Owner-Intruder contests with correlated resource values

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Abstract. We study a conflict of two individuals over a valuable resource. We construct a sequential game where the first individual, the Owner, decides whether to defend the resource and the other individual, the Intruder, decides whether to attack and try to steal the resource. Individuals know the value of the resource to themselves. This provides the individuals an estimate of the value of the resource to the opponent. We build a mathematical model that allows us to quantify and vary the accuracy of this estimate. We study how the outcomes of the game depend on the accuracy. We show that, in
our setting, the accuracy does not matter to the Intruder but it does to the Owner. For resources of a large value, the Owner benefits from a smaller accuracy. However, for resources of a small value, the Owner benefits from a bigger accuracy.

Keywords: Kleptoparasitism; Owner-Intruder; Resource Valuation; Game Theory; Incomplete information

1. Introduction

Kleptoparasitism, the parasitism by theft [5], is a very common phenomenon observed across different species, including insects [11], fish [15] and most notably birds [19], see [17] for an extensive review.

Many mathematical models of kleptoparasitic interactions have already been built, see [16] for a recent review. Some models have explored detailed and realistic interactions [14, 20, 21] while other models focus on specific aspects such as brood parasitism [10, 8], stochastic modelling in finite population [6], incomplete information [7], or territorial structure [13].

A common way to model kleptoparasitism is an Owner-Intruder game [12]. In this paper, we will follow the Owner-Intruder game presented in [9] and [4]. One individual, the Owner, is in possession of a resource that another individual, the Intruder, wants for themselves. The Owner and Intruder both know the value of the resource for themselves and that the values correlate with each other. However, neither the Owner nor the Intruder know exactly the resource value to the opponent. The mathematical model where we quantify this uncertainty about the opponent’s values is presented in Section 2. The analysis is done in Section 3. The results and numerical simulations can be found in Section 4. We conclude by a discussion in Section 5.
We model the conflict between the Owner and the Intruder as a sequential game as shown in Figure 1. All parameters and symbols are summarized in Table 1.

The Owner is in possession of a resource when it identifies an Intruder. The Owner has two options: to flee and abandon the resource, leaving it in the possession
of the Intruder, or to stay and defend the resource. If the Owner decides to defend, the Intruder then has two options: to flee, abandoning their attempt to gain the resource and leaving it in the Owner’s possession, or to attack the Owner for the resource. If the Intruder flees, the Owner keeps the resource and neither individual pays any cost. If the Intruder attacks, a fight results. The Owner wins the fight with probability $a$; the Intruder wins with probability $1 - a$. The winner will gain the resource, the loser will gain nothing. Both individuals will have to pay a cost $c$ for the fight.

The same resource may have a different value to different individuals. Let $U_O$ denote the utility of the resource to the Owner and $U_I$ the utility of the same resource to the Intruder. To model the fact that knowing $U_O$ provides a reasonably accurate estimate of $U_I$ (and vice versa), we will assume that

$$U_O = wV + (1 - w)S_O$$
$$U_I = wV + (1 - w)S_I$$

where $V$ could be thought of as the objective value for the resource for both the Owner and Intruder, for example, a calorie content, $S_O$ and $S_I$ the subjective values for the Owner and Intruder, respectively, and $w$ the measure of the accuracy. We assume that $S_O$ and $S_I$ are independent, identically distributed random variables with a normal distribution. Figure 2 demonstrates the relationship between $w$ and the correlation between $U_O$ and $U_I$, given by

$$\text{corr}(U_O, U_I) = \frac{E\left[(U_O - E[U_O])(U_I - E[U_I])\right]}{\sigma_{U_O} \sigma_{U_I}}$$

where $E[X]$ is the expected value of a random variable $X$, and $\sigma_X$ denotes the variance of $X$. Decreasing the variance of $S_O$ and $S_I$ corresponds to increasing $w$ and vice versa; we can thus assume that the variance is 1. Tuning $w$ from 0 to 1
allows us to smoothly transition from the partial information case (not knowing anything about the opponents value) to the full information case (knowing the opponent’s value exactly) which were both studied in [4], [9] and [8].

Throughout this paper, we will consider that the Owner knows $U_O$, $w$, $c$, and $a$ and the Intruder knows $U_I$, $w$, $c$, and $a$.

3. Analysis

The Owner has two options, to defend or to flee. If it does not defend the resource, the Owner will get 0. If the Owner defends the resource, its payoff will then depend on the action of the Intruder. So we have to find the Intruder’s optimal behavior first. As in [9] and [4], the Intruder should attack only if $0 < (1 - a)U_I - c$, i.e. if the payoff for not attacking (0) is smaller than the expected payoff of attacking (paying the cost $-c$ and winning $U_I$ with probability $1 - a$). This is equivalent to

$$\frac{c}{1 - a} < U_I. \quad (3.1)$$

Note that the Intruder does not need to know $U_O$. 

Fig. 2: Correlation between the Owner and Intruder values. Left: relationship between $w$ and corr($U_O$, $U_I$). Right: specific example of 100 random draws of $U_O$ and $U_I$ for $w = 0.6$. In both cases, $U_O$ and $U_I$ are given by (2.1) and (2.2) with $V$, $S_O$ and $S_I$ all being random variables following a normal distribution with a mean 3 and variance 1.
Let $\pi_I$ denote the probability that the Intruder will attack (given the Owner defends), i.e.

$$\pi_I = \text{Prob} \left( \frac{c}{1-a} < U_I \right). \quad (3.2)$$

While the Owner does not know $U_I$, they know $U_O$ and can thus estimate $U_I$. By (2.1) and (2.2),

$$U_I = U_O + (1-w)(S_I - S_O) \quad (3.3)$$

and thus

$$\pi_I = \text{Prob} \left( \frac{1}{1-w} \left[ \frac{c}{1-a} - U_O \right] < S_I - S_O \right). \quad (3.4)$$

Since $S_O$ and $S_I$ are independent normally distributed random variables with the same mean and variance 1, their difference $(S_I - S_O)$ is a normally distributed random variable with mean 0 and variance 2 [18]. Consequently, formula (3.4) is a way for the Owner to evaluate $\pi_I$ from the known $U_O, a, c, w$.

Let us now consider what happens when the Owner defends. If the Intruder does not attack (which happens with probability $1 - \pi_I$), then the Owner keeps the resource without a fight and gets $U_O$. If the Intruder is going to attack (which happens with probability $\pi_I$), there will be a fight. With probability $a$ the Owner will win the fight and get $U_O - c$. With probability $1 - a$, the Owner will lose the fight and get $-c$. Consequently, the Owner should defend if

$$U_O (1 - (1-a)\pi_I) - \pi_I c > 0. \quad (3.5)$$

In particular, since $\pi_I \leq 1$ and the left-hand side of (3.5) is decreasing in $\pi_I$, the Owner should defend if $0 < U_O (1 - (1-a)) - c = aU_O - c$. 

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We note that the expected Owner’s payoff (given the Owner already knows the value of the resource $U_O$) is given by

$$P_O = \max\left\{0, U_O \left(1 - (1 - a)\pi_I \right) - \pi_I c\right\}.$$  \hfill (3.6)
4. Results

We show the results from the perspective of the Owner, i.e. dependent on the values of $U_O$, $c$, $a$ and $w$.

The numerical simulations for the probability that the Intruder will attack, $\pi_I$, given in (3.4), are shown in Figures 3a-3c. It follows from (3.4) that $\pi_I$ is constant along the lines $w = 1 - K\left(c/(1-a) - U_O\right)$ where $K \neq 0$ is an arbitrary non-zero constant. All such lines go through the point $U_O = c/(1-a)$ and $w = 1$. Clearly, for $w > 0$, $\pi_I$ is increasing in $U_O$ (the larger the value of the resource for the Owner, the larger the value of the resource is for the Intruder and so the more likely it is that the Intruder is willing to fight for it). Also, $\pi_I$ is decreasing in $w$ when $U_O < c/(1-a)$ and increasing in $w$ otherwise (as the correlation between $U_O$ and $U_I$ grows, the value of $U_O$ predicts the value of $U_I$ with greater accuracy and thus the Owner can predict with less uncertainty whether the Intruder will attack or not). Finally, $\pi_I$ is decreasing in $a$ (the more likely it is that the Owner is going to win the fight, the less likely it is that the Intruder will try to fight), and in $c$ (the larger the cost of the fight, the less inclined the Intruder is to fight).

Next, let us study the Owner’s payoff, $P_O$, given by (3.6). First, and foremost, $P_O$ is increasing in $w$ when $c/(1-a) > U_O$ and decreasing in $w$ if $c/(1-a) < U_O$, see Figures 3d-3f. This is because $\frac{dP_O}{dw} = \left(\frac{\partial P_O}{\partial \pi_I}\right) \left(\frac{d\pi_I}{dw}\right)$ and $\frac{\partial P_O}{\partial \pi_I} < 0$.

Second, $P_O$ is increasing in $a$. This is because $\pi_I$ and $1-a$ are decreasing in $a$, and therefore the terms $-\pi_I$ as well as $-(1-a)\pi_I$ in (3.6) are increasing in $a$. This corresponds to the facts that, with increasing $a$, the Intruder is less likely to attack (i.e. the Owner keeps the resource more often as $a$ increases), and, when the Intruder attacks, the Owner is more likely to win.

The dependence of $P_O$ on $c$ is slightly more complex. As $c$ increases, $\pi_I$ decreases
from $\pi_I \approx 1$ when $c \approx 0$ to $\pi_I \approx 0$ when $c$ is large enough. Consequently, $P_O \approx aU_O - c$ for small $c$ (and thus it is decreasing in $c$) and $P_O = U_O$ for large $c$. For intermediate $c$, $P_O$ is increasing in $c$.

Finally, $P_O$ is increasing in $U_O$ for small $U_O$ ($\pi_I$ is small and thus the Intruder does not attack) and large $U_O$ ($\pi_I$ is large and the Intruder always attacks, the individuals always fight, and the payoff is given by $aU_O - c$). For intermediate $U_O$, $P_O$ is decreasing, especially for large $w$ or small $a$. This is caused by $\pi_I$ being increasing in $U_O$, i.e. the Owner having to fight more often while the increase in rewards ($aU_O$) not being able to compensate enough for the costs of the fights, see Figures 3g-3i.

The Owner should defend under most circumstances, especially when $a \geq 0.5$, see Figures 3d-3f. There are only two scenarios when the Owner should flee: (1) when $U_O$ is small and $w$ is not too large, and (2) when $a < 0.5$, $U_O$ is medium (roughly between $c/(1-a)$ and $c/a$), and $w$ is large. The first case corresponds to the findings for the partial information case in [9, 4]. The second case corresponds to the full information case, because for high $w$, $U_I \approx U_O$ and thus the Owner essentially has information about the resource value for the Intruder.

We note that the Owner should defend when $w$ is relatively high and $U_O$ is relatively low, approximately $U_O < c/(1-a)$. This is because as demonstrated in Figures 3a-3c $U_I$ and so the probability of the Intruder attacking are also low. This means that the Owner is bluffing. They know that by pretending to be prepared to fight, they are likely not going to be attacked as the Intruder sees too little value in the resource to attack.
5. Conclusions and discussion

In this paper we studied the Owner-Intruder game that models conflict between two individuals over a valuable resource. Our model built on and extended the work previously done in [9] and [1]. The individuals knew the value of the resource to themselves. This provided them a reasonable estimate of the value of the resource to the opponent. We built a mathematical model that allowed us to quantify the accuracy of the estimate. We studied how this accuracy influences the individual behaviors and outcome of the game. The Intruder does not need to know the resource value to their opponents and so the accuracy does not affect their decisions. At the same time, there is a clearly defined threshold that determines how the Owner’s payoff depends on the accuracy. If the value of the resource for the Owner is larger than the threshold, then the Owner benefits from smaller accuracy. However, if the value of the resource is smaller than the threshold, the Owner’s payoff increases as the accuracy increases.

Typically, the more information that an individual has, the more beneficial it should be for their outcome. It thus seems counter-intuitive that in our case the Owner benefits from bigger uncertainty. The reason behind this is that when the uncertainty is small, the Intruder values the resource roughly in the same way as the Owner. When the accuracy is high and the resource is valuable to the Owner, it is valuable to the Intruder as well who is thus more likely to attack, decreasing the payoffs to the Owners. When the accuracy is low, the Intruder’s value may be small and the Intruder may thus attack less often.

There are numerous ways in which we can extend and build on our model. Field studies demonstrate that in species where the resource is a territory, the Owners are more likely to win the fights, simply because to become the Owner, they had to
win the territory from the previous Owner [2, 1]. The actual value of the territory may also be objectively different to the Owner and the potential Intruders. For example, the Owners may be more familiar with various aspects of the territory making a territory more valuable to the Owners than to the Intruders. Conversely, the Owner could have already utilized all that was in their abilities, making the territory potentially more valuable to the Intruders than to the Owners. All these plausible scenarios warrant the creation and studies of further and more detailed models.

References


REFERENCES


