Piero della Francesca's Diagonal Construction

Lecture notes for MATH 121
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   Piero della Francesca (1412–1492) invented a number of different ways to render three-dimensional objects in perspective. This handout describes one of those methods, called the *Diagonal Construction*. It extends Alberti's idea of the diagonal check to a method of drawing various three-dimensional objects in perspective. Figure 1 shows two diagrams from Piero's book *De Prospectiva Pingendi* (On perspective for Painting) showing views of a cube and an octagonal prism constructed with this method.
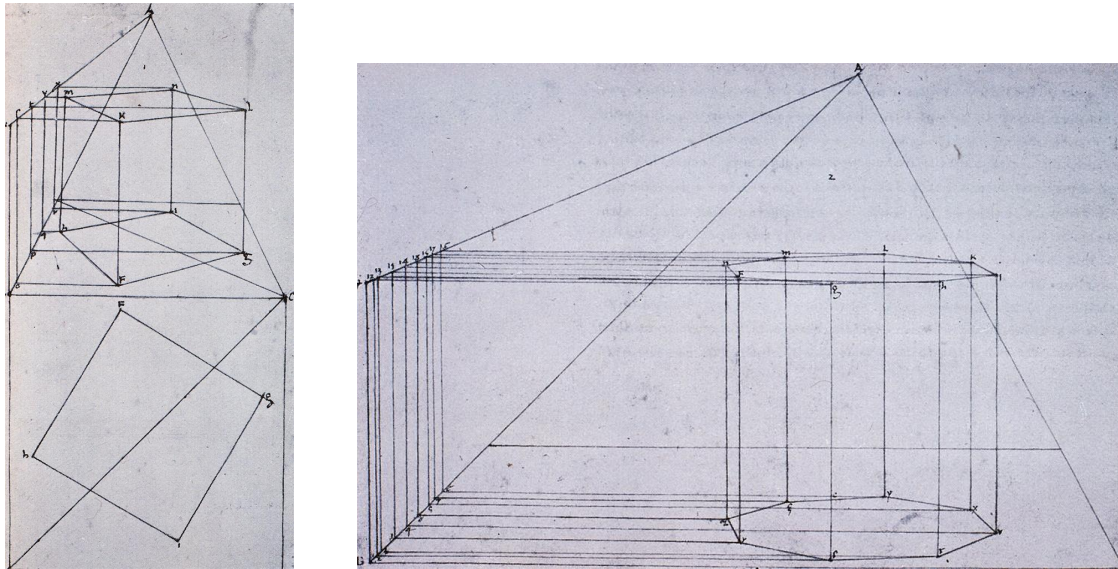

Figure 1: Simple objects rendered with Piero's Diagonal Construction

   Piero saw basic objects like these as building blocks for more complex objects, as illustrated in Figure 2. Being able to draw the simple objects meant that he could draw arbitrarily complex objects as well.
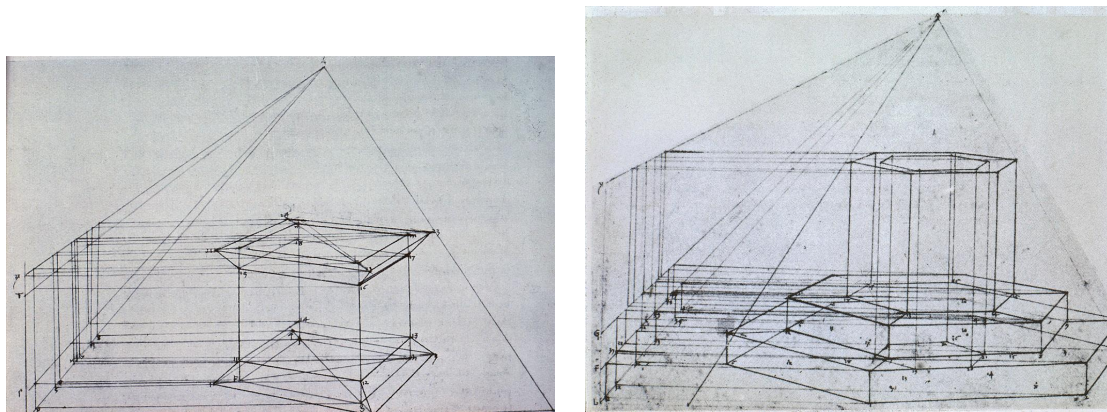

Figure 2: More complex objects built from simple objects

   This handout describes how to use the Diagonal Construction to draw simple objects like cubes in correct perspective. If you find it useful, you can then use the method in your own work to build up complex objects and scenes.

Piero's Diagonal Construction involves creating a kind of "stage" on which objects can be placed and then rendered. The basic setup, sketched on the right, involves a kind of split-veiw diagram of the floor-plan of the stage combined with a picture plane on which is drawn the perspective image of the floor-plan. The square $GRTS$ is a floor-plan of of the stage. Think of the segment GR as the front of the stage, and ST as the back. The upper rectangle $GRCB$ is a picture plane which features a perspective view of the floor-plan $GRTS$ done with the alternative to Alberti's Method. (See class notes.) The segment $S'T'$ is the perspective view of the back edge ST of the stage. The *ground-line GR* is common to both the floor-plan and its image. (Imagine folding along line GR until ST matches up with $S'T'$.)
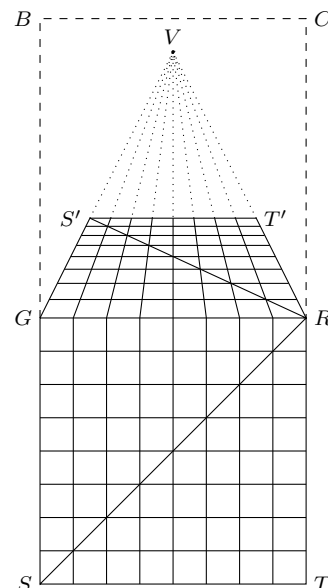
One of the major pieces of Piero's Diagonal involves answering the following question. Given a point P on the floor-plan $GRTS$ (see Figure 3a), where on the image $GRT'S'$ of the floor-plan should the image of P be located?
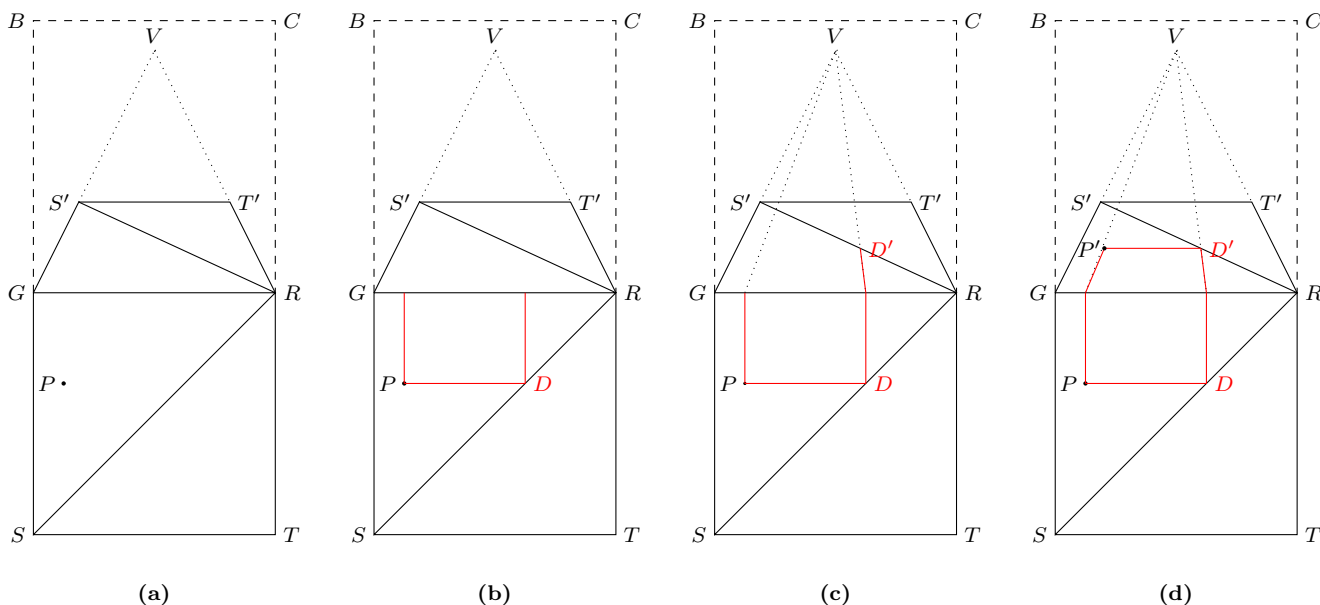

Figure 3: Locating the image of a point P

Piero answered the question as follows. From P, extend a line vertically to line GR. Then extend another line horizontally from P until it touches the diagonal SR at a point D. (See Figure 3b.) From D extend another line vertically to meet GR, thus forming a rectangle whose bottom corners are P and D and whose top edge lies on GR.

Next (Figure 3c) lightly draw lines from the vanishing point V to the upper corners of the rectangle from the previous step. One of these lines crosses the diagonal $S'R$ at a point D'. From D', extend a line horizontally to form a point P' as indicated in Figure 3d. Point P' is the perspective image of P.

Piero's Diagonal Construction employs multiple applications of this procedure for finding P'. It can be divided into two parts: constructing the base of the object being drawn, and constructing the elevation. We'll treat these two parts separately.

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1 Constructing the Base

Imagine that you place a box on the stage and it touches the floor at the rectangle illustrated in the diagram on the right. Your goal is to draw a perspective image of the box in the picture plane $GRCB$. (Note: This method will work equally well for shapes that are more complicated than a rectangular box, but we're using a box to make the discussion easier.)

The first step in drawing the perspective image of the box is to draw its base in perspective. This can be done by using the method on the previous page to find the locations of the images of the four corners of the box's base, and then connect the corners by lines.

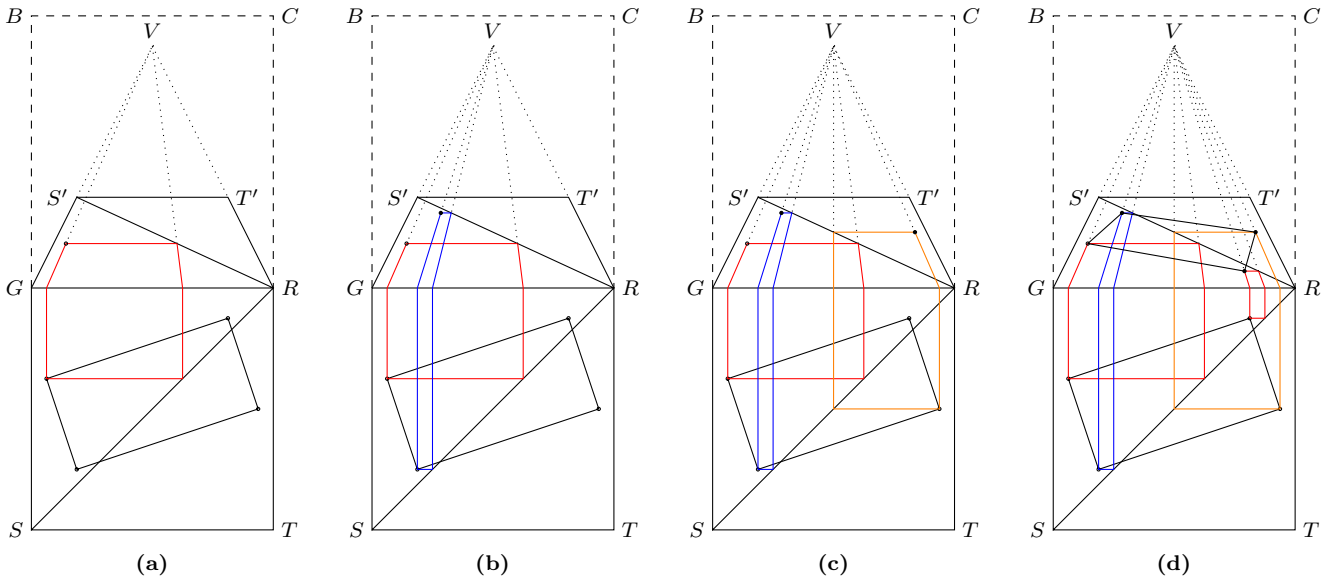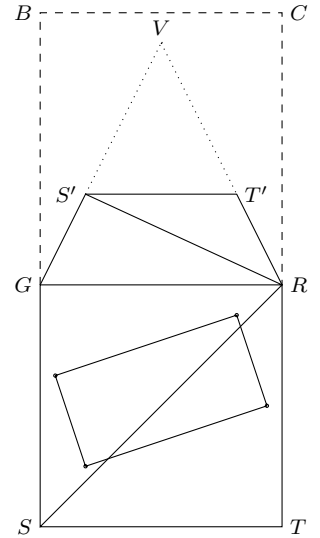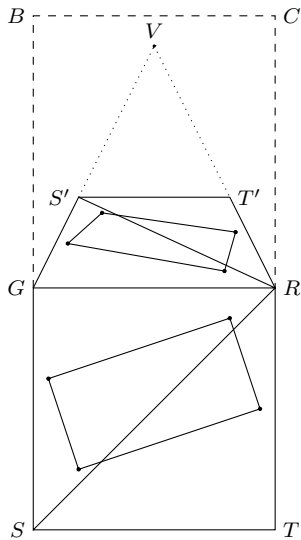This is illustrated in four steps in figures 4a–d.

Figure 4: Drawing the base

When we erase all the intermediate lines, we get the image shown on the left. The base of the box is now drawn in correct perspective in the upper part of the diagram.

At this point we are ready to draw the upper part (i.e. the elevation) of the box.

2   Drawing the Elevation

Here is the procedure for drawing the upper part of the cube. Begin by drawing a vertical segment GK that is equal to the box's height (as measured on the picture plane). From K, draw a segment towards V, and form a "wall" on the left of the diagram, as illustrated.

The next steps are indicated in Figure 5a–d.

From a corner of the base of the box, draw a horizontal line to the left wall. From there draw a vertical line to the top of the wall, then move horizontally to a point directly over the point you started from. The starting and ending points form an edge of the box, so draw a line between them.

Repeat the above procedure for each of the four corners of the base of the box. Then connect the lines at the top of the box.
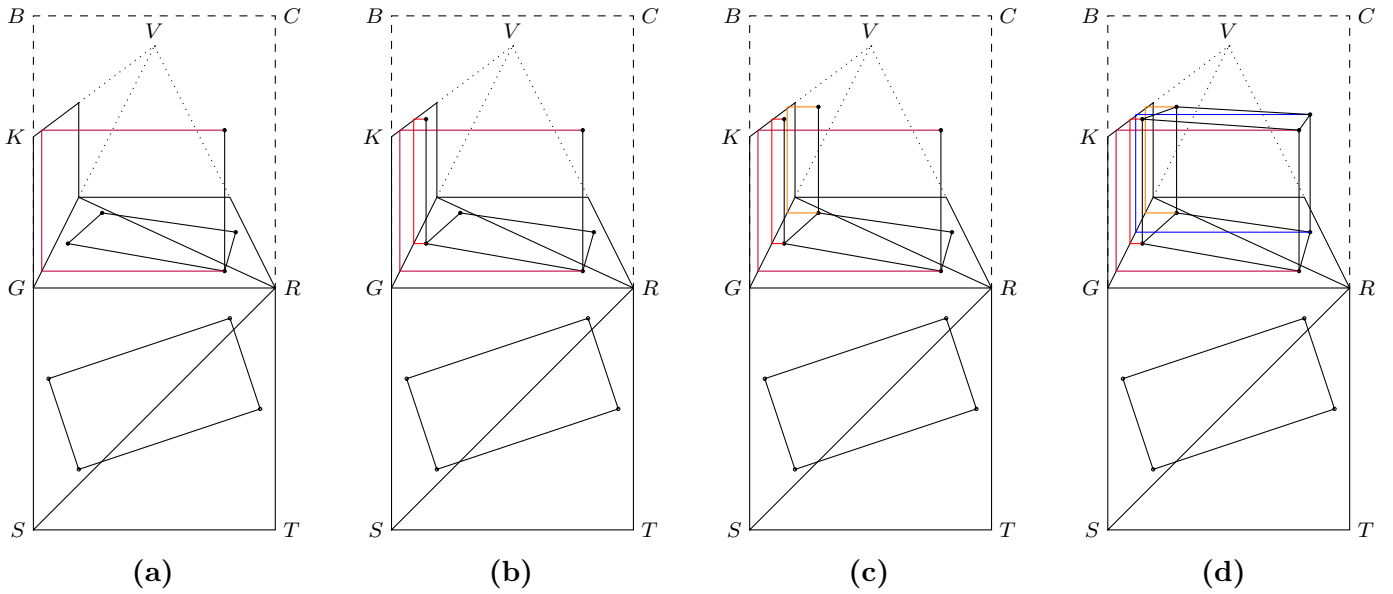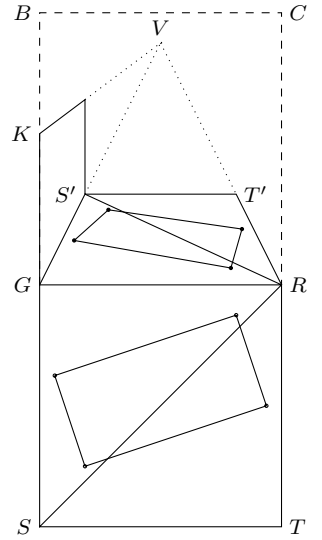

| (a) | (b) | (c) | (d) |

Figure 5: Drawing the elevation

Now you can erase all of the construction lines. You have a drawing of the box in correct perspective.

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