Frieze Symmetries

MATH 122 Lecture notes by R. Hammack

**There** are five basic symmetry operations that can be applied to a frieze pattern: Translation (T), Glide reflection (G), Rotation (R), Vertical reflection (V) and Horizontal reflection (R). These are illustrated below. We say that a frieze pattern has a particular symmetry if that symmetry operation can be applied to the pattern without changing the appearance of the pattern.

T	(Translation)	
G	(Glide reflection)	
R	(Rotation)	
V	(Vertical reflection)	
H	(Horizontal reflection)	
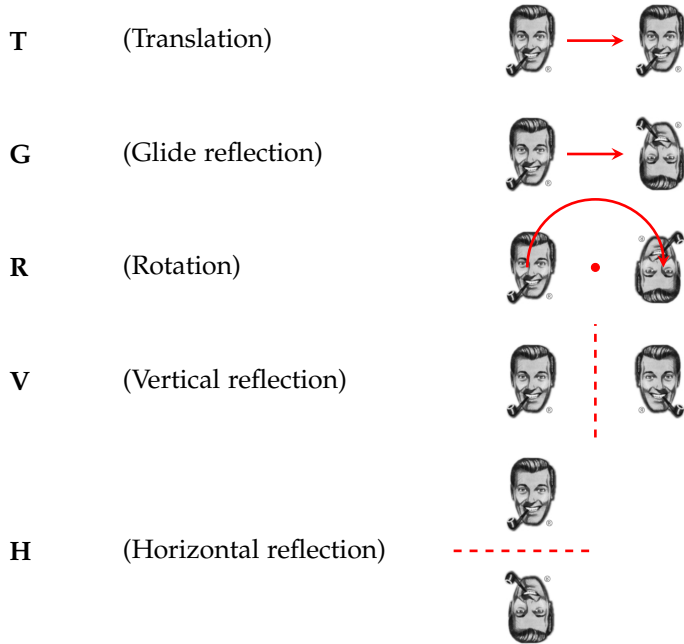
Any given frieze pattern may have some of these symmetries but not others. However, a frieze pattern can have only one of **seven** possible combinations of symmetries, illustrated below.

1.	T	
2.	TG	
3.	TGH	
4.	TV	
5.	TR	
6.	TGRV	
7.	TGRVH	
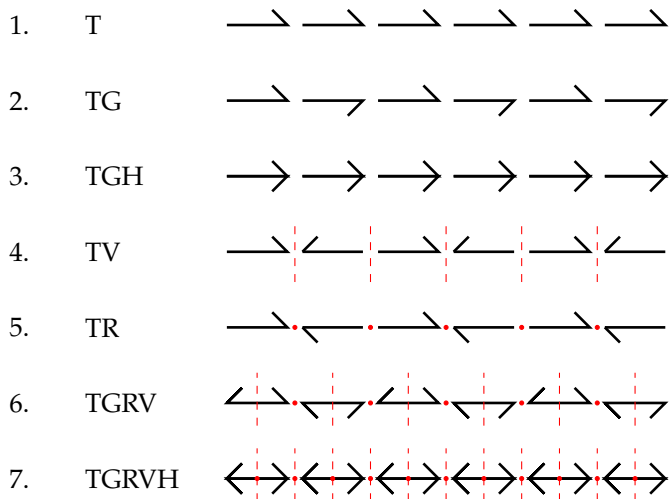
Further examples are shown on the next page.

The Seven Frieze Patterns

1. T	
2. TG	
3. TR	
4. TV	
5. TGRV	
6. TGH	
7. TGRVH	

KEY:

T = Translation   **G** = Glide Reflection

R = Rotation by $180°$

V = Vertical Reflection   **H** = Horizontal Reflection