Humboldt Universität zu Berlin | January 25, 2010

Algebraic Geometry I

Lectures by Prof. Dr. R. N. Kloosterman

Exercises by N. Tarasca

Exercise Sheet 12

Hand in solutions not later than Monday, February 1.

Exercise 1. Let x, y, z be homegeneous coordinates on $\mathbb{P}^2$. Determine the Hilbert function of $V(xy) \subset \mathbb{P}^2$.

Exercise 2. Let X be a prevariety over an algebraically closed field $\mathbb{K}$, and let $P \in X$ be a closed point of X. Let D be defined as $\operatorname{Spec} \mathbb{K}[x]/(x^2)$. Show that $X(D) \cong T_{X,P}$.

Exercise 3. Define X as $\operatorname{Spec} \mathbb{K}[x, y, z]/(x^2, y^2, xy, xz, yz)$. Show that there exist two subschemes S and T of X such that $S \cup T = X$, $\dim T = 0$ and $S = X_{red}$, i.e. S and X are the same considered as topological space

Exercise 4. *[Gathmann's notes, Ex. 5.6.13]* Let X be an affine variety, let Y be a closed subscheme of X defined by the ideal $I \subset A(X)$, and let $\widetilde{X}$ be the blow-up of X at I. Show that:

i) $\widetilde{X} = \operatorname{Proj}(\bigoplus_{d \geq 0} I^d)$, where we set $I^0 := A(X)$;

ii) The projection map $\widetilde{X} \to X$ is the morphism induced by the ring homomorphism $I^0 \to \bigoplus_{d \geq 0} I^d$;

iii) The exceptional divisor of the blow-up, i.e. the fiber $Y \times_X \widetilde{X}$ of the blow-up $\widetilde{X} \to X$ over Y, is isomorphic to $\operatorname{Proj}(\bigoplus_{d \geq 0} I^d/I^{d+1})$.