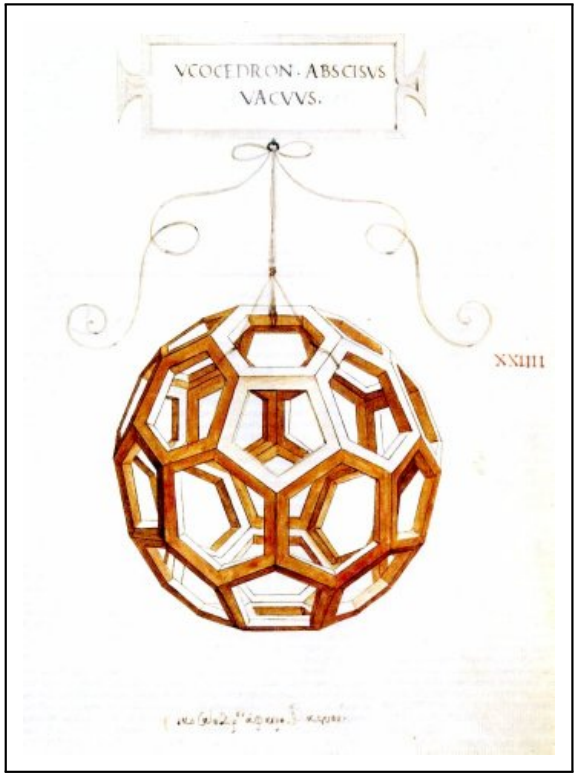
Contemporary

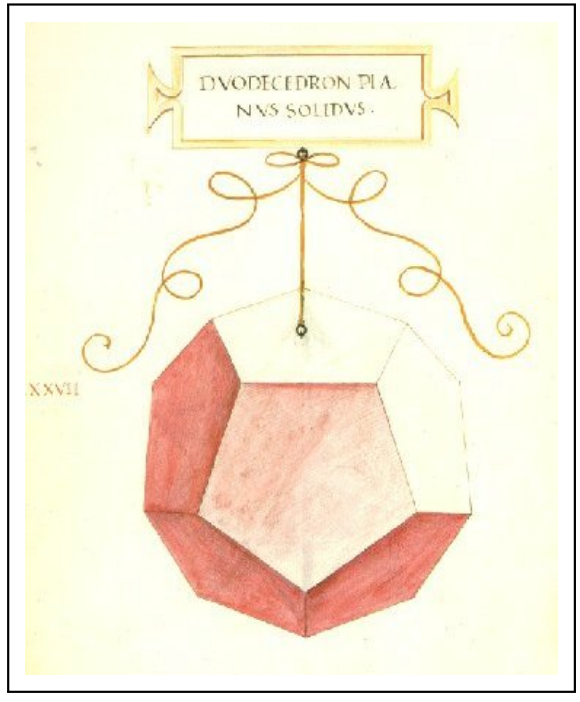
Leonardo da Vinci's illustrations
for *The Divine Proportion*

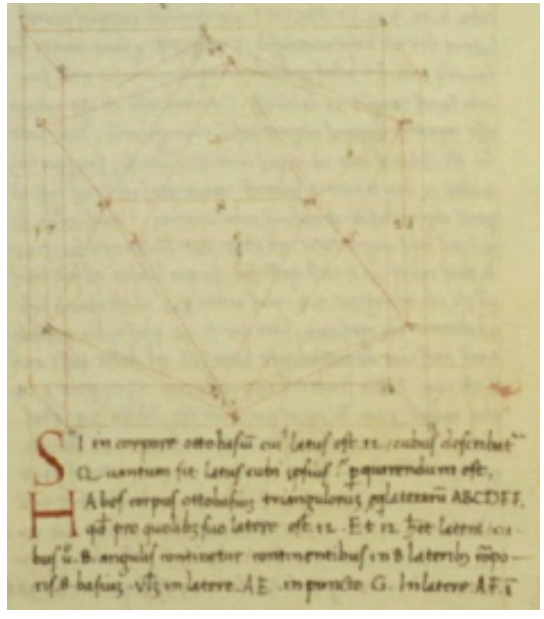




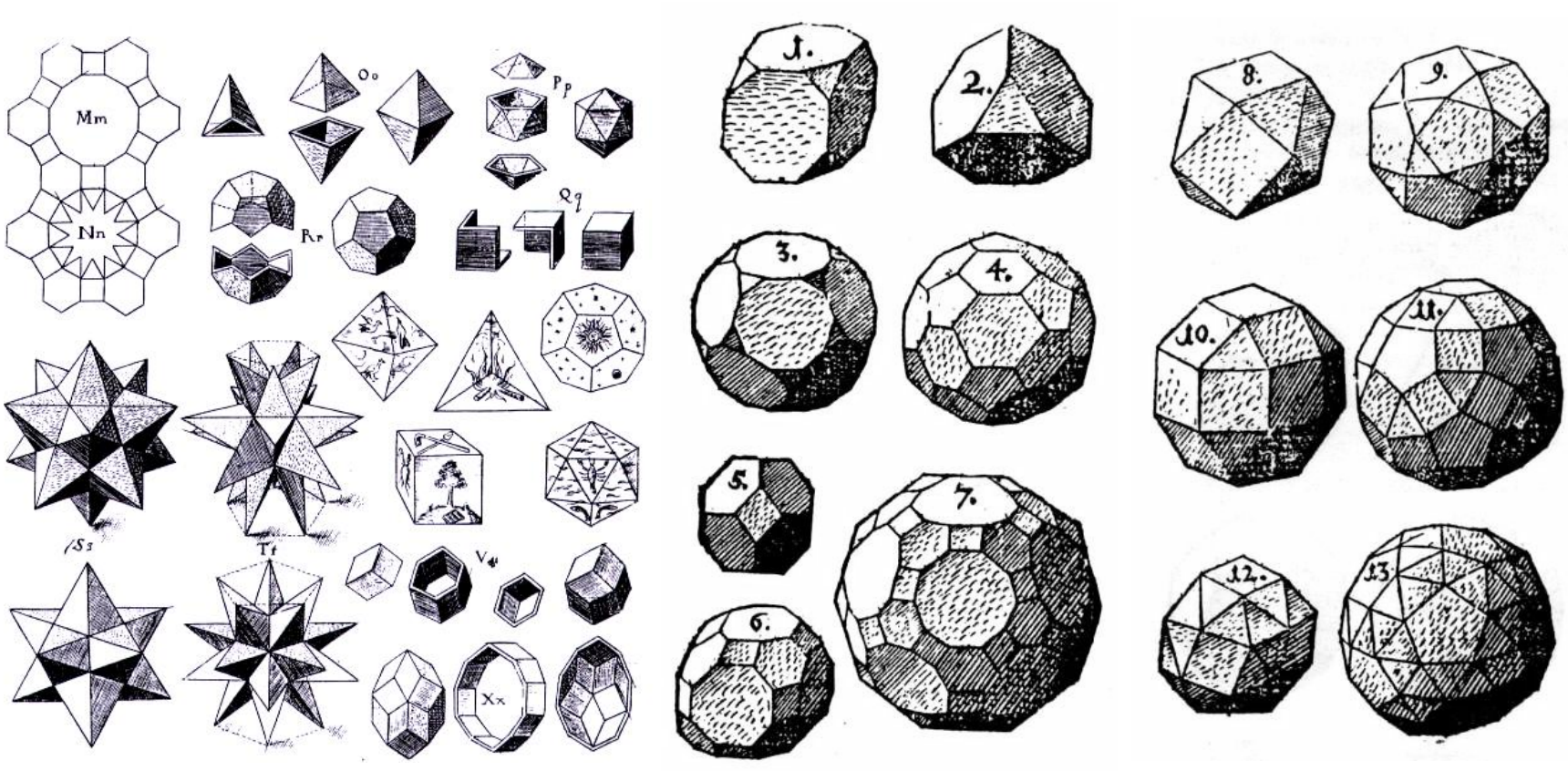
Leonardo da Vinci's illustrations
for *The Divine Proportion*



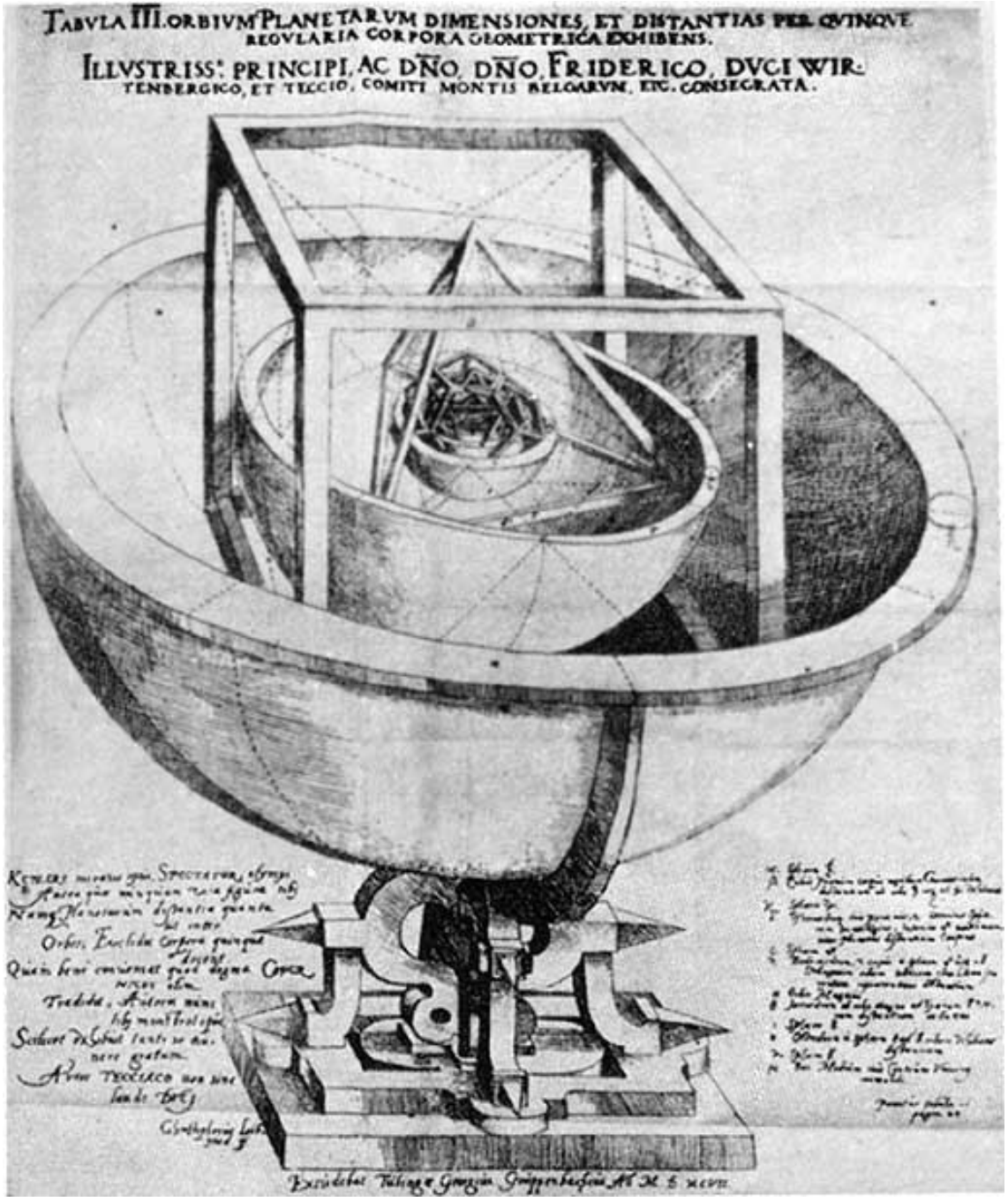




Piero della Francesca



Illustrations from Johannes Kepler's Harmonices Mundi, 1619



Johannes Kepler's (early) model of the solar system

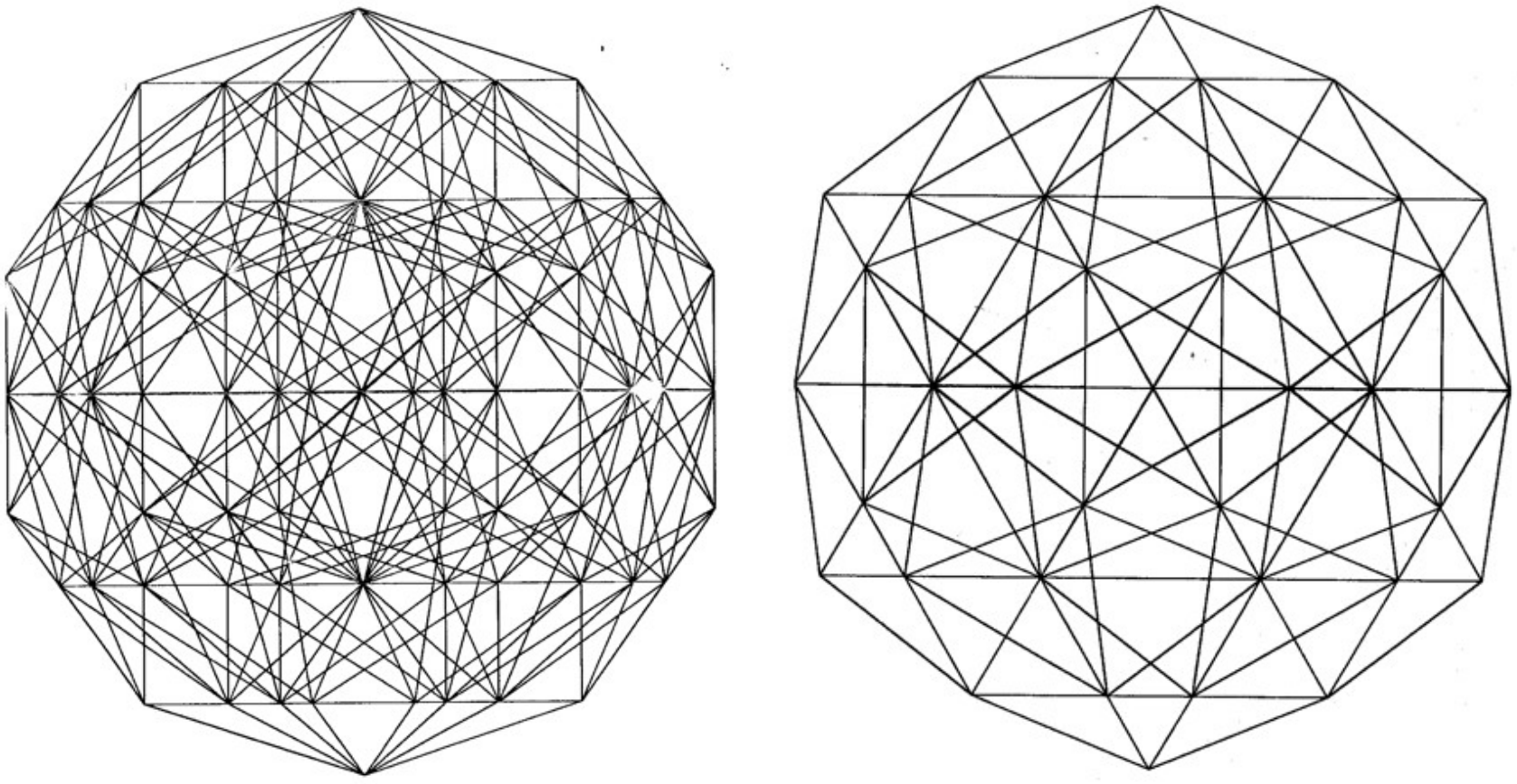
Four Dimensional Platonic Polyhedrons

The 4-D Tetrahedron

The 4-D Cube

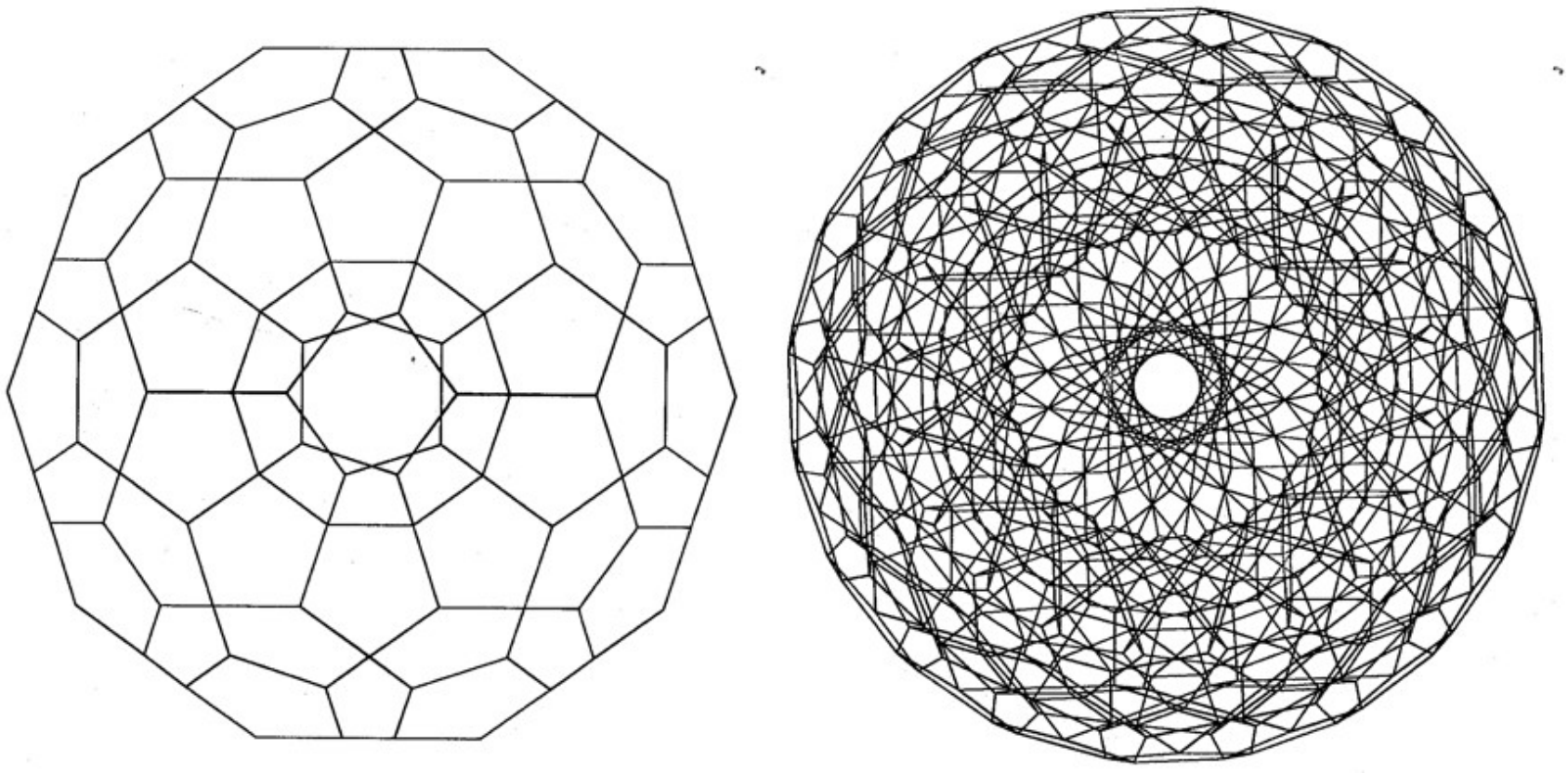
The 4-D Octahedron

The 24-Cell



The 4-D Icosahedron

Has 600 tetrahedron cells



The 4-D Dodecahedron

Has 120 dodecahedron cells

The 24-Cell

Next Time

How the idea of the fourth dimension influenced cubism