Owner-Intruder contests with information asymmetry

Jay Bisen*, Faheem Farooq*, Manaeil Hasan*, Akhil Patelb, Jan Rychtář*, Dewey T. Taylorc

*Department of Biology, Virginia Commonwealth University, Richmond, VA 23284-2012, USA

bDepartment of Biomedical Engineering, Virginia Commonwealth University, Richmond, VA 23284-3068, USA

cDepartment of Mathematics and Applied Mathematics, Virginia Commonwealth University, Richmond, VA 23284-2014, USA

Abstract

We consider kleptoparasitic interactions between two individuals - Owner and Intruder - and model the situation as a sequential game in an extensive form. Owner is in a possession of a valuable resource when it spots Intruder. Owner has to decide whether to defend the resource; if the Owner defends, the Intruder has to decide whether to fight with the Owner. The individuals may value the resource differently and we distinguish three information cases: (a) both individuals know resource values to both of them, (b) individuals know only their own valuation, (c) individuals do not know the value at all. We solve the game in all three cases. We find that it is typically beneficial for the individuals to know as much information as possible. However, we identify several scenarios where knowing less seems better. We also show that an individual may or may not benefit from their opponent knowing less. Finally, we consider the same kind of interactions but with the reversed order of decisions. We find that typically the individual initiating the interaction has an advantage. However, when individuals know only their own valuation and not the valuations to their opponents, it is sometimes better when the opponent initiates.

Keywords: Game theory, Kleptoparasitism, Resource valuation, Information asymmetry

1. Introduction

In nature, kleptoparasitism, the act of stealing and fighting over resources is common among birds (Spear et al., 1999; Steele and Hockey, 1995; Triplet et al., 1999); but it occurs across many types of species such as insects (Jeanne, 1972), fish (Grimm and Klinger, 1996) and mammals (Kruuk, 1972). There are many different behaviors involved in kleptoparasitic interactions. Sometimes individuals will defend and fight for the resource and sometimes the resource is simply forfeited without any conflict (Iyengar, 2008).

There are many models of kleptoparasitic interactions with varying degree of complexity, see for example Broom et al. (2008), Broom and Rychtář (2009), Broom et al. (2010), Barker et al. (2012), Galanter et al. (2017). Many recent models contain a high degree of details and realism, see for example Garay et al. (2017, 2018), Varga et al. (2019a, b). An Owner-Intruder game is a
common way to model kleptoaparasitic interactions; it can account for many different situations and
assumptions while yielding clear and testable predictions (Caraco and Giraldeau 1991, Eshel and
Sansone 1995, Dubois et al. 2003, Dubois and Giraldeau 2005, Mesterton-Gibbons and Sherratt
2014, Cressman and Krivan 2019), see also Sherratt and Mesterton-Gibbons (2015) or Hinsch and
Komdeur (2017) for recent reviews.

The value of the resource has a substantial influence on the interaction (Broom et al. 2014,
Sykes and Rychtar 2017, Broom et al. 2018). Determining the resource value is not an easy task.
For example, the value of a territory depends on factors such as access to food (Maher and Lott
1995), mating opportunities (Sinervo and Lively 1996), refuge from predators (Cowlishaw 1997)
and many others. The subjective value of the resource may also depend on the individual’s Resource
Holding Potential (RHP), (Parker 1974). When the individuals are competing for resources to
survive, the desperate individuals value the resource more than those that are well-fed; this is seen
in the case of Olrog’s Gull Larus atlanticus (Delhey et al. 2001). On the other hand, when the
individuals compete for access to mates, the stronger individuals may value the resource more.
This is seen in the case of the ant colonies with multiple queens (Vehrencamp 1983, Reeve 1993).

When there is a difference in resource valuation, several cases have to be distinguished. First,
individuals may know the resource value to themselves and also to the opponent. For example, the
male ray-finned fish sand goby, Pomatoschistus minutus, compete for nest sites; the value of the site
can be quickly assessed by its size and so can be the opponent’s RHP (Lindstrom 1992). A similar
situation was observed in male paper wasps, Polistes fuscatus, fighting over territory (Polak 1994).
Second, individuals may know how much they value the resource themselves, but do not know the
value to the opponent. This is the case when RHP cannot be easily assessed, for example in the con-
tests between Mozambique mouthbrooder, Oreochromis mossambicus, (Turner and Huntingford
1986), or damselflies, Calopteryx maculata, (Marden and Waage 1990); see Mesterton-Gibbons
et al. (1996) for more details. Finally, individuals may not know the resource value for themselves
or to their opponent. Such scenarios are likely not common, but may theoretically still happen
among animals like the great tit, Parus major, (Krebs 1982) when the contest happens between
the intruder and a new territory owner that does not yet know the true value of the territory.

In this paper, we consider a scenario where one individual, the Owner, has a valuable resource
that another individual, the Intruder, may want to steal. In Section 2, we set up the Owner-Intruder
game. In Section 3, we solve the game for all three information cases. In Section 4, we compare the
outcomes between the cases to see the effect of extra information. In Section 5, we investigate the
affect of the order of individual actions. We conclude our paper by a discussion in Section 6.

2. Model

We model the conflict between the Owner and the Intruder as a sequential game in extensive
form as shown in Figure 1. The notation is summarized in Table 1.

The Owner is in the possession of a resource when it spots an Intruder. The Owner has two
options: O1) it can either flee the area which will result in the Intruder taking over the resource, or
O2) it can stay and defend the resource. In the latter case, the Intruder then has two options: I1)
to flee the area, or I2) to attack the Owner. If the Intruder leaves, the Owner keeps the resource
and no individual pays any cost. If the Intruder attacks, it results in a fight. The Owner wins
the fight with probability \( a \); the Intruder wins with probability \( 1 - a \). The winner will gain the
resource, the loser will gain nothing. Both individuals will have to pay a cost \( c \) for the fight.
The same resource may have a different value to different individuals. Let $V_O$ denote the value of the resource to the Owner and $V_I$ the value of the same resource to the Intruder. We will distinguish three cases: 1) the full information case when the Owner and the Intruder both know the values of $V_O$ and $V_I$, 2) the partial information case when the Owner knows $V_O$, the Intruder knows $V_I$, but neither individual knows the resource value for the opponent, and finally 3) the no information case when neither individual knows $V_O$ nor $V_I$ (but they both know the expected values $E[V_O]$ and $E[V_I]$).

Throughout this paper, we will assume that $V_O$ and $V_I$ are independent identically distributed random variables; in particular $E[V_O] = E[V_I]$. For illustrative and comparison purposes, we assume $V_O$ and $V_I$ are uniformly distributed on $[0, 4]$ as in [Broom et al.] (2013).

3. Analysis

In this section, we analyze the Owner-Intruder game by backward induction, see for example [Broom and Rychtář] (2013) p.187). We will treat each information case separately.
3.1. Full information case

In the full information case, both the Owner and the Intruder know $V_O$ and $V_I$. The Owner has two options - defend the resource or flee the area and give up the resource. The payoff to the Owner depends on the Intruder’s action. So, we first need to find Intruder’s optimal behavior.

If the Owner flees, the Intruder does not need to decide anything, it simply takes the resource. Now, assume that the Owner defends. The Intruder then has two options - to attack or to flee. If the Intruder does not attack, its payoff will be 0. If the Intruder attacks, the individuals will fight. The Intruder will lose the fight (and get payoff $-c$) with probability $a$; the Intruder will win the fight (and get payoff $V_I - c$) with probability $1 - a$. Hence, the Intruder’s expected payoff when attacking is $a(-c) + (1 - a)(V_I - c) = (1 - a)V_I - c$. Consequently, the Intruder should attack only if $0 < (1 - a)V_I - c$ which is equivalent to

$$\frac{c}{1 - a} < V_I. \quad (1)$$

Note that (1) means that the Intruder should fight only if the reward for winning the fight, $V_I$, is more than the cost, $c$, of a single fight, times the expected number of fights until the first win, $\frac{1}{1 - a}$.

Also, note that the Intruder does not need to know $V_O$. All that is relevant to the Intruder is whether or not the Owner decided to defend the resource. We will use this important observation later in Section 4.

Now, we will analyze the decision of the Owner. The Owner has two options, to defend or to flee. If it does not defend the resource, the Owner will get 0. If the Owner defends the resource, its payoff will then depend on the action of the Intruder. If the Intruder does not attack (which, by (1), happens exactly when $\frac{c}{1 - a} > V_I$), then the Owner keeps the resource without a fight and gets $V_O$. In this case, the Owner should thus defend. However, if the Intruder is going to attack (which, by (1), happens exactly when $\frac{c}{1 - a} < V_I$), there will be a fight. With probability $a$ the Owner will win the fight and get $aV_O - c$. With probability $1 - a$, the Owner will lose the fight and get $-c$. Thus, the Owner’s expected payoff is $a(V_O - c) + (1 - a)(-c) = aV_O - c$. Consequently, when $\frac{c}{1 - a} < V_I$, the Owner should fight only if $aV_O - c > 0$ which is equivalent to

$$\frac{c}{a} < V_O. \quad (2)$$

Unlike in the case of the Intruder’s decision, the Owner needs to know the value of the resource for its opponent to make the optimal decision. Similarly to the Intruder’s decision, the inequality (2) means that the Owner should defend only if the reward for winning the fight, $V_O$, is more than the cost of a single fight, $c$, times the expected number of fights until the first win ($\frac{1}{a}$ when Intruders are fighting, or 0 when Intruders flee).

The behavioral outcomes and corresponding payoffs are summarized in Table 2, see also Figure 2. It follows from the Table 2 that the payoffs to the individuals depend on the relationship between the resource values $V_O$ and $V_I$ and the quantities $\frac{c}{a}$ and $\frac{c}{1 - a}$.

3.2. Partial information case

In this section, we assume that (a) the Owner knows $V_O$ but not $V_I$ and (b) the Intruder knows $V_I$ but not $V_O$. We will again proceed with the analysis by backward induction.
Figure 2: Behavioral outcomes of the game for the different information cases. The payoffs depend on the behavior of the Owner and Intruder and are as follows: 1) when Owner defends and Intruder flees: $P_O = V_O$, $P_I = 0$, 2) when Owner defends and Intruder attacks: $P_O = aV_O - c$, $P_I = (1-a)V_I - c$, 3) when Owner flees and Intruder takes the resource: $P_O = 0$, $P_I = V_I$.

### Table 2: Summary of behavioral outcomes and payoffs depending on the information case and conditions on $V_I$ and $V_O$.

<table>
<thead>
<tr>
<th>Behavior and Payoffs</th>
<th>Full information</th>
<th>Partial information</th>
<th>No information</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Owner</strong></td>
<td><strong>Intruder</strong></td>
<td>$\frac{c}{1-a} &gt; V_I$</td>
<td>any $V_O$</td>
</tr>
<tr>
<td>Defends</td>
<td>Flees</td>
<td>$\frac{c}{1-a} &lt; V_I$</td>
<td>$\frac{\pi_I c}{1-\pi_I (1-a) \pi_I} &lt; V_O$</td>
</tr>
<tr>
<td>$V_O$</td>
<td>0</td>
<td>$\frac{c}{1-a} &lt; V_I$</td>
<td>$\frac{\pi_I c}{1-\pi_I (1-a) \pi_I} &gt; V_O$</td>
</tr>
<tr>
<td>Defends $aV_O - c$</td>
<td>Attacks $(1-a)V_I - c$</td>
<td>$\frac{c}{1-a} &gt; V_I$</td>
<td>$\frac{\pi_I c}{1-\pi_I (1-a) \pi_I} &gt; V_O$</td>
</tr>
<tr>
<td>0</td>
<td>Takes over $V_I$</td>
<td>$\frac{c}{1-a} &gt; V_I$</td>
<td>any $V_I$</td>
</tr>
</tbody>
</table>
If the Owner decides to flee, the Intruder simply takes over the resource. If the Owner stays and defends, then the Intruder has to decide whether to attack or not. Exactly as in the previous section, we conclude that the Intruder should attack only if

\[
\frac{c}{1-a} < V_I. \tag{3}
\]

Now, we will analyze the decision of the Owner. Let \( \pi_I \) be the probability that the Intruder is going to attack. By (3), we get

\[
\pi_I = \text{Prob}\left(\frac{c}{1-a} < V_I\right). \tag{4}
\]

The Owner has two options, to defend or to flee. If it flees, it will get 0. If the Owner defends, its payoff will then depend on the action of the Intruder. The Intruder does not attack with the probability \( 1 - \pi_I \). In this case, the Owner keeps the resource without a fight and gets \( V_O \). The Intruder attacks with probability \( \pi_I \). Then, with probability \( a \), the Owner wins the fight and gets \( aV_O - c \). With probability \( 1-a \), the Owner loses the fight and gets \( -c \). Thus, the Owner’s expected payoff is

\[
(1 - \pi_I)V_O + \pi_I(a(V_O - c) + (1 - a)(-c)) = V_O(1 - (1-a)\pi_I) - \pi_Ic. \tag{5}
\]

Consequently, the Owner should defend only if \( V_O(1 - (1-a)\pi_I) - \pi_Ic > 0 \) which is equivalent to

\[
\frac{\pi_Ic}{1 - (1-a)\pi_I} < V_O. \tag{6}
\]

The behavioral outcomes and corresponding payoffs are summarized in Table 2, see also Figure 2.

### 3.3. No information case

In this section we assume that neither the Owner nor the Intruder knows the value of the resource (neither for themselves, nor for their opponent). They do know, however, that the expected value of the resource for either one of them is \( E[V] \).

The analysis follows the steps in Section 3.1. The only difference is that here we need to use \( E[V_I] \) instead of \( V_I \) and \( E[V_O] \) instead of \( V_O \) whenever the individual decisions are considered. While \( E[V_O] = E[V_I] \), we will keep the distinction between the two to make it clearer which condition relates to which decision.

As in Section 3.1, the Intruder should attack only if \( 0 < (1-a)E[V_I] - c \) which is equivalent to

\[
\frac{c}{1-a} < E[V_I] \tag{7}
\]

and the Owner should defend only if \( aE[V_O] - c > 0 \) which is equivalent to

\[
\frac{c}{a} < E[V_O]. \tag{8}
\]

The behavioral outcomes and corresponding payoffs are summarized in Table 2, see also Figure 2.
4. Comparison between different information cases

In this section we provide the comparison between the full, partial, and no information cases for Owners and Intruders. The summary is provided here, the details are then provided in Section 4.1 and Section 4.2. The payoffs to the Owner (resp. Intruder) in the full, partial, and no information case will be denoted $P^F_O$, $P^P_O$, $P^N_O$ (resp. $P^F_I$, $P^P_I$, $P^N_I$).

Under all information cases we have $P^F_I \geq P^O_I$ i.e. for the Owner, once it knows the value of $V_O$, it is always beneficial to know the value of $V_I$.

The situation is different for the Intruder. Note that since the Intruder does not need to know the value of $V_O$ for its decision, the comparison between the full and partial information cases is effectively a comparison between a case when the opponent knows more and a case when the opponent knows less.

If $V_I$ is small, it is better for the Intruder if the Owner does not know $V_I$. Indeed, in the partial information case, if the Owner does not know that $V_I$ is small (i.e. not worth it for the Intruder to fight), the Owner will give up when $V_O$ is small because it does not want to risk a fight. In the full information case, the Owner would know that Intruder would not fight and thus Owner would “bluff” by being ready to defend.

Using a similar argument, if $V_I$ is large, it is better for the Intruder if the Owner knows $V_I$. Indeed, in this case the Owner knows that the Intruder is going to fight and thus the Owner may decide to give up. In the partial information case, the Owner does not know $V_I$ and may thus defend which would force the Intruder to fight rather than get the resource for free.

Now we will compare against the no information cases. For the Owner, the no information case is best when the cost of the fight is relatively high, specifically when $c > (1-a)E[V_I]$. In this case, the Intruder is not going to risk the fight. The Owner knows it and consequently bluffs by defending the item that may not be beneficial to defend in a real fight. However, when the cost of the fight is low, no information case is not beneficial for the Owner.

For the Intruder, the no information case is best only when $a < 0.5$ and only when the fight costs are intermediate. Specifically, when $aE[V_O] < c < (1-a)E[V_I]$, the cost is high enough for the Owner not to fight but low enough for the Intruder to fight if needed. For other fight costs, other information cases are better for the Intruder.

4.1. Comparison between full and partial information cases

First, assume that $\frac{c}{1-a} > V_I$. In this case, $P^F_O = V_O$. At the same time, $P^P_O$ is either $V_O$ if $\frac{\pi F}{1-(1-a)\pi_I} < V_O$ or it is 0 if $\frac{\pi F}{1-(1-a)\pi_I} > V_O$. Similarly, $P^F_I = 0$ and at the same time, $P^P_I$ is either 0 if $\frac{\pi F}{1-(1-a)\pi_I} < V_O$ or it is $V_I$ if $\frac{\pi F}{1-(1-a)\pi_I} > V_O$. In either case, when $\frac{c}{1-a} > V_I$, we get that $P^F_O \leq P^P_O$ and $P^F_I \geq P^P_I$.

Second, assume that $\frac{c}{1-a} < V_I$ and $\frac{\pi F}{1-(1-a)\pi_I} < V_O$. Then $P^F_O = aV_O - c$. At the same time, $P^P_O$ is either $aV_O - c$ if $\frac{c}{a} < V_O$ or it is 0 if $\frac{c}{a} > V_O$. In the latter case, $P^F_O = aV_O - c < a\frac{c}{a} - c = 0 = P^P_O$.

Similarly, $P^F_I = (1-a)V_I - c$ and at the same time, $P^P_I$ is either $(1-a)V_I - c$ if $\frac{c}{a} < V_O$ or it is $V_I$ if $\frac{c}{a} > V_O$. In the latter case, $P^F_I = (1-a)V_I - c < V_I = P^F_I$. So, in either case, when $\frac{c}{1-a} < V_I$ and $\frac{\pi F}{1-(1-a)\pi_I} < V_O$, we get that $P^F_O \leq P^P_O$ and $P^F_I \leq P^P_I$.

Finally, assume that $\frac{c}{1-a} < V_I$ and $\frac{\pi F}{1-(1-a)\pi_I} > V_O$. Then $P^F_O = 0$. Since $\frac{c}{1-a} \geq \frac{\pi F}{1-(1-a)\pi_I}$ with the equality occurring only if $\pi_I = 1$, we have that $\frac{c}{a} > V_O$ and thus $P^P_O = 0$. Similarly, $P^F_I = V_I$.

Since $\frac{c}{1-a} \geq \frac{\pi F}{1-(1-a)\pi_I}$ with the equality occurring $\pi_I = 1$, we have that $\frac{c}{a} > V_O$ and thus $P^P_I = V_I$.

Consequently, when $\frac{c}{1-a} < V_I$ and $\frac{\pi F}{1-(1-a)\pi_I} > V_O$, $P^F_O = P^P_O$ and $P^F_I = P^P_I$. 7
To summarize, by considering that the above three cases exhaust all the options, we get that $P^F_O \leq P^F_I$ under all circumstances. Also, $P^F_I \geq P^F_F$ if $V_I < \frac{c}{1-a}$ and $P^F_P \leq P^F_F$ if $V_I > \frac{c}{1-a}$.

In particular, it is always beneficial for the Owner to know the value of $V_I$. At the same time, it may sometimes be beneficial and sometimes detrimental for the Intruder if the Owner knows the value of $V_I$. On average, however, $P^F_P \leq P^F_F$. Figure 3 shows the mean payoffs to the Owners and Intruders under different scenarios.

Note that $P^F_O \geq 0$. However, we saw that it is possible to have $P^F_O < 0$ when $\frac{c}{1-a} < V_I$ and $\frac{\pi_I c}{1-(1-a)\pi_I} < V_O$. In fact, as shown in Figure 4, there can be quite a significant difference between the mean and the median payoff in the full information case as well as in the partial information case. Note that $P^F_I \geq 0$ and $P^F_F \geq 0$.

### 4.2. Comparison between the no information case and the other cases

First assume that $c$ is large, specifically $c > (1 - a) \max\{V_I\}$. Then we always have $\frac{c}{1-a} > V_I$ and thus $P^F_O = V_O$ and $P^P_O = 0$. Also, $\pi_I = 0$ and so $P^F_O = V_O$ and $P^P_O = 0$. We also have $\frac{c}{1-a} > E[V_I]$ and so $P^F_O = V_O = P^F_O = P^F_F$ and $P^N_O = 0 = P^N_F = P^N_I$.

Second, assume that $(1-a)E[V_I] < c < (1-a) \max\{V_I\}$. Then $\frac{c}{1-a} > E[V_I]$ and so $P^F_O = V_O$ and $P^N_I = 0$. Consequently, $P^N_O \geq P^F_O \geq P^P_O$ and $P^P_I \leq P^F_F$ and $P^N_I \leq P^F_F$. As shown in Figure 3 on average, here
Figure 4: Percentiles of payoffs to the Owner under different information cases. Here \( c \) varies, \( a = 0.4 \) (top) and \( a = 0.6 \) (bottom), \( V_O \) and \( V_I \) are drawn from the uniform distribution on \([0, 4]\). All three information cases are shown: full information (left), partial information (center) and no information (right).

Figure 5: Percentiles of payoffs to the Intruder under different information cases. Here \( c \) varies, \( a = 0.4 \) (top) and \( a = 0.6 \) (bottom), \( V_O \) and \( V_I \) are drawn from the uniform distribution on \([0, 4]\). All three information cases are shown: full information (left), partial information (center) and no information (right).
$$V_O = P_O^N = E[P_O^N] > E[P_O^F] > E[P_I^F],\quad$$ (9)

$$0 = P_I^N = E[P_I^N] < E[P_I^F] < E[P_I^P].\quad$$ (10)

Third, assume that $aE[V_O] < c < (1 - a)E[V_I]$. Here, $P_O^N = 0$ and $P_I^N = V_I$. At the same time, we always have $P_O^P \geq 0$ and $P_I^F \leq V_I$ so $P_O^F \leq P_I^F$ and $P_I^P \geq P_I^N$. The inequalities can be strict. Also, on this interval, $P_I^P$ can be either positive or negative depending on the exact values of the resource. As shown in Figure 3, on average, here

$$0 = P_O^N = E[P_O^N] < E[P_O^P] < E[P_I^F],\quad$$ (11)

$$V_I = P_I^N = E[P_I^N] > E[P_I^F] > E[P_I^P].\quad$$ (12)

Fourth, assume that $c < aE[V_O]$. We will distinguish three cases to understand the relationship between $P_O^N$, $P_I^N$ and $P_I^F$, $P_I^P$. Case a: If $V_I < \frac{c}{1-a}$, then $P_I^F = V_O$ and $P_I^F = 0$ and thus $P_I^F > P_I^N$ and $P_I^F = 0 > (1 - a)V_I - c = P_I^N$. Case b: If $V_I > \frac{c}{1-a}$ and $V_O > \frac{c}{a}$, then $P_I^F = aV_O - c$ and $P_I^F = (1 - a)V_I - c$ and thus $P_O^F = P_O^N$ and $P_I^F = P_I^N$. Case c: if $V_I > \frac{c}{1-a}$ and $V_O < \frac{c}{a}$, then $P_O^N < 0 = P_O^F$ and $P_I^N < V_I = P_I^F$. Consequently, under any of the three cases, $P_O^N \leq P_O^F$ and $P_I^N \leq P_I^F$; and the inequalities can be strict.

Finally, still assume $c < aE[V_O]$. To understand the relationship between $P_O^N$, $P_I^N$ and $P_I^P$, $P_I^P$, we will distinguish three cases. Case a: If $\frac{\pi c}{1 - (1 - a)\pi I} < V_O$ and $\frac{\pi c}{1 - a} > V_I$, then $P_O^N = V_O$ and $P_I^P = 0$ and thus $P_O^P \geq P_O^N$. Also $P_I^P = 0 > (1 - a)V_I - c = P_I^N$. Case b: If $\frac{\pi c}{1 - (1 - a)\pi I} < V_O$ and $\frac{\pi c}{1 - a} > V_I$, then $P_O^P = aV_O - c$ and thus $P_I^P = P_O^N$. Similarly, $P_I^P = P_I^N$. Case c: If $\frac{\pi c}{1 - (1 - a)\pi I} > V_O$, then $P_O^P = 0$ and $P_I^P = V_I$. Since $\frac{c}{a} \geq \frac{\pi c}{1 - (1 - a)\pi I}$, we can have $V_O < \frac{c}{a}$ (in which case $P_O^N < 0$) or $V_O > \frac{c}{a}$ in which case $P_O^N > P_O^P$. We also have $P_I^P > P_I^N$.

Consequently, when $c < aE[V_O]$, $P_O^N$ can be larger or smaller than $P_O^P$. However, as seen from Figure 3, on average we have $E[P_O^N] < E[P_O^P]$.

5. Changing the order of the decisions

In this section, we compare our results with the results of Broom et al. (2013) who considered a similar model but with individuals making decisions in the reverse order. The results of Broom et al. (2013), in our current terminology and notation, are summarized in Figure 6.

Let us start by comparing the outcomes in the full information case. When $V_O > \frac{c}{a}$ or $V_I > \frac{c}{1-a}$, the order of decision does not matter. The individuals will take comparable actions and receive the same payoff as if the order of the decisions is reversed. However, when $V_O < \frac{c}{a}$ and $V_I < \frac{c}{1-a}$, the order matters. When Owners decide first, they defend while potential Intruders flee. Consequently, $P_O = V_O$ and $P_I = 0$. When Intruders decide first, they attack while the Owners flee, i.e. $P_O = 0$ and $P_I = V_I$. In summary, for the full information case, the individual deciding first has an advantage over the second individual.

The situation in the no information case is similar to the full information case. Specifically, when $E[V_O] > \frac{c}{a}$ or $E[V_I] > \frac{c}{1-a}$, the order of decision does not matter. The individuals will do comparable actions and receive the same payoff as if the order of the decisions is reversed. However, when $E[V_O] < \frac{c}{a}$ or $E[V_I] < \frac{c}{1-a}$, the order matters and the first individual to decide receives the full resource value while the second individual gets 0. In summary, for the no information case, the individual deciding first has an advantage over the second individual.
The situation is more complex in the partial information case, see Figure 7. There are four regions of parameter values where the order of the decisions will matter and we discuss the regions below.

Region I: Here $V_O < \frac{c a V_I}{1-(1-a)\pi_I}$ and $V_I < \frac{c a V_O}{1-(1-a)\pi_O}$. If the Owner decides first, it flees and receives $P_O = 0$ (and the Intruder receives $P_I = V_I$). However, if the Intruder decides first, the Intruder flees and receives $P_I = 0$, while the Owner receives $P_O = V_O$. In this region, the first individual does not want to risk the fight and thus chooses to flee. Consequently, it is better to be the second individual.

Region II: Here $\frac{c a V_I}{1-(1-a)\pi_I} < V_O < \frac{c a V_O}{1-(1-a)\pi_O}$ and $V_I < \frac{c a V_O}{1-(1-a)\pi_O}$. If Owners decide first, they stay while the Intruders flee. Consequently, $P_O = V_O$ and $P_I = 0$. However, when Intruders decide first, they attack and Owners then flee, i.e. $P_I = V_I$ and $P_O = 0$. So, in this region, the first individual makes the choice has the advantage. It bluffs by pretending to be ready to fight and it is too costly for the second individual to call the bluff.

Region III: Here $\frac{c a V_O}{1-(1-a)\pi_O} \leq V_O < \frac{c a V_I}{1-(1-a)\pi_I}$ and $\frac{c a V_O}{1-(1-a)\pi_O} < V_I < \frac{c a V_O}{1-(1-a)\pi_O}$. If Owners decide first, they stay while the Intruders flee. Consequently, $P_O = V_O$ and $P_I = 0$. However, when Intruders decide first, they attack and Owners consequently defend, i.e. $P_I = (1-a)V_I - c < 0$ and $P_O = aV_O - c > 0$ (but $P_O < V_O$). In this case, it is better for the Owner to go first and for the Intruders to go second.

Region IV: Here $\frac{c a V_I}{1-(1-a)\pi_I} < V_O < \frac{c a V_I}{1-(1-a)\pi_I}$ and $\frac{c a V_I}{1-(1-a)\pi_I} < V_I < \frac{c a V_I}{1-(1-a)\pi_I}$. If Owners decide first, they stay and Intruders then attack. Consequently, $P_O = aV_O - c < 0$ and $P_I = (1-a)V_I - c < V_I$. If Intruders decide first, they stay and Owners then leave. Consequently, $P_I = V_I$ and $P_O = 0$. In this region, it is better for the Intruder to go first and it is better for the Owner to go second.

6. Conclusions and Discussion

In this paper, we investigated the effect of information on behavior during interactions between an Owner of a valuable resource and an Intruder. We distinguished three information cases - (1)
the full information case when the individuals know the resource values for themselves as well as
their opponents, (2) the partial information case when the individuals know the resource values for
themselves but not for their opponents, and (3) the no information case when the individuals do
not know the resource values at all. For each information case, we determined when it is optimal
for the Owner to defend their resource and for the Intruder to fight for it.

We observed that the actual contests occur only when the cost of the fight is relatively low
compared to the resource value. This is in agreement with previous experiments. For example,
fights among group living pholcid spiders, *Holocnemus pluche*, are more common over larger (more
valuable) prey, without any observable increase in the fight cost (Jakob, 1994).

Not surprisingly, under most circumstances, it is beneficial for the individual to know more
rather than to know less. In particular, Owner’s payoff in full information case is larger than in
the partial information case. Such a phenomenon was also observed before. For example, Enquist
and Leimar (1987) used a sequential assessment game to model a situation where the owner of
the resource knows its subjective value while the individual attempting to steal it may or may
not know its value. Both individuals were making choices to fight or to flee. It was shown that
the chances of owner’s victory increased with the increasing value of the resource. This result was
attributed to the owner having an advantage provided by the extra knowledge.

However, when the average value of the resource is smaller than the expected cost of fighting
for the Intruder, the no information case is best for the Owner (because the Intruder will flee and
not fight). Similarly, when the average value of the resource is small enough for the Owner to
fight yet large enough that the Intruder is willing to fight, then no information case is best for
the Intruder. In both cases though, the advantage seems to come from the opponent knowing less
rather than the focal individual knowing more. We believe that a more detailed model is needed to make the proper distinction.

We saw that increasing the opponent’s knowledge may be helpful in some instances and detrimental in others. Specifically, contestants prefer opponents to know that they are willing to fight. They also prefer to hide that they are not going to fight when challenged. This may be the case of bald eagles contesting over a prey who often assess the size and hunger level of their opponents and attack those most likely to retreat (Hansen 1986). In general, the fact that an individual may benefit from opponent’s knowledge may be a factor behind the evolution of signalling, see for example Payne and Pagel (1996).

We also studied the effect of the order in which the individuals take actions. We saw that the individual initiating the contest may have an advantage in the full and no information cases but only when the fight costs are high. In that case, the initiator bluffs by showing a willingness to engage in a fight. Should the fight really happen, it would be too costly for them. However, the fight is costly also for the second individual who thus flees and no fight will take a place. When the fight cost is not so high, the order does not matter. The situation is more complex in the partial information case. When the cost is high, the initiator is in a disadvantage. It may need to give up because it does not want to risk that the cost is not high enough for the second individual to engage in the fight. When the cost is intermediate, the initiator has an advantage as in the no information case. It would be interesting to test these predictions.

There is a natural extension of our model that should be considered in the future. We assumed no correlation between the resource values to individual contestants. This may not always be the case. For example, the value of a food item correlates with its calorie content, the value of a territory correlates with its size etc. So while values for the Owner and Intruder may still be different, they are likely to be correlated as well.

Acknowledgements

J.B., F.F., M.H., and A.P. worked on this manuscript as part of the course MATH/BIOL 380 - Introduction to mathematical biology. They acknowledge the help and support of their classmates.

References


