MATH 122
(Day 3)

The Seven Frieze Pattern Types

Richard Hammack

http://www.people.vcu.edu/∼rhammack/Math122/

Today's Goals

- ▶ Review the 5 types of symmetry that a frieze pattern may possess.

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► Examine how combinations of symmetries force other symmetries.

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▶ Analyze examples (from Richmond's architecture).

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▶ New assignment

There are five baic symmetries for frieze patterns:

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T   (Translation)

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T   (Translation)

G   (Glide reflection)

There are five baic symmetries for frieze patterns:
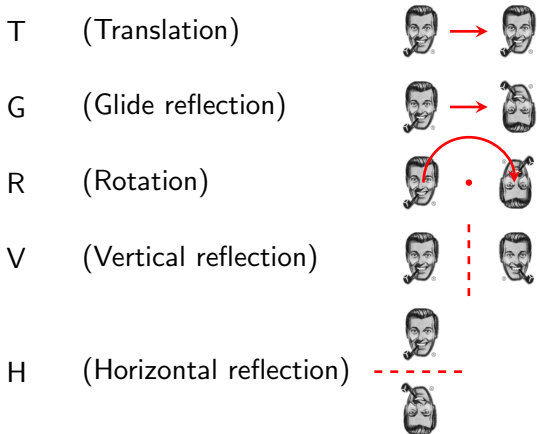
T   (Translation)

G   (Glide reflection)

R   (Rotation)

There are five baic symmetries for frieze patterns:

T   (Translation)

G   (Glide reflection)

R   (Rotation)

V   (Vertical reflection)

There are five baic symmetries for frieze patterns:

T   (Translation)

G   (Glide reflection)

R   (Rotation)

V   (Vertical reflection)

H   (Horizontal reflection)

Certain combinations of symmetries force other symmetries

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T (always)

Certain combinations of symmetries force other symmetries

T (always)   T T T T T T

Certain combinations of symmetries force other symmetries

T (always)   T T T T T T $\Big\}$ Can't avoid T.

Certain combinations of symmetries force other symmetries

T (always)   T T T T T T $\Big\}$ Can't avoid T.

$H \Longrightarrow G$

Certain combinations of symmetries force other symmetries

T (always) $\qquad\qquad$ T T T T T T $\quad \Big\}$ Can't avoid T.

$H \Longrightarrow G$ $\qquad\qquad$ H H H H H H

Certain combinations of symmetries force other symmetries

T (always)   T T T T T T $\Big\}$ Can't avoid T.

$H \implies G$ H H H H H H $\Big\}$ Choose H,
                                                    get G too.

Certain combinations of symmetries force other symmetries

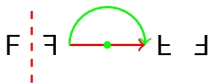T (always)   T T T T T T $\Big\}$ Can't avoid T.

$H \Longrightarrow G$ H H H H H H $\Big\}$ Choose H, get G too.

$R + G \Longrightarrow V$

Certain combinations of symmetries force other symmetries

T (always)   T T T T T T $\Big\}$ Can't avoid T.

$H \Longrightarrow G$ H H H H H H $\Big\}$ Choose H, get G too.

$R + G \Longrightarrow V$ F • ⅃

Certain combinations of symmetries force other symmetries

T (always)

T T T T T T $\Big\}$ Can't avoid T.

$H \Longrightarrow G$

H H H H H H $\Big\}$ Choose H, get G too.

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Certain combinations of symmetries force other symmetries

T (always)   T T T T T T $\Big\}$ Can't avoid T.

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Certain combinations of symmetries force other symmetries
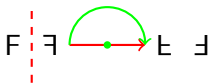
T (always)   ⊤ ⊤ ⊤ ⊤ ⊤ ⊤   } Can't avoid T.

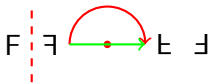$H \implies G$ H H H H H H   } Choose H, get G too.

$R + G \implies V$

$V + G \implies R$

Certain combinations of symmetries force other symmetries

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Certain combinations of symmetries force other symmetries

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⊤ ⊤ ⊤ ⊤ ⊤ ⊤ } Can't avoid T.

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Certain combinations of symmetries force other symmetries
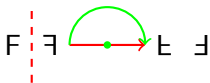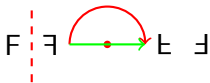
T (always)   T T T T T T $\}$ Can't avoid T.

$H \Longrightarrow G$ H H H H H H $\}$ Choose H, get G too.

$R + G \Longrightarrow V$

$V + G \Longrightarrow R$

Certain combinations of symmetries force other symmetries

T (always)   T T T T T T $\big\}$ Can't avoid T.

$H \Longrightarrow G$ H H H H H H $\big\}$ Choose H,
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$R + G \Longrightarrow V$

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Certain combinations of symmetries force other symmetries

T (always)   ⊤ ⊤ ⊤ ⊤ ⊤ ⊤   } Can't avoid T.

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$V + G \Longrightarrow R$

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$R + G \Longrightarrow V$

$V + G \Longrightarrow R$

$V + R \Longrightarrow G$

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T (always)   ⊤ ⊤ ⊤ ⊤ ⊤ ⊤   } Can't avoid T.

$H \Longrightarrow G$ H H H H H H   } Choose H, get G too.

$R + G \Longrightarrow V$

$V + G \Longrightarrow R$

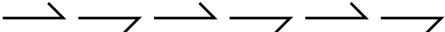$V + R \Longrightarrow G$

} Choose any two: $\{R, V, G\}$ Get all three.
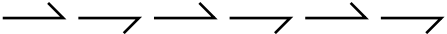
There are seven possible combinations of frieze symmetries:

There are seven possible combinations of frieze symmetries:

1. T   —➤ —➤ —➤ —➤ —➤ —➤
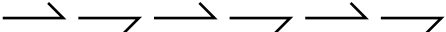
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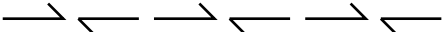
1.   T

2.   TG

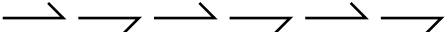**There are seven possible combinations of frieze symmetries:**
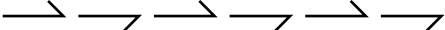
1. T $\longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow$

2. TG $\longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow \longrightarrow$

3. TR $\longrightarrow \longleftarrow \longrightarrow \longleftarrow \longrightarrow \longleftarrow$

There are seven possible combinations of frieze symmetries:

1.   T

2.   TG

3.   TR

There are seven possible combinations of frieze symmetries:
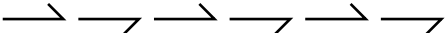
1.   T

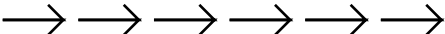2.   TG

3.   TR

4.   TV

There are seven possible combinations of frieze symmetries:

1. T

2. TG

3. TR

4. TV

There are seven possible combinations of frieze symmetries:

1. T

2. TG

3. TR

4. TV

5. TGH

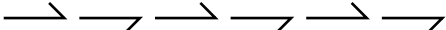**There are seven possible combinations of frieze symmetries:**

1. T

2. TG

3. TR

4. TV

5. TGH

6. TGRV

There are seven possible combinations of frieze symmetries:

1. T

2. TG

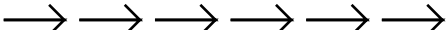3. TR

4. TV

5. TGH

6. TGRV

There are seven possible combinations of frieze symmetries:

1. T

2. TG

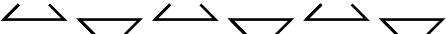3. TR

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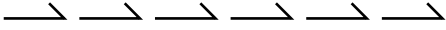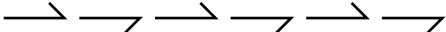
1. T

2. TG

3. TR

4. TV

5. TGH

6. TGRV

7. TGRVH

There are seven possible combinations of frieze symmetries:

1. T

2. TG

3. TR

4. TV
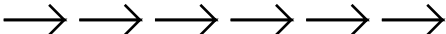
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6. TGRV

7. TGRVH

There are seven possible combinations of frieze symmetries:

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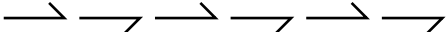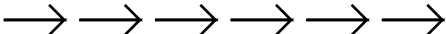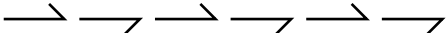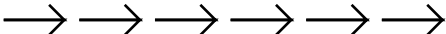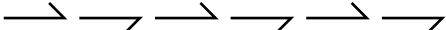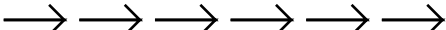
5. TGH

6. TGRV

7. TGRVH

Example:
What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:

What is the symmetry type?

Translation?   YES

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	YES

Example:
What is the symmetry type?

Translation?   YES
Glide Reflection?   NO
Rotation?   YES
Vertical Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	YES
Vertical Reflection?	NO

Example:
What is the symmetry type?

Translation?   YES
Glide Reflection?   NO
Rotation?   YES
Vertical Reflection?   NO
Horizontal Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	YES
Vertical Reflection?	NO
Horizontal Reflection?	NO

Example:

What is the symmetry type?

Translation?   YES
Glide Reflection?   NO
Rotation?   YES
Vertical Reflection?   NO
Horizontal Reflection?   NO

Symmetry Type: **TR**

Example:
What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:

What is the symmetry type?

Translation?   YES

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?   NO

Rotation?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	YES

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?   NO

Rotation?   NO

Vertical Reflection?   YES

Horizontal Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	YES
Horizontal Reflection?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	YES
Horizontal Reflection?	NO

Symmetry Type: **TV**

Example:
What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:
What is the symmetry type?

Translation?   YES

Example:
What is the symmetry type?

Translation?   YES
Glide Reflection?

Example:

What is the symmetry type?

Translation?　　　　　　YES

Glide Reflection?　　　　YES

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?   YES

Rotation?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO

Example:

What is the symmetry type?

Translation?   YES
Glide Reflection?   YES
Rotation?   NO
Vertical Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?   YES

Rotation?   NO

Vertical Reflection?   NO

Horizontal Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	YES

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	YES

Symmetry Type: **TGH**

Example:
What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:
What is the symmetry type?

Translation?   YES

Example:
What is the symmetry type?

Translation?   YES

Glide Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	NO

Symmetry Type: **TG**

Example:
What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:
What is the symmetry type?

Translation?   YES

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?

Example:

What is the symmetry type?

Translation?   YES

Glide Reflection?   NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	NO

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	NO
Rotation?	NO
Vertical Reflection?	NO
Horizontal Reflection?	NO

Symmetry Type: **T**

Example:
What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:

What is the symmetry type?

Translation?   YES

Example:
What is the symmetry type?

Translation?   YES
Glide Reflection?

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	

Example:

What is the symmetry type?

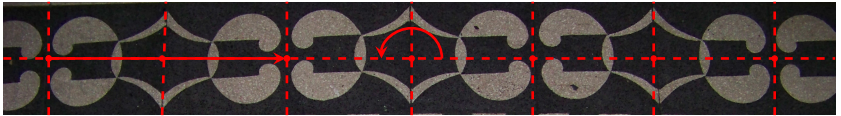|	
Translation?	YES
Glide Reflection?	YES
Rotation?	YES

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES
Horizontal Reflection?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES
Horizontal Reflection?	YES

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES
Horizontal Reflection?	YES

Symmetry Type: **TGRVH**

Example:
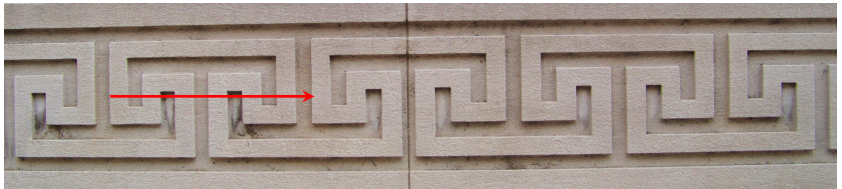What is the symmetry type?

Example:
What is the symmetry type?

Translation?

Example:

What is the symmetry type?

Translation?   YES

Example:

What is the symmetry type?

Translation?   YES

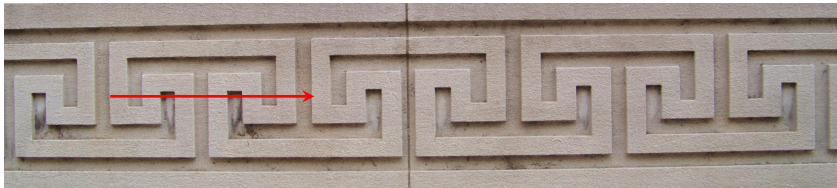Glide Reflection?

Example:

What is the symmetry type?

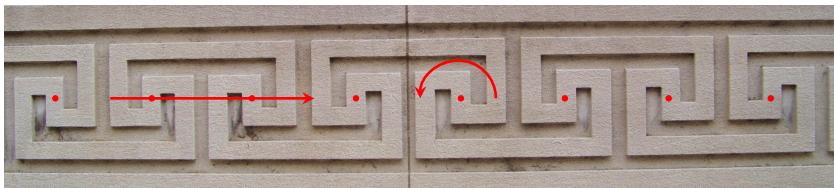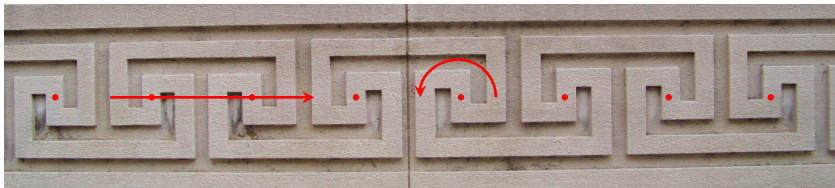| Translation? | YES |
| Glide Reflection? | YES |

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES

Example:

What is the symmetry type?

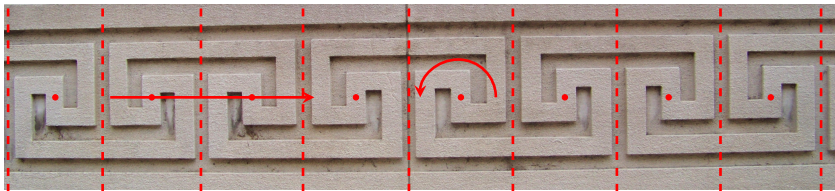|	
Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	

Example:
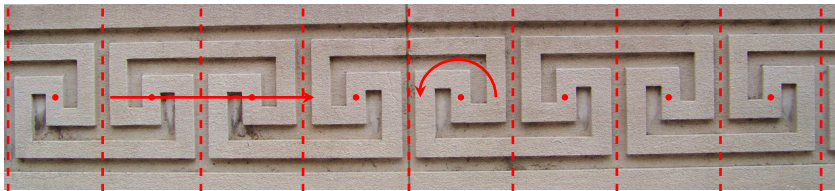
What is the symmetry type?

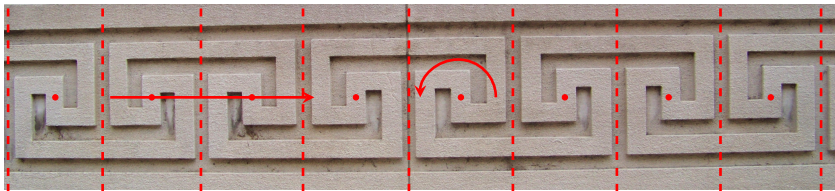|	
Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES

Example:

What is the symmetry type?

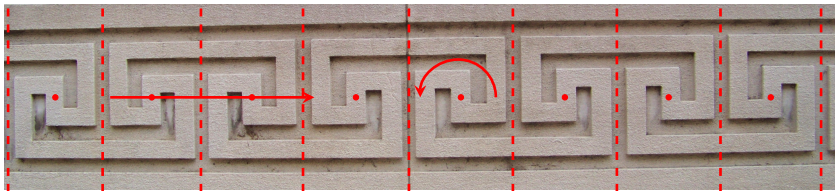|	
Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES
Horizontal Reflection?	

Example:

What is the symmetry type?

Translation?   YES
Glide Reflection?   YES
Rotation?   YES
Vertical Reflection?   YES
Horizontal Reflection?   NO
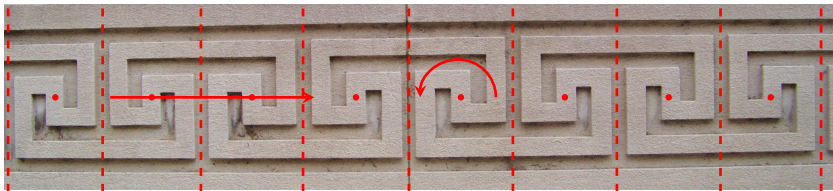
Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES
Horizontal Reflection?	NO

Symmetry Type: **TGRV**

Example:

What is the symmetry type?

Translation?	YES
Glide Reflection?	YES
Rotation?	YES
Vertical Reflection?	YES
Horizontal Reflection?	NO

Symmetry Type: **TGRV**

Assignment #2

http://www.people.vcu.edu/~rhammack/Math122/