The smooth transition autoregressive target zone model with the Gaussian stochastic volatility and TGARCH error terms with applications

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Abstract

This paper proposes to model the error term in smooth transition autoregressive target zone model as Gaussian with stochastic volatility (STARTZ-SV) or as Student-t with GARCH volatility (STARTZ-TGARCH). Using the dynamics of Norwegian krone exchange rate index, we show that both models produce standardized residuals that are closer to assumed distributions and do not produce a hump in the estimated marginal distribution of exchange rate which is more consistent with theoretical predictions. We apply developed models to test whether the dynamics of oil price can be well approximated by the Krugman's target zone model. Our estimates of conditional volatility and marginal distribution reject the target zone hypothesis.

Keywords: *target zone, oil price, exchange rate, stochastic volatility, griddy Gibbs, smooth transition*
JEL Classification Numbers:

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1 Introduction

Policies for keeping fluctuations of an economic variable close to the specified target have been a reality for exchange rates (Exchange Rate Mechanism (ERM), Hungary, Scandinavian and South American Countries) and inflation (New Zeland, Canada, UK, Sweden, Australia and Emerging Economies). In a seminal paper Krugman (1991) posited that the theoretical model of the exchange rate within credible bands has distinct, nonlinear characteristics. This observation started a vast theoretical and empirical literature though with a limited empirical success. Recent references include theoretical developments by Bartolini and Prati (1999), Koedijk, Stork, and de Vries (1998), Taylor (1995) and empirical investigations by Lundberg and Terasvirta (2005) (LT), Forbes and Kofman (2000), Li (1999), Bekaert and Gray (1998) among many others.

The testable implications of basic Krugman model is that conditional mean and variance of the process depend on the position of the variable inside the target zone. Most of empirical models focus on the implications of Krugman model for the conditional mean,[1] while only few papers consider both the conditional mean and variance. Bekaert and Gray (1998) assume a Gaussian error term and allow the variance of the error term to follow GARCH (1,1) process augmented with a variable characterizing the position of exchange rate within the band. They model the relationship between the conditional variance of the process and the position within the band as linear. Lundberg and Terasvirta (2005) model the same relationship as nonlinear smooth transition process. They propose smooth transition autoregressive target zone (STARTZ) model in which the conditional volatility follows a GARCH(p,q) process (STARTZ-GARCH), but its dynamics changes to constant variance when the process approaches the boundaries of the target corridor. Both Bekaert and Gray (1998) and Lundberg and Terasvirta (2005) find evidence in support of basic Krugman model. However, Lundberg and Terasvirta (2005) indicate that STARTZ-GARCH model that they estimate produces standardized residuals with excess kurtosis. The estimated model also produces a hump in the center of the marginal distribution erroneously implying intramarginal interventions by Norges Bank.

To address these problems, we propose to model the error term in STARTZ model as Gaussian with stochastic volatility (STARTZ-SV) or as Student-t with GARCH volatility (STARTZ-TGARCH). We show benefits of alternative models using dynamics of Norwegian krone. Next, we use both models to test the existence of target zone regime for oil price dynamics.

A researcher may be interested in the extension of standard, Gaussian GARCH model to TGARCH model or SV model because of several reasons. First, Bai, Russell, and Tiao (2003) point out that Gaussian GARCH(1,1) model is inconsistent with large leptokurtosis typically observed in the asset price returns. As a result, a standard GARCH(1,1) model requires a distribution with fat tails (for example, t-distribution, generalized t-distribution, or a mixture of normal distributions). Carnero, Pena, and Ruiz (2004) note that

[1] For example, Forbes and Kofman (2000) and Li (1999) assume that the conditional volatility is independent of the position in the target zone.

gaussianity assumption for the error term in SV model is usually adequate.[2] These authors argue that SV model is more flexible than the GARCH(1,1) model with conditional normal and fat-tailed distributions to simultaneously represent the values of the persistence of volatility, high kurtosis of the time series, and slow decay of the autocorrelations of squared residuals to zero.

Second, SV model is attractive because it may be a better discrete time approximation to the continuous model of Krugman (1991) that is used to represent the behavior of exchange rate and other time-series processes. For financial models, Brown, Wang and Zhao (2003) show that SV models are asymptotically equivalent to their diffusion limits at the basic frequency of their construction, while multiplicative GARCH models match to the diffusion limits only for observations singled-out at frequencies lower than the square root of the basic frequency of construction.

Third, Danielsson (1994), Geweke (1994), and Kim, Shephard and Chib (1998) argue that TGARCH and SV models have better empirical performance than Gaussian GARCH models in terms of the in-sample fitting of volatility, while Yu (2002) shows that SV models compare favorably in terms of the out-of-sample forecasting.

Based on Jacquier, Polson, and Rossi (1994, 2004) and Bauwens and Lubrano (1998), we develop MCMC algorithms to estimate STARTZ models with GARCH, TGARCH, and SV process for volatility. We evaluate alternative models for the dynamics of Norwegian krone, recently analyzed by Lundberg and Terasvirta (2005). The analysis of Norwegian krone suggests that STARTZ-SV and STARTZ-TGARCH models give better empirical performance than STARTZ-GARCH model for this example. The summary statistics, in particular kurtosis of standardized residuals are close to the values of assumed distributions. The estimated marginal distribution of exchange rate does not have a hump in the middle of distribution in both STARTZ-TGARCH and STARTZ-SV models which indicates that STARTZ-GARCH model may be misspecifed because it produces weaker evidence of marginal interventions and stronger evidence of intramarginal interventions.

The analysis of the oil price dynamics in the presence of target band has received considerably less attention than the analysis of exchange rates and is still a question open to a debate. Theoretical foundation of oil target zone can be found in papers by Hammoudeh (1996) and Hammoudeh and Madan (1996). Tang and Hammoudeh (2002) provide some empirical evidence. They conclude that the oil target band was credible and interventions were marginal which is consistent with crucial assumptions of basic Krugman model. An opposite view is advocated by Alhajji and Huettner (2000) who argue that OPEC never defended oil prices and lacks appropriate mechanisms to do that. OPEC does not have cash or buffer stocks to prevent very high and low oil prices. Cuts in the daily output ceiling to defend the lower bound are not strictly

[2] See the papers by Shephard (1996), Ghysels, Harvey, and Renault (1996), Kim, Shephard, and Chib (1998) and additional references in Carnero et al. (2004). Malmsten and Terasvirta (2004) compare the autoregressive SV model with the Gaussian GARCH and the exponential GARCH models and find that none of the models dominates the others in reproducing the stylized facts of financial time series.

complied by all members of OPEC and negatively affect OPEC's market share.

We let the data speak on this issue and provide statistical evidence from STARTZ-TGARCH and STARTZ-SV models on whether the target zone model is an appropriate assumption for oil prices. Our methodology is different in several respects from the analysis of Tang and Hammoudeh (2002). First, we estimate the implicit target zone, while Tang and Hammoudeh (2002) set the target zone bounds arbitrarily. STARTZ model allows us to estimate the implicit target zones for the period 1986.01 - 2000.01 when OPEC announced only the target price. Second, we relax the assumption of perfectly credible target zone and allow the oil price to move outside the estimated target zone because the oil price is outside the announced target corridor very often for the period 2000.02 - 2004.11. Third, we allow the variance of oil prices vary across time and depend on the position of oil price in the target corridor which is consistent with the literature on target zones.

We do not find support for the Krugman's type target zone model in the dynamics of oil prices. The estimated models do not reveal evidence of $\cap$-shaped form for the conditional volatility or the evidence of $\cup$-shaped marginal distribution implying the absence of marginal interventions from OPEC in order to keep oil prices in the implicit target band.

The structure of the paper is as follows. In Section 2 we present the econometric STARTZ models for target bands with GARCH, TGARCH, and SV processes for conditional volatility and explain the estimation of the model. In Section 3 we report our estimates of the target zone models of Norwegian krone. Section 4 reports our estimates for alternative models of oil prices. Concluding remarks are in Section 5. Detailed description of MCMC algorithms for GARCH type and SV models is presented in Appendices A and B.

2 The Econometric Models of Target Zones and Estimation

We start this section by describing the smooth transition target zone model with Gaussian GARCH, TGARCH and SV error terms. Then we discuss the estimation methodology and specify prior distributions for the parameters of interest.

2.1 STARTZ model with the Gaussian GARCH and TGACRH error term

We are interested in estimating the target zone model for the time series y_t, $t = 1, 2, ..., T$, with lower and upper bounds denoted y^L and y^U. The boundaries of the target zone are either announced by policymakers, explicit, or not announced but still defended with interventions, implicit. In either case, we estimate target zone boundaries.

We follow Lundberg and Terasvirta (2005), who develop a model that allows a change in dynamics for the conditional mean and conditional variance when the process approaches the boundary of the target zone.

The degree of change in the conditional mean and variance depends nonlinearly on the distance between the value of the process and the center of the target band. A similar assumption about the dynamics of the time-series process was made by Bekaert and Gray (1998) and Forbes and Kofman (2000) who introduce a variable characterizing the position of the process in the zone.

The model of Lundberg and Terasvirta (2005) with GARCH(1,1) process for the conditional volatility may be written as follows:

$$y_t = m_t + e_t \tag{1}$$

$$h_t = \eta' w_t + (\delta - \eta' w_t) G^L(z_t, \gamma_b, \theta_b, \mu y^L) + (\delta - \eta' w_t) G^U(z_t, \gamma_b, \theta_b, \mu y^U) \tag{2}$$

$$m_t = \psi' x_t + (\mu y^L - \psi' x_t) G^L(z_t, \gamma_a, \theta_a, \mu y^L) + (\mu y^U - \psi' x_t) G^U(z_t, \gamma_a, \theta_a, \mu y^U), \tag{3}$$

where $e_t = \sqrt{h_t}\varepsilon_t$, $x_t = [1, y_{t-1}, ..., y_{t-p}]'$, $w_t = [1, e^2_{t-1}, h_{t-1}]'$, $\psi = [\phi_0, \phi_1, ..., \phi_p]'$, $\eta = [\alpha_0, \alpha_1, \beta_1]$, z_t is a variable characterizing the position of time series y in the target band. In LT model the error term, ε_t, follows Gaussian process, Gaussian GARCH, while we also consider the model where the error term ε_t follows Student-t distribution with v degrees of freedom, TGARCH. LT set the variable $z_t = y_{t-1}$ for the exchange rate dynamics, but we do not make this restriction at this stage because it may be more appropriate to use different specifications of z_t for the analysis of other time-series processes. Functions $G^L(s, \gamma, \theta, c)$ and $G^U(s, \gamma, \theta, c)$ are defined as follows:

$$\begin{aligned} G^L(s,\gamma,\theta,c) &= (1 + exp(-\gamma(c - s)))^{-\theta}, \quad \gamma > 0,\ \theta > 0 \\ G^U(s,\gamma,\theta,c) &= (1 + exp(-\gamma(s - c)))^{-\theta}, \quad \gamma > 0,\ \theta > 0. \end{aligned} \tag{4}$$

where $G^L(s,\gamma,\theta,c)$, $G^U(s,\gamma,\theta,c)$ are generalized logistic functions (Sollis, Leybourne, and Newbold (1999)). The parameter θ is introduced for the possible asymmetry in the transition process. When $\theta = 1$, functions in equations (4) change monotonically from 0 to 1, with the change being symmetric around c. Sollis et al. (1999) point out that as θ approaches zero, extreme asymmetry is generated. For positive γ and $\theta < 1$, a transition starts more slowly than it finishes, but it is the opposite for negative γ. The speed of transition of s depends on the value of γ. Notice that $\frac{\partial G^L(s,\gamma,\theta,c)}{\partial \gamma} = \theta(c-s)(1+exp(-\gamma(c-s)))^{-(\theta+1)}exp(-\gamma(c-s))$ and the value of the generalized logistic function increases as γ increases if $s < c$. Therefore, large value of γ are associated with rapid transition in the dynamics of the process y_t. For $\gamma = 0$, the model (1) - (3) becomes the standard (T)GACRH model and one may use this fact to design a test of the nonlinear (T)GACRH model in (1) - (3) against the standard (T)GARCH model.

Even though we assume in equation (2) that the conditional volatility h_t follows (T)GARCH(1,1) model, the MCMC algorithm that we develop allows estimation of any (T)GACRH(p,q) model at additional computational cost. The reason for using (T)GACRH(1,1) model is that Hansen and Lunde (2005) show that the parsimonious GARCH(1,1) model performs as well as more complex alternatives. Another reason is that it is easy to insure the stationarity of the conditional volatility for the (T)GACRH(1,1) process. This volatility model was also used by Bekaert and Gray (1998) and Lunbderg and Terasvirta (2005) in their empirical analysis.

Lundberg and Terasvirta (2005) call model (1) - (3) the smooth transition autoregressive target zone model (STARTZ). To emphasize that the conditional volatility follows (T)GARCH process, we call model (1) - (3) STARTZ-(T)GARCH model. For sufficiently large values of γ_a and γ_b, when the time series process is near the center of the target zone then $G^U \approx 0$, $G^L \approx 0$, the dynamics of the time series is approximately characterized by an autoregressive process with the conditional mean $\psi' x_t$ and the GARCH(1,1) process for the volatility. When the time series process approaches the upper boundary, then $G^U \to 1$, $G^L \to 0$ and there is a smooth transition from the autoregressive behavior represented by $\psi' x_t$ with (T)GARCH(1,1) process for the volatility toward white-noise like behavior around the mean μy^U with variance δ. When the variable is near the lower bound of the target zone, the dynamics of the time-series process is approximately described by white-noise behavior around μy^L with variance δ. The variance δ of the analyzed time series process is the same around the upper and lower bounds, but this assumption may be relaxed if there are reasons to suspect that the volatility of process around the upper bound and the lower bound are different.

2.2 The STARTZ model with the SV error term

We modify model (1) - (3) by allowing the conditional volatility h_t to follow the stochastic volatility process (STARTZ-SV):

$$y_t = m_t + e_t \quad (5)$$

$$\ln h_t = \tilde{\eta}' \tilde{w}_t + (\rho - \tilde{\eta}' \tilde{w}_t) G^L(z_t, \gamma_b, \theta_b, \mu y^L) + (\rho - \tilde{\eta}' \tilde{w}_t) G^U(z_t, \gamma_b, \theta_b, \mu y^U) + \sigma_v v_t, \quad (6)$$

where $\tilde{\eta} = [\tilde{\alpha} \quad \phi]'$, $\tilde{w}_t = [1 \quad \ln h_{t-1}]'$, $e_t = \sqrt{h_t}\varepsilon_t$, $(\varepsilon_t \quad v_t)' \sim N(0, I_2)$ and all other variables and parameters are defined similarly to the STARTZ-(T)GARCH model (1) - (3). The process y_t follows an AR process with the mean $\psi' x_t$ and the SV error term in the center of the target zone. When the time series approaches one of the boundaries of the target zone, y_t is described by the white noise type process around an appropriately defined mean (μy^L or μy^U) with the variance following the stochastic process of the following form:

$$\ln h_t = \rho + \sigma_v v_t.$$

2.3 Estimation

Several estimation procedures have been proposed to estimate the basic SV model, including the Generalized Method of Moments (GMM) used by Melino and Turnbull (1990), the Quasi Maximum Likelihood (QML) approach followed by Harvey, Ruiz and Shephard (1994), Ruiz (1994), and Lundberg and Terasvirta (2005), the Efficient Method of Moments (EMM) applied by Chernov, Gallant, Ghysels and Tauchen (2003), and Markov-Chain Monte Carlo (MCMC) procedures popularized by Jacquier et al. (1994, 2004) and Kim et al. (1998), the Efficient Importance Sampling (EIS) used by Liesenfeld and Richard (2003). Broto and Ruiz (2004) survey estimation methods for SV models.[3] Forbes and Kofman (2000), Li (1999) develop MCMC algorithms for estimation of exchange rate target zone models.

In this paper, estimation of the STARTZ-(T)GARCH model and STARTZ-SV model is done in Bayesian framework using the combination of the Metropolis-Hasting algorithm and the griddy Gibbs sampling algorithm. Using a Monte Carlo study, Jacquier et al. (1994) show that MCMC based algorithm is more efficient that QML and GMM estimators for the basic SV model. The main attraction of MCMC procedures is that they permit to obtain simultaneously sample inference about the parameters, smooth estimates of the unobserved volatilities, and predictive distributions of the multistep forecasts of volatility.

Jacquier et al. (1994) develop an MCMC algorithm for the basic stochastic volatility model, while Jacquier et al. (2004) extend this algorithm for models with the correlated error terms and fat tails distributions. For conjugate situations, when the vector of parameters (θ) can be split into several groups θ_1, θ_2,...,θ_k and the analytical conditional posterior densities are known for all parameters, Gibbs sampling is simple. However, in some cases the analytical formulas for conditional posteriors are lacking for at least one of the parameters and a researcher has to use other methods to sample parameters (MH algorithm, the importance-sampling algorithm, or the griddy Gibbs sampling). The analytical formulas for conditional posteriors for many parameters of STARTZ-SV or STARTZ - (T)GARCH models are not available. As a result, we combine the Metropolis Hasting algorithm, Gibbs algorithm, and griddy Gibbs sampling algorithm to evaluate the posterior distribution of model parameters. The draw of autoregressive parameters ψ is done using the MH step, the draw of σ_v is implemented using the Gibbs sampling and the remaining parameters are drawn using the griddy Gibbs algorithm. The details of the proposed algorithms are presented in Appendices A and B.

We draw nine parameter using Griddy-Gibbs sampling for the STARTZ-GARCH model (ten parameters

[3]Asai, McAleer and Yu (2005) survey estimation methods for multivariate SV models.

for STARTZ-TGARCH model) and five parameters for the STARTZ-SV model. This implies that for a example with 40^4 grid points and 1000 draws, there are 360,000 posterior function evaluations plus one has to recursively compute the conditional volatilities h_t at each draw of parameters. As a result, the MCMC algorithm is computationally intensive.

2.4 Priors

We use a Normal - Gamma prior for parameters ψ and σ_v as in standard Bayesian analysis of regression models. For σ_v, we use an inverse gamma with $v_0 = 1$ degrees of freedom and a very small sum of squares. This is a very flat prior over the relevant posterior range. We follow Jacquier et al. (2004) and set $\tilde{\alpha} \sim N(0, 100)$ and $\phi \sim N(0.9, 0.1)$. Flat priors are also assumed for the parameters of GARCH and TGARCH processes. Bauwens and Lubrano (1998) use flat priors for estimation of the fat-tailed GARCH model. The degree of freedom parameter v in Student-t distribution is assumed to follow uniform discrete distribution on the interval $[5, 14]$. All parameters in the STARTZ-SV and STARTZ-GARCH are a priori independent with the exception of the parameters α_1 and β_1 in the GARCH(1,1) model. To insure the stationary of the conditional volatility in the STARTZ-GARCH model, we impose the following restriction $\alpha_1 + \beta_1 < 1$. The stationarity for $\ln h_t$ is controlled by truncating the prior of ϕ to be less than one in absolute value. In Table 1 we collect prior distributions for all parameters used in the estimations.

3 Norwegian Exchange Rate Index

In this section we show the differences between STARTZ-SV, STARTZ-TGARCH, and STARTZ-GARCH models by examining daily Norwegian exchange rate index from October 1, 1986 to June 17, 1988, Figure 1.[5] Because we always estimate smooth transition autoregressive models, we remove STARTZ part from the abbreviation of the model to simplify notation. For example, we use SV to denote STARTZ-SV model. During the analyzed period exchange rate index was allowed to fluctuate within $\pm 2.25\%$ from its central parity and one of the crucial assumption of Krugman (1991) model, intervention at the boundaries of a target zone (marginal interventions), was satisfied. According to Mundaca (2001) Norges Bank (Central Bank of Norway) intervened at the boundaries of target zone between October 1, 1986 and June 17, 1988. In addition, the same data set was analyzed by Lundberg and Terasvirta (2005) using the maximum likelihood estimation (MLE) of GARCH model. This allows the comparison of not only the SV and TGARCH models with the GARCH model but also the comparison of Bayesian estimates of GARCH model with MLE estimates

[4]The number and placement of grid points is important for the quality of the approximation of the posterior densities, but there is a trade-off between the precision and computational speed. Increasing the number of grid points will increase the precision of the estimated density function at the cost of more function evaluations. As a result, gains in precision from increasing the number of nodes are associated with the loss of speed.

[5]Source: Norges Bank, http://www.norges-bank.no/english/statistics/exchange/ .

obtained by Lundberg and Terasvirta (2005). Results in this subsection are not meant to argue that SV or TGARCH models are better models for all applications of target zone modeling but rather highlight possible differences between these models for a particular application.

Panel A of Table 2 reports the maximum likelihood estimates of GARCH model obtained by LT, our Bayesian estimates of GARCH, TGARCH, and SV model. We include two autoregressive lags and allow for uncertainty around the parameters γ_a and γ_b in the estimation of (T)GARCH model. In a Bayesian framework, we are also able to estimate the GARCH parameter β_1 for the conditional volatility.

Comparison of parameter estimates leads us to the following observations. First, the maximum likelihood and Bayesian estimates of GARCH model are close to each other. The only exception is the estimate of θ_b which is five times lower for the maximum likelihood method, but in both methods this parameter is not significant. Second, estimates of mean are very close in all three models. The estimates of ψ_1 are very close, while the estimates of ψ_2 are insignificant in the GARCH, TGARCH models and weakly significant in the SV model. Third, the estimated implicit bounds are close to announced bounds in all models. The estimate of μ is 0.96 in the TGARCH and SV models and it is 0.95 in the GARCH model. Fourth, the estimate of degrees of freedom in the TGARCH model is $\hat{v} = 5.27$ which is indication of fat tail distribution of error terms. Finally, the persistence of the volatility is high and significant in the SV model, $\phi = 0.74$, while there is no evidence of persistence in the (T)GARCH model. Notice that the sum of α_1 and β_1 is 0.27 for the GARCH model, 0.34 for the TGARCH model. Moreover, the individual parameters α_1 and β_1 are not significantly different from zero rejecting (T)GARCH effects in volatility and implying that the only source of time variation is due to the position of the exchange rate within the target zone through the generalized logistic functions G_t^L and G_t^U.

Panel A of Table 3 reports the summary statistics for the estimated standardized residuals. As in the Table 2, we reproduce LT maximum likelihood estimates in the second column of the table. Just like for the parameter estimates, one may notice that residuals estimated using Bayesian GARCH and MLE GARCH model have similar properties. However, properties of estimated standardized residuals of GARCH model are different from SV model even though for both models we assume standard Normal distribution. Notice that the minimum and maximum values of residuals in the SV model are -2.93 and 3.00, while in GARCH models these numbers are -4.50 and 4.70 for MLE estimation and -3.89 and 3.66 for Bayesian estimation. The probability of observing values less than -3.90 or greater than 4.7 for standard Normal distribution is 5×10^{-5} and 1×10^{-6} respectively. Because the estimate of the degree of freedom parameter implies fat tails in the error term, even though the maximum and minimum residuals for TGRACH model are -5.59 and 6.59 the probability of observing such values for t_5 distribution is reasonable for our sample size, 1.2×10^{-3} and 6×10^{-4} respectively.

The estimate of excess kurtosis, which is a difference between theoretical and estimated kurtosis, is 2.10

for the GARCH model estimated using MLE, 1.66 for the GARCH model estimated using MCMC algorithms, -2.32 or 0.67 for the TGARCH model, depending on whether we round the estimate of degrees of freedom parameter $\hat{v} = 5.25$ to $v = 5$ or to $v = 6$ respectively, and -0.31 for the SV model. Also, notice that the skewness estimate of residuals in the SV model is close to zero.

Our analysis of standardized residuals indicates that residuals in SV and TGARCH models are closer to assumed standard Normal or Student-t distributions and thus more appropriate for modeling dynamics of Norwegian exchange rate index.

The estimates of conditional volatility using the Bayesian GARCH, TGARCH models and SV model are presented in Figure 2. One may notice a much higher volatility in the SV model relative to the GARCH and TGARCH models. The scatter plots of conditional volatility against the deviations from the central parity are presented in Figure 3. These graphs reveal that the conditional variance is $\cap$-shaped in all models, which is in line with theoretical predictions of Krugman's target zone model. The difference between the models is that in (T)GARCH model $\cap$ is empty, while in SV model $\cap$ is full. When the exchange rate index is close to the target, volatility seems to be limited in downward movement in the GARCH model, and to some degree in the TGARCH model, while in tge SV model the range of fluctuations increases without limiting downward fluctuations.

Marginal densities of exchange rate deviations, which we present in Figure 4, are simulated from the estimated models by generating 100,000 observations. One may notice that the simulated marginal densities are $\cup$ - shaped in all models which is in line with Krugman's target zone model. In the GARCH model, second graph from the top, there is a hump in the center of the target zone. The hump indicates that the process spends significant time in the center, near the target, which in turn implies that the model produces weaker evidence of marginal interventions and stronger evidence of intramarginal interventions (exchange rate interventions in the center of target zone). The Krugman's model with marginal interventions predicts a $\cup$-shaped marginal distribution without a hump in the center. Neither data, top graph, nor TGARCH or SV model have the hump in the center.

A question may arise as to where the hump comes from in the GARCH model. All three models have very similar persistence of conditional mean thus the source of the difference must be the error term, $e_t = \sqrt{h_t}\varepsilon_t$. If one looks at the estimated volatilities in Figure 2, one may notice that most increases in the GARCH estimated volatility, $\hat{h}_t$, are considerably lower than increases in the SV estimated volatilities. These volatility spikes in the SV model make large moves in exchange rate more likely. On the other hand, estimated volatilities in the GARCH model in Figure 2 are higher than estimates in the TGARCH model. The fact that distribution of standardized error terms, ε_t, in TGARCH model has fat tails makes large movement in exchange rate for this model more likely. Thus both SV model, through high volatilities, and TGARCH model, through higher probability of large errors, make exchange rate to move faster from the center of the target zone toward the boundaries.

To conclude, the SV and TGARCH models give better empirical performance for the Norwegian exchange rate then GARCH. The estimated standardized residuals in SV and TGARCH models are closer to assumptions. While all three models produce strong evidence in favor of Krugman's target zone model, SV and TGARCH models are better at reproducing historical marginal distribution and more in line with Krugman's predictions. The obtained results for the Bayesian GARCH model are similar to Lundberg and Terasvirta (2005) MLE results.

4 Crude Oil Price

In this section we apply STARTZ-TGARCH and STARTZ-SV models to test whether the dynamics of oil price can be well approximated by the Krugman's target zone model.

Recently, Hammoudeh (1996) and Tang and Hammoudeh (2002) argue that a standard Krugman target zone model is appropriate to characterize the OPEC influence on the dynamics of oil price. Even though there were no official bounds on oil price (since 1986, the oil target price was $21 but there were no explicit limits for the deviation of oil price from the target price, only in 2000 OPEC increased the target price to $25 and introduced the target zone with the lower limit $22 and the upper limit $28), authors claim that market share considerations and costs of new explorations create a credible upper limit, while the fact that oil is a major source of budget revenue for OPEC countries creates a credible lower limit. Authors attribute the ability of OPEC to keep the oil price within the target zone to cutting production quotas when the price is low, and increasing production when the price is high. This argument is supported by an excess capacity of main OPEC members (Saudi Arabia, Kuwait, UAE). Tang and Hammoudeh (2002) conclude that the oil target band is consistent with assumptions of Krugman's model: interventions are marginal and credible.

On the other hand, there are several arguments against OPEC's ability to keep prices between the lower and upper limits of target zone. First, cuts in the daily output ceiling to defend the lower bound are not strictly complied by all members of OPEC which undermines the commitment to defend the lower limit of price corridor.[6] Moreover, OPEC does not have an enforcement mechanism for countries who cheat on production quotas.

Second, Alhajji and Huettner (2000) claim that OPEC can not defend the oil price band because OPEC does not have cash and buffer stocks to prevent high or low prices. The objective of buffer stocks is to keep price within a target zone when it reaches the upper limit, while cash is needed to buy excess supply of oil and increase oil inventories when price approaches the lower limit. Ideally, when the price is below the price floor, the cartel needs to decrease the oil production and use cash to increase oil stocks. When the price is

[6] For more details on compliance within the OPEC, one may look at a paper by Kohl (2002).

above the price ceiling, the cartel needs to increase oil production and sell some of its inventories in order to decrease the price. Kohl (2002) also points out that OPEC works with limited policy instruments, which are inadequate to exercise the market power and to keep the oil price within the desired target zone. OPEC has only one instrument, changes in production, to react to seasonal demand changes, production and demand shocks, price movements, and shifts in economic conditions.

Third, the commitment of OPEC to defend the lower bound of target zone by cutting production may negatively affect its market share. The market share of OPEC decreased from 55.8% in 1973 to 30% in 1985 as OPEC decreased the production from 30.8 million barrels per day in 1973 to 16 milion barrels per day in 1985 and the price of West Texas Intermediate increased in nominal terms from $3.56 in 1973 to $27.6 in 1985. If increasing or keeping the market share constant is considered to be more important by OPEC's members, their commitment to defend the lower bound of the corridor is not credible. Indeed the market share of OPEC increased from 30% in 1985 to 43% in 1998 when the OPEC abondoned its policy of maintaining high oil prices and the WTI price dropped from $27.6 in 1985 to $11.28 in 1998.[7]

To examine the ability of OPEC to keep prices within a Krugman's type target zone, we use STARTZ-SV and STARTZ-TGARCH models for empirical analysis of oil price dynamics.

Data on crude oil price have been obtained from US Department of Energy.[8] We use weekly data in the analysis for West Texas Intermediate (WTI) crude oil prices for the time period from April 1991 to November 2004.[9] We exclude the period of Gulf war. The obtained results and conclusions are qualitatively similar for OPEC reference basket price, thus we present results for WTI price only.

We divide the sample into two periods 1991.04 - 2000.01 (period I) and 2000.02-2004.11 (period II) and estimate the target zone for each subsample. OPEC announced the explicit target band for oil prices in February of 2000, which may lead to a structural break in the dynamics of oil prices and volatility. Also, examination of Table 4 suggests that OPEC may have had two distinct goals in 1991.04-2000.01 and 2000.02-2004.11. While in the first period the main challenge for OPEC was defending the lower bound of the implicit target band (the mean deviation of the oil price from the target price is -$1.87), the main problem in the second period was defending the upper bound of the target zone (the mean deviation of the oil price from the target price is $5.68). One may also notice that oil prices are consistently higher in the second period and the kurtosis of oil prices for two periods seems to be different.

Price deviations from the target price have high kurtosis (kurtosis = 6.91 for period I and 5.08 for period II, Table 4) implying that the SV or TGARCH model for volatility may be more appropriate compared to the Gaussian GARCH model.

Figure 5 reveals that there are time periods when oil prices stay close to announced target (1992-1993

[7]Check the paper by Alhajji and Huenttner (2000) for more details.
[8]The data can be accessed at http://www.eia.doe.gov/neic/historic/hpetroleum2.htm#Gasoline.
[9]To obtain weekly data, we averaged the appropriate daily observations.

or 1995-1998), but there are periods when we observe quite large deviations from the target price (1994, 1998-1999, or 2004). For the period 2000.02 - 2004.11 the oil price was outside the announced target band very often. After the OPEC Conference in January of 2005, the organization noted that "... prices have remained outside the band for over a year due to market changes that have rendered the band unrealistic and has, therefore, decided to temporarily suspend the current price band, pending completion of further studies on the subject." OPEC president Kuwaiti Oil Minister Sheikh Ahmad Fahd al-Sabah said on January 28, 2005 that a price band of 32-35 dollars would "be a good price", remarking that the 22-28 range was "effectively defunct". Such dynamics of oil prices in itself is a very strong argument against OPEC's ability to keep price within specified range. It leads us to estimating the implicit target band which may be larger than the announced zone.[10]

In the analysis of oil price target zone, we set $z_t = \frac{1}{n}\sum_{i=1}^{n} y_{t-i}$, $n = 11$, rather then setting $z_t = y_{t-1}$ as is the case for Norwegian exchange rate. This implies that the conditional mean and variance of the oil price change, when the eleven week average approaches the boundaries of the target zone. The reason we look at eleven week price average instead of last week price is that a decision making process on interventions at OPEC is slow compared to Norges Bank. The OPEC countries are required by the charter to have at least two conferences per year, typically held in June and November plus some "extraordinary" meetings. Horan, Peterson and Mahar (2004) point out that in addition to the larger conferences and extraordinary meetings, the Ministerial Monitoring Committee and Ministerial Monitoring Sub-committee are closely watched by market participants and have an effect on oil prices. As a result, there are four OPEC meetings per year on average which implies that OPEC members may be looking at the history of oil price for the last 2-3 months in making the decision about production quotas.

The announced oil target price for the period 1991.04 - 2000.01 is set to \$21 and for the period 02/01/2000-11/01/04 the target is \$25 with the lower and upper bounds set at \$22 and \$28 respectively. We set the upper bound for oil price deviations from the target price at 40% and the lower bound is set at 100% for the first period. For the second period the announced lower and upper bounds on deviatons from the target price are 12%, but we set those bounds at 45% and 70% because the announced bounds were broken very often. The bounds are set asymmetrically because it seems that OPEC is more concerned with downward deviations of oil price from the target price in period I and with upward deviations from the target price in period II.

Parameter estimates for the oil target band models for both periods are presented in Table 2. Estimates of conditional mean are very close for two periods (as in previous section to simplify notation we omit STARTZ part from abbreviation of the models), but the dynamics of conditional volatility is different. In

[10] Because we analyze the WTI price instead of OPEC reference basket price, we expect the oil price to be outside the target zone often because the average spread between the WTI price and the OPEC reference basket price is 13.9% for the considered period.

the SV model the estimate of $\tilde{\alpha}$ is significant for period I, but it is insignificant for period II. In the TGARCH model the estimate of ARCH coefficient is higher in the first period, $\alpha_1 = 0.20$, than in the second period, $\alpha_1 = 0.037$. Estimates of the parameter μ which determines the implicit target band are close to 0.9 in both periods indicating that implicit target zone is close to the assumed target band. Estimate of the degrees of freedom is 12.29 for period I and 10.59 for period II implying that the distribution of residuals is not very different from Normal distribution.

The properties of standardized residuals are summarized in panel B of Table 3. The SV model produces standardized residuals in period I that are close to a standard Normal distribution. This can be seen from the estimated minimum and maximum values of standardized residuals (-2.72 and 3.13) and the estimate of kurtosis (kurtosis = 2.92). The estimates of standard deviation for both periods are close to unity. However, this model is less successful in the second period in reproducing the fat tails of residuals. The minimum and maximum values of residuals are -3.48 and 3.10 respectively with kurtosis 3.66. While the estimate of excess kurtosis for SV model is -0.07 for period I and 0.66 for period II, the excess kurtosis for TGARCH model is much higher: 1.67 for period I and 2.40 for period II (rounding degrees of freedom to lower integer).

Figure 6 implies that deviations of oil price from the target have an asymmetric effect on volatility, i.e. increase in volatility of oil price dynamics is greater when the oil price is below the target price than when it is above the target. In the period I, the volatility is fairly constant when the oil price is close to the the target. However, it increases considerably when deviations from the target are higher than 30%. Just like in period I, in period II we observe a negative relationship between volatility and deviations from the target: positive deviations from the target have lower volatility than negative. The asymmetry is lower in TGARCH model. We believe that the fact that cuts in the daily output ceiling are not strictly complied by all members of OPEC[11] can explain this finding. Oil export revenue is the major source of budget revenue for many OPEC countries. It is more difficult for OPEC to cut oil production, defending lower bound, then to increase it. Decreasing oil production when price is below the target leads to shortfalls in the budget revenues.

We do not find enough evidence to support the Krugman target zone model with marginal interventions for oil prices for period I or for the period II. Figure 6 depicts the scatter plot of the estimated conditional volatility and the deviations of oil price from the target price. There is no a $\cap$-shaped distribution for conditional volatility in either SV or TGARCH models for period I or period II implying that target zone model with marginal interventions is not appropriate for oil prices. The estimated marginal distributions are presented in Figure 7. One may notice that there is no $\cup$-shape form for the marginal distribution rejecting the Krugman type target zone model.

[11] For more details on compliance within the OPEC, one may look at a paper by Kohl (2002).

5 Conclusions

In this paper we propose a Gaussian stochastic volatility model and Student-t GARCH model for the error process in STARTZ model. In these models the dynamics of the conditional volatility is described by SV or GARCH process in the center of the target zone and it is described by white noise process process or Student-t process when the time series approaches the boundaries. We develop MCMC algorithms which combine the Metropolis-Hasting and griddy Gibbs sampling to estimate STARTZ models with the GARCH, TGARCH and SV error terms. We apply proposed models to evaluate the target zone model for dynamics of Norwegian krone and the crude oil price.

Our first application shows that the SV and TGARCH models give better empirical performance for Norwegian krone compared to the Gaussian GARCH model. The estimated standardized residuals are closer to assumed distributions. The SV and TGRACH models do not produce the excess kurtosis of error process unlike the Gaussian GARCH model. While all models give strong evidence in favor of Krugman's original model, the SV and TGARCH models are better at reproducing the historical marginal distribution. They do not have a hump in the center of simulated marginal distribution, which is in accord with historical marginal distribution and is in line with theoretical prediction.

In our second application we test whether dynamics of oil price can be well approximated by target zone model with marginal interventions. Even prior to estimation, we have a strong argument against Krugman's type model, which is the fact that during 2000.02 - 2004.11 oil price was outside announced target zone for over a year. The estimated results from SV and TGARCH models for two periods (1991.04 - 2000.01 and 2000.02 - 2004.11) also do not provide empirical support the target zone model. We do not find neither the $\cap$-shaped form for the conditional volatility nor $\cup$-shaped marginal distribution for oil prices.

MCMC algorithms for STARTZ-(T)GARCH and STARTZ-SV Models

STARTZ-(T)GARCH

Let $\Theta = (\psi, \eta, \mu, \gamma_a, \theta_a, \gamma_b, \theta_b, \delta)$, where η defined under equation (3), and let $\Theta_{-\psi}$ denote all parameters in Θ with the parameters ψ excluded, $\Theta_{-\eta}$ denote all parameters in Θ with the parameters η excluded and so on. The MCMC algorithm for estimation of model (1) - (3) consists of several steps.

1. Draw of the parameters ψ.

   Given the draw $\Theta_{-\psi}$ and assuming that the conditional volatility h_t is known and does not depend on the parameters ψ, the model (1) - (3) can be written:

$$h_t^{-1/2}(y_t - a_{1t} - a_{2t}) = \psi'(h_t^{-1/2}x_t) - \psi'(G_t^L h_t^{-1/2}x_t) - \psi'(G_t^U h_t^{-1/2}x_t) + \varepsilon_t$$

   where $\varepsilon_t \sim N(0,1)$, $G_t^L = G^L(z_t, \gamma_a, \theta_a, \mu y^L)$, $G_t^U = G^U(z_t, \gamma_a, \theta_a, \mu y^U)$, $a_{1t} = \mu y^L G_t^L$, $a_{2t} = \mu y^U G_t^U$. Or as

$$\tilde{y}_t = \psi' \tilde{x}_t + e_t, \tag{7}$$

   where $\tilde{y}_t = h_t^{-1/2}(y_t - a_{1t} - a_{2t})$, $\tilde{x}_t = (1 - G_t^L - G_t^U)h_t^{-1/2}x_t$. Model (7) is a linear regression model and the draw of parameters ψ in this model is straightforward. However, because the conditional volatility h_t depends on the parameters ψ, we introduce the Metropolis-Hasting step.

2. Draw of the parameters γ_a, θ_a.

   Given the draw $\Theta_{-\gamma_a,-\theta_a}$, we can rewrite the model (1) - (3) as:

$$\tilde{y}_t = d_{1t}(1 + exp[-\gamma_a(\mu y^L - z_t)])^{-\theta_a} + d_{2t}(1 + exp[-\gamma_a(z_t - \mu y^U)])^{-\theta_a},$$

   where $\tilde{y}_t = y_t - \psi' x_t$, $d_{1t} = \mu y^L - \psi' x_t$, $d_{2t} = \mu y^U - \psi' x_t$. The analytical conditional posteriors for the parameters γ_a and θ_a can not be derived, but they can be evaluated over the grid of points. This allows a researcher to compute the corresponding distribution function using a deterministic integration rule. Then, one generates a draw of γ_a or θ_a by inversion of the distribution at a random value sampled uniformly in $[0, 1]$ interval. This draw of parameters γ_a, θ_a is called the griddy Gibbs sampling.[12]

3. Draw of the parameters η.

   Given the draw of parameters $\Theta_{-\eta}$, the model (1) - (3) can be written as:

$$\begin{aligned} u_t &= \sqrt{h_t}\varepsilon_t, \\ h_t &= (1 - G_t^L - G_t^U)\eta' w_t + (G_t^L + G_t^U)\delta, \end{aligned}$$

[12] One may check the papers by Ritter and Tanner (1992) or Bauwens and Lubrano (1998).

where $u_t = y_t - m_t$, $G_t^L = G^L(z_t, \gamma_b, \theta_b, \mu y^L)$, $G_t^U = G^U(z_t, \gamma_b, \theta_b, \mu y^U)$. To parameters η, we use the Griddy Gibbs sampling algorithm of Bauwens and Lubrano (1998) developed for GARCH(1,1) model with error terms following t-distribution. To insure that volatilities are positive, we impose restrictions on model parameters to guarantee that $1 - G_t^L - G_t^U \geq 0$.

4. Draw of the parameters γ_b, θ_b, δ.

   Given the draw $\Theta_{(-\gamma_b,-\theta_b,-\delta)}$, the model (1) - (3) can be written as:

$$\begin{aligned} u_t &= \sqrt{h_t}\varepsilon_t \\ \tilde{h}_t &= (\delta - \eta' w_t)\left[1 + exp(-\gamma_b(\mu y^L - z_t))\right]^{-\theta_b} \\ &\quad +(\delta - \eta' w_t)\left[1 + exp(-\gamma_b(z_t - \mu y^U))\right]^{-\theta_b}, \end{aligned} \tag{8}$$

   where $\tilde{h}_t = h_t - \eta' w_t$. The draw of parameters γ_b, θ_b, δ is done using the Griddy Gibbs sampling.

5. Draw of the parameter μ.

   Given the draw $\Theta_{-\mu}$, the model (1) - (2) can be written as:

$$\begin{aligned} \tilde{y}_t &= (\mu y^L - \psi' x_t)[1 + exp(-\gamma_a(\mu y^L - z_t))]^{-\theta_a} \\ &\quad +(\mu y^U - \psi' x_t)\left[1 + exp(-\gamma_a(z_t - \mu y^U))\right]^{-\theta_a} + \sqrt{h_t}\varepsilon_t \\ h_t &= \eta' w_t + (\delta - \eta' w_t)\left[1 + exp(-\gamma_b(\mu y^L - z_t))\right]^{-\theta_b} + (\delta - \eta' w_t)\left[1 + exp(-\gamma_b(z_t - \mu y^U))\right]^{-\theta_b}. \end{aligned}$$

   The draw of the parameter μ is done using the Griddy Gibbs algorithm.

The algorithm is the same as for STARTZ-TGARCH model with two modifications. First, the likelihood function is modified based on t_v distribution of the error term. Second, the draws of degrees of freedom parameter v are done using the Gibbs sampling algorithm of Jacquier et al. (2004).

STARTZ-SV

To explain the algorithm for estimation of model (5) - (6), we rewrite equation (6) as follows:

$$\begin{aligned} \ln h_t &= \rho(G_t^L + G_t^U) + \tilde{\alpha}(1 - G_t^L - G_t^U) + \phi(1 - G_t^L - G_t^U)\ln h_{t-1} + \sigma_v v_t \\ \ln h_t &= \rho_t + \alpha_t + \phi_t \ln h_{t-1} + \sigma_v v_t, \end{aligned} \tag{9}$$

where $\rho_t = \rho(G_t^L + G_t^U)$, $\alpha_t = \tilde{\alpha}(1 - G_t^L - G_t^U)$, $\phi_t = \phi(1 - G_t^L - G_t^U)$, $G_t^L = G^L(z_t, \gamma_b, \theta_b, \mu y^L)$, $G_t^U = G^U(z_t, \gamma_b, \theta_b, \mu y^U)$, $z_t = \bar{y}_n = \frac{1}{n}\sum_{i=1}^{n} y_{t-i}$. Notice that when the time series y_t approaches the upper limit of the target zone, then $G_t^U \approx 1$ and $G_t^L \approx 0$ and the process for the conditional log-volatility becomes:

$$\ln h_t = \rho + \sigma_v v_t.$$

The parameter ρ determines the mean of conditional volatility process near the bounds of the target zone. We augment parameter space with unobserved process for $\ln h_t$, then the likelihood function for the model (5) - (6) is written as follows:

$$\begin{aligned} L(\cdot) &\propto \prod_{t=1}^{T} \frac{1}{h_t^{1/2}} exp\left(-\frac{(y_t - m_t)^2}{2h_t}\right) \\ &\times \prod_{t=1}^{T} \frac{1}{\sigma_v} exp\left(-\frac{(\ln h_t - \rho_t - \alpha_t - \phi_t \ln h_{t-1})^2}{2\sigma_v^2}\right). \end{aligned} \tag{10}$$

The MCMC algorithm for estimation of model (5) - (6) consists of several steps.

1. The draw of the autoregressive parameters ψ is the same as for the STARTZ-GARCH model.

2. The draw of parameters γ_a, θ_a is done using the Griddy Gibbs sampling algorithm. The posterior density evaluated over the grid of points in this case is

   $$f(\gamma_a, \theta_a) \propto p(\gamma_a)p(\theta_a) \prod_{t=1}^{T} \frac{1}{h_t^{1/2}} exp\left(-\frac{(y_t - m_t)^2}{2h_t}\right)$$

   where m_t depends on γ_a and θ_a, $p(\gamma_a)$ and $p(\theta_a)$ are the prior densities for γ_a and θ_a. Notice that the second part of the likelihood is dropped because α_t, ϕ_t do not depend on the values of γ_a and θ_a.

3. The draw of parameters γ_b, θ_b is done using the Griddy Gibbs sampling algorithm. The posterior density is

   $$f(\gamma_b, \theta_b) \propto p(\gamma_b)p(\theta_b) \prod_{t=1}^{T} \frac{1}{\sigma_v} exp\left(-\frac{(\ln h_t - \rho_t - \alpha_t - \phi_t \ln h_{t-1})^2}{2\sigma_v^2}\right)$$

   where α_t, δ_t depend on γ_b and θ_b, $p(\gamma_b)$ and $p(\theta_b)$ are prior densities for γ_b and θ_b. Notice that the first part of the likelihood is dropped because m_t does not depend on the values of γ_b and θ_b.

4. The draw of parameter μ is done using the Griddy Gibbs sampling algorithm. The evaluated posterior density is

   $$f(\mu) \propto p(\mu) \prod_{t=1}^{T} \frac{1}{h_t^{1/2}} exp\left(-\frac{(y_t - m_t)^2}{2h_t}\right)$$

$$\times \quad \prod_{t=1}^{T} \frac{1}{\sigma_v} exp\left(-\frac{(\ln h_t - \rho_t - \alpha_t - \phi_t \ln h_{t-1})^2}{2\sigma_v^2}\right)$$

where $p(\mu)$ is the prior for μ.

5. To draw the vector of log-volatilities, we follow Jacquier et al. (2004, 1994) and break $p(h|\cdot)$ into T univariate conditional distributions

$$p(h_t|\cdot) \quad \propto \quad h_t^{-1/2} exp\left(-\frac{\tilde{y}_t^2}{2h_t}\right) \times h_t^{-1} exp\left(-\frac{(\ln h_t - \mu_t)^2}{2\sigma_t^2}\right)$$

where

$$\begin{aligned} \tilde{y}_t &= y_t - m_t \\ \sigma_t^2 &= \frac{\sigma_v^2}{1+\phi_{t+1}^2}, \\ \mu_t &= \frac{1}{1+\phi_{t+1}^2}\left[(\rho_t - \phi_{t+1}\rho_{t+1}) + (\alpha_t - \phi_{t+1}\alpha_{t+1}) + \phi_t \ln h_{t-1} + \phi_{t+1} \ln h_{t+1}\right] \end{aligned}$$

In choosing the generating function, we follow Jacquier et al. (2004) and choose the inverse gamma distribution $IG(s, \lambda)$ where

$$\begin{aligned} s &= \frac{1 - 2exp(\sigma_t^2)}{1 - exp(\sigma_t^2)} + 0.5 \\ \lambda &= (s-1)exp(\mu_t + 0.5\sigma_t^2) + 0.5\tilde{y}_t^2 \end{aligned}$$

To draw the log-volatilities, we combine the accept/reject and the MH sampling.

6. To draw parameters $[\rho, \tilde{\alpha}, \phi]$ and σ_v, we consider only the process for volatilities and write model (9) as follows:

$$\tilde{h}_t = \tilde{\beta}' \tilde{x}_t + \sigma_v v_t \tag{11}$$

where $\tilde{\beta} = [\rho, \quad \tilde{\alpha}, \quad \phi]'$, $\tilde{h}_t = \ln h_t$, $\tilde{x}_t = \left[G_t^L + G_t^U, 1 - G_t^L - G_t^U, (1 - G_t^L - G_t^U)log(h_{t-1})\right]'$. Model (11) is a linear regression and the draw of parameters $\tilde{\beta}$ and σ_v is done using standard formulas.

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Table 1: The prior distributions

	(T)GARCH	SV, er	SV, oil
ψ	N(ψ_0, Σ_ψ)	N(ψ_0, Σ_ψ)	N(ψ_0, Σ_ψ)
γ_a	N(300, 10)	N(300, 10)	N(300, 10)
γ_b	N(300, 10)	N(300, 10)	N(300, 10)
θ_a	U(0,1)	U(0,1)	U(0,1)
θ_b	U(0,1)	U(0,1)	U(0,1)
μ	U(0,1)	U(0,1)	U(0,1)
α_0	N(0,10)	-	-
α_1	N(0,10)	-	-
β_1	N(0,10)	-	-
δ	U(0,10)	-	-
v	U(5,14)	-	-
$\tilde{\alpha}$	-	N(0,10)	N(0,10)
ϕ	-	N(0,10)	N(0.9,0.1)
ρ	-	U(-10,10)	N(-2,10)
σ_v	-	IG(1,0.005)	IG(1,0.005)

Notes: GARCH - STARTZ-GARCH model, SV,er - STARTZ-SV model for Norwegian exchange rate, SV, oil - STARTZ-SV model for oil price. Parameters α_1 and β_1 are truncated $\alpha_1 + \beta_1 < 1$ to insure the stationary of the conditional volatility in the STARTZ-GARCH model. The parameter ϕ is truncated $|\phi| < 1$ to guarantee the stationary of the conditional volatility in the STARTZ-SV model. $\psi_0 = (0, 0.75, 0)'$ and $\Sigma_\psi = I_3$.

Table 2: Parameter estimates

	A. Norwegian Korone				*B. Oil Price*			
	GARCH(LT)	GARCH	T-GARCH	SV	T-GARCH(I)	T-GARCH (II)	SV(I)	SV(II)
ψ_0					-0.003	0.010	-0.001	0.008
					(0.002)	(0.004)	(0.001)	(0.004)
ψ_1	0.75	0.79	0.78	0.74	1.10	1.12	1.11	1.13
	(0.065)	(0.104)	(0.095)	(0.070)	(0.060)	(0.080)	(0.048)	(0.065)
ψ_2	-	0.001	0.041	0.12	-0.13	-0.17	-0.14	-0.16
		(0.075)	(0.082)	(0.06)	(0.061)	(0.083)	(0.048)	(0.065)
γ_a	300	300	300	300	300	300	300	300
	(.)	(3.18)	(3.24)	(3.31)	(3.18)	(3.18)	(3.15)	(1.04)
γ_b	300	300	300	300	300	300	300	300
	(.)	(3.17)	(3.15)	(3.17)	(3.21)	(3.19)	(3.17)	(1.03)
θ_a	0.0049	0.0061	0.0063	0.0069	0.0059	0.055	0.228	0.28
	(0.00092)	(0.001)	(0.001)	(0.0007)	(0.0075)	(0.009)	(0.099)	(0.15)
θ_b	0.032	0.179	0.019	0.12	0.23	0.21	0.26	0.35
	(0.018)	(0.21)	(0.018)	(0.17)	(0.073)	(0.08)	(0.11)	(0.11)
μ	0.96	0.95	0.96	0.96	0.91	0.89	0.88	0.91
	(0.0049)	(0.0067)	(0.0060)	(0.0032)	(0.046)	(0.065)	(0.064)	(0.05)
α_0	0.047	0.041	0.027		0.001	0.001		
	(0.013)	(0.011)	(0.011)		(0.0003)	(0.0007		
α_1	0.10	0.12	0.15		0.20	0.037		
	(0.11)	(0.07)	(0.09)		(0.073)	(0.037)		
β_1	-	0.15	0.19		0.03	0.041		
		(0.11)	(0.13)		(0.035)	(0.044)		
δ	0.0048	0.0058	0.0032		0.080	0.079		
	(0.00089)	(0.0013)	(0.0017)		(0.049)	(0.049)		
v			5.27		12.29	10.59		
			(0.64)		(1.69)	(2.45)		
$\tilde{\alpha}$				-0.88			-0.31	-0.13
				(0.10)			(0.14)	(0.09)
ϕ				0.74			0.95	0.98
				(0.03)			(0.020)	(0.014)
ρ				-6.08			-2.52	-2.90
				(0.28)			(0.70)	(3.47)
σ_v				1.30			0.27	0.17
				(0.09)			(0.058)	(0.046)

Notes: GARCH(LT) - MLE estimates reported by Lundberg and Terasvirta (2005), GARCH, T-GARCH, and SV denote the estimates obtained using the Bayesian MCMC algorithms for STARTZ-GARCH, STARTZ-TGARCH, and STARTZ-SV model respectively, T-GARCH(I) and SV(I) denote the estimates of STARTZ-TGARCH and STARTZ-SV models for crude oil prices for 1991.04 - 2000.01; T-GARCH(II) and SV(II) denote the estimates of STARTZ-TGARCH and STARTZ-SV models for crude oil prices for 2000.02-2004.11. Standard deviations of parameters are reported in parenthesis.

Table 3: Characteristics of standardized residuals

	A. Norwegian Krone			
	GARCH(LT)	GARCH	T-GARCH	SV
min	-4.5000	-3.8910	-5.5990	-2.9832
max	4.7000	3.6605	6.5950	2.9965
mean	-0.0280	-0.0208	0.0012	-0.0214
std	1.0000	0.9492	1.2054	1.0449
skewness	0.4400	0.2874	0.3646	0.0297
kurtosis	5.1000	4.6662	6.6774	2.6887
excess kurtosis	2.1000	1.6662	[-2.32, 0.68]	-0.3113
	B. Oil Price			
	T-GARCH(I)	T-GARCH(II)	SV(I)	SV(II)
min	-4.2431	-4.8039	-2.7272	-3.4804
max	4.7851	4.0639	3.1347	3.1044
mean	-0.0261	-0.0528	-0.0219	-0.0430
std	0.9491	0.9764	1.0046	1.0107
skewness	0.1071	-0.7554	0.1455	-0.4327
kurtosis	5.3370	6.4044	2.9284	3.6653
excess kurtosis	[1.58,1.67]	[2.25,2.40]	-0.0716	0.6653

Notes: GARCH(LT) - MLE estimates reported by Lundberg and Terasvirta (2005); GARCH, T-GARCH and SV denote the estimates obtained using the Bayesian MCMC algorithms for STARTZ-GARCH, STARTZ-TGARCH, and STARTZ-SV model respectively; SV(I) denotes the estimates of STARTZ-SV model for crude oil prices for 1991.04 - 2000.01; SV(II) denotes the estimates of STARTZ-SV model for crude oil prices for 2000.02-2004.11. The estimate of excess kurtosis for exchange rate for TGARCH model depends on whether we round the estimate of degrees of freedom parameter to upper or lower integer, e.g., $\hat{v} = 5.27$ to 5 (kurtosis = 9, excess kurtosis = -2.32) or to 6 (kurtosis = 6, excess kurtosis = 0.68).

Table 4: Descriptive Statistics for oil price

	P_{oil}		$\triangle P_{oil}$		$P_{oil} - P_{target}$	
	I	II	I	II	I	II
Mean	19.12	30.68	0.0015	0.002	-1.87	5.68
Median	19.01	29.64	0.0012	0.009	-1.98	4.64
Max	39.88	54.42	0.245	0.087	18.88	29.42
Min	11.00	18.27	-0.19	-0.19	-10.00	-6.72
Skewness	1.13	1.09	0.11	-1.02	1.13	1.09
Kurtosis	6.91	5.08	6.52	5.30	6.91	5.08
Autocorr	0.96	0.96	0.05	0.15	0.96	0.96

Notes: period I refers to the period 1991.04 - 2000.01; period II refers to the period 2000.02 - 2004.11. I AM NOT SURE IF WE NEED THIS TABLE. IT IS OK FOR PRESENTATION.

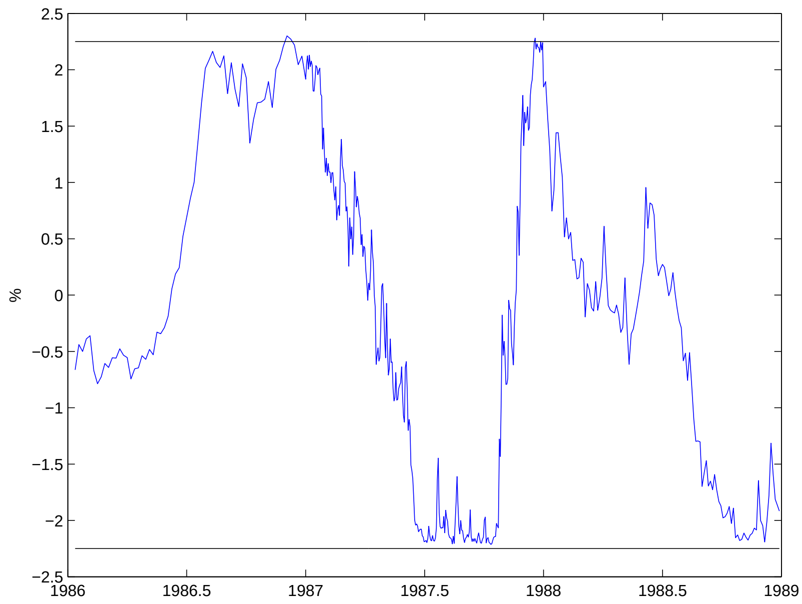

Figure 1: Norwegian exchange rate index. Percentage deviations from central parity. Target zone ±2.25%.

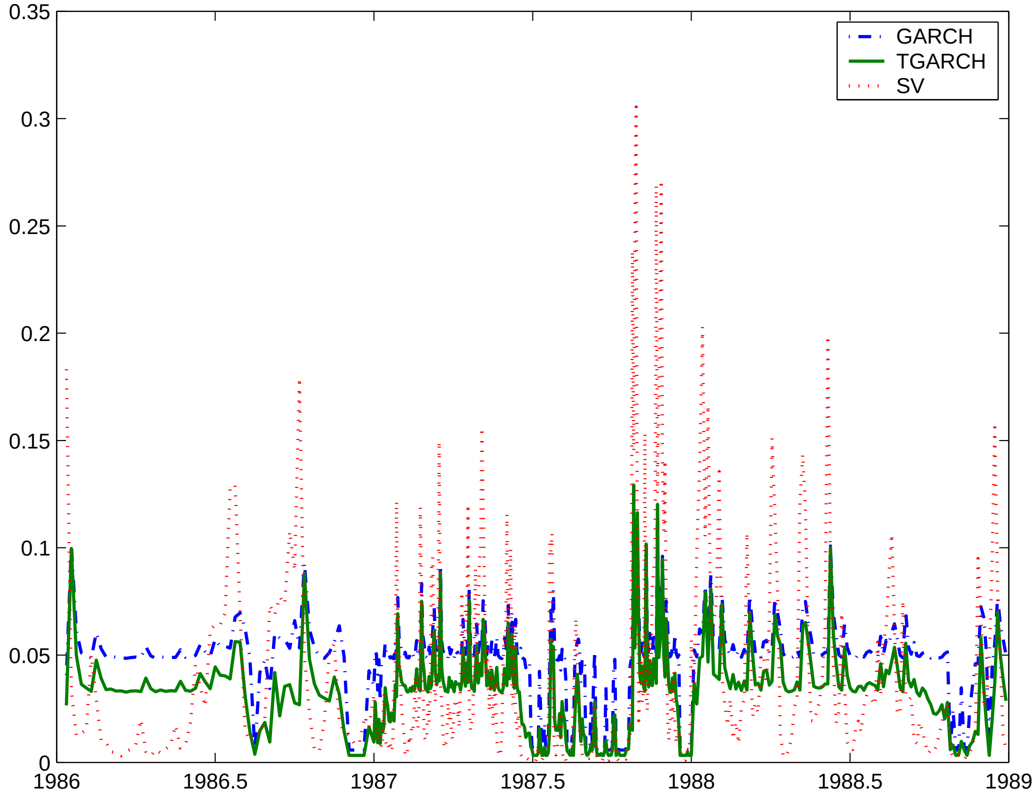

Figure 2: The estimated conditional volatility h_t from the Bayesian STARTZ-GARCH and STARTZ-SV models.

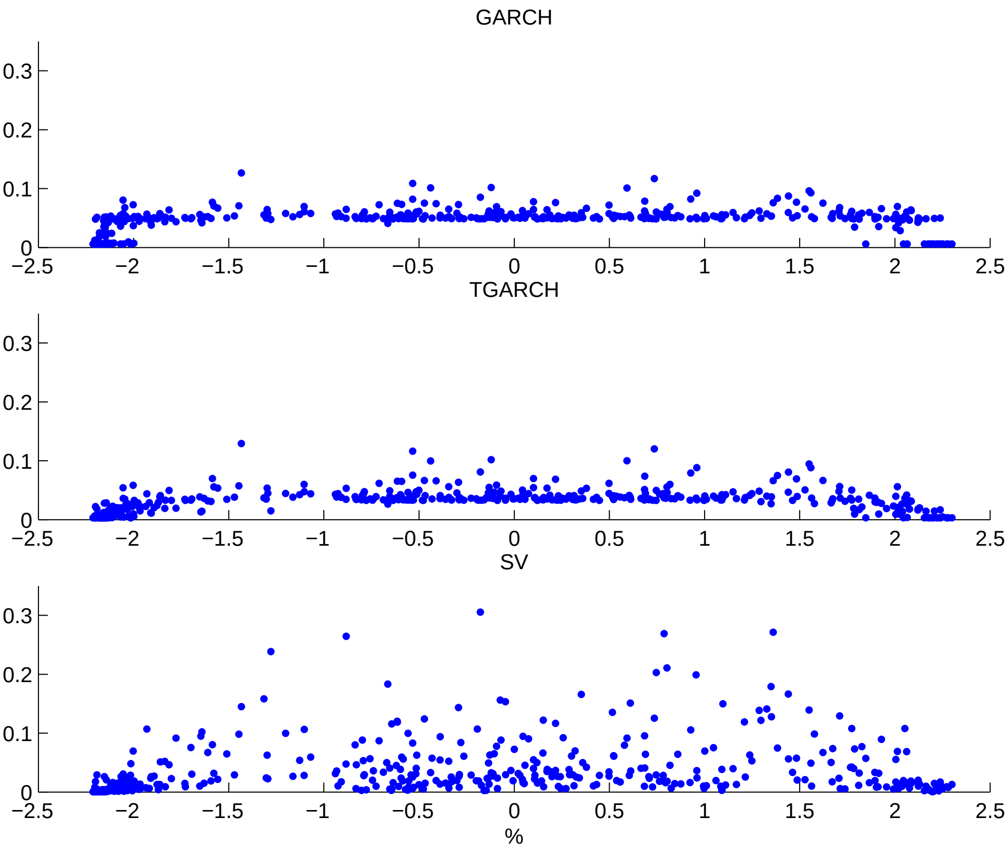

Figure 3: Conditional variance, h_t, on the y-axis is plotted against the observed deviation from the central parity (in percent) on the x-axis.

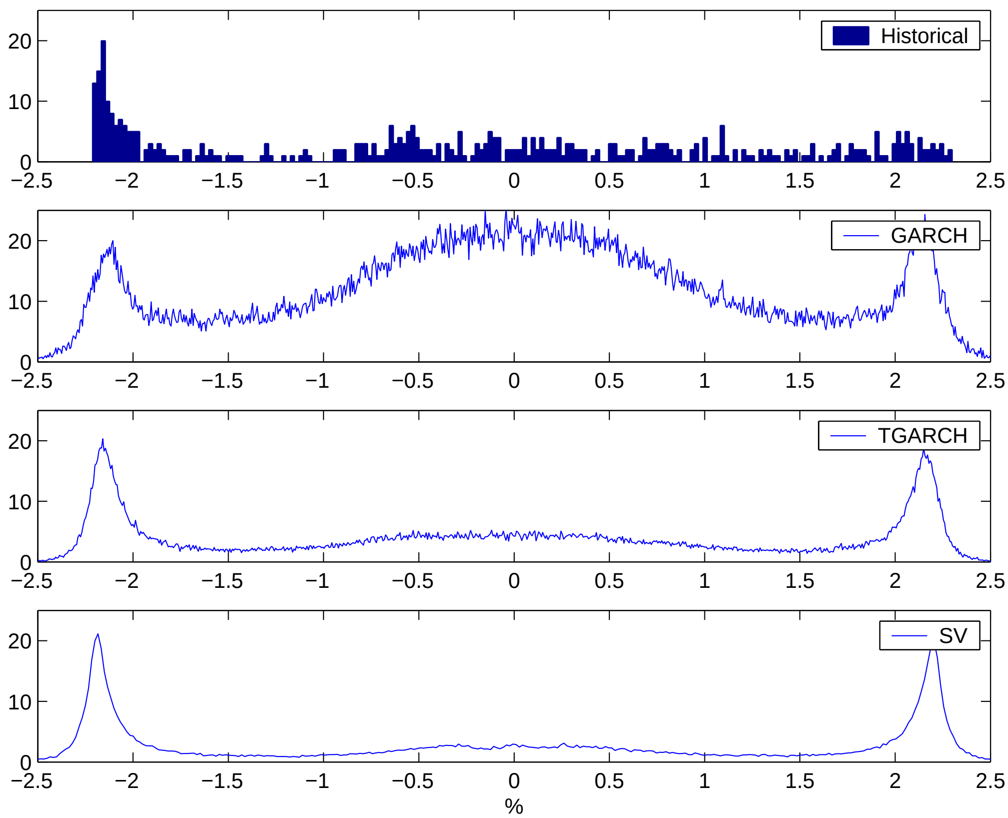

Figure 4: Historical and simulated marginal density of Norwegian exchange rate index. A histogram of the marginal density based on 100000 generated data points is plotted in the figure.

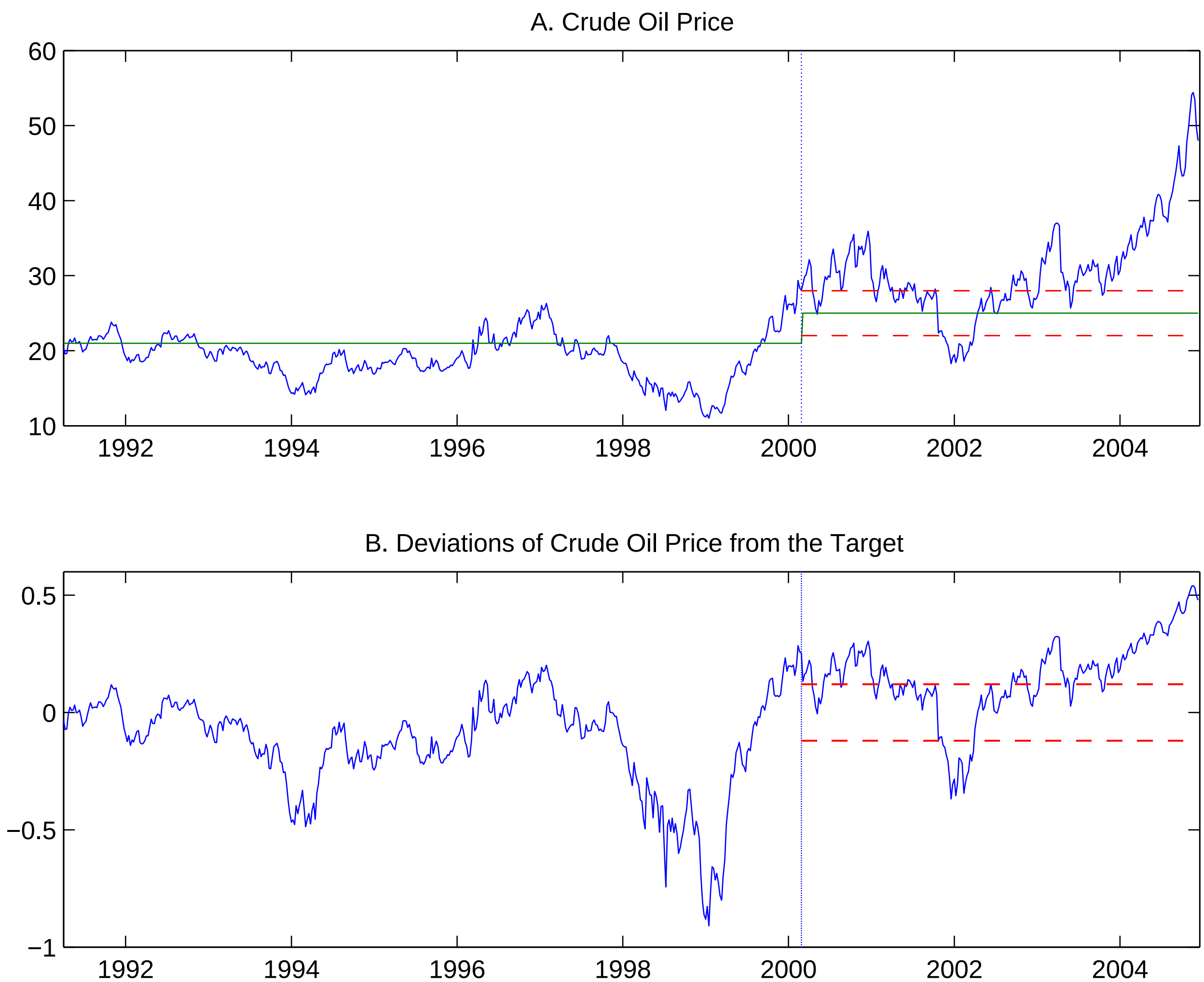

Figure 5: Panel A presents crude oil price data; Panel B depicts the deviations of oil price from the announced target price. OPEC announced the target band (\$22, \$28) in February of 2000 with the target price of \$25.

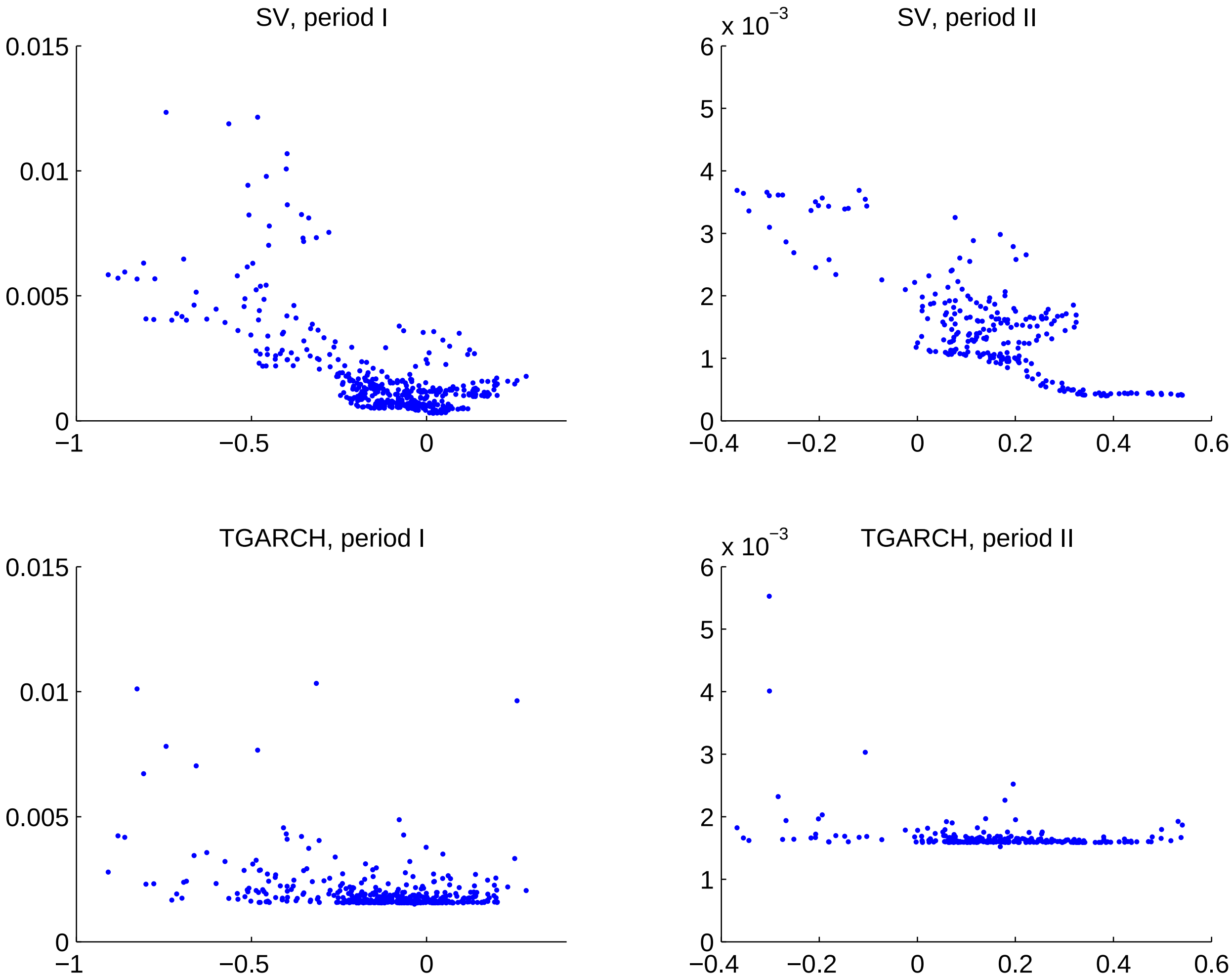

Figure 6: The estimated conditional volatility against the deviations of oil price from the announced target price.

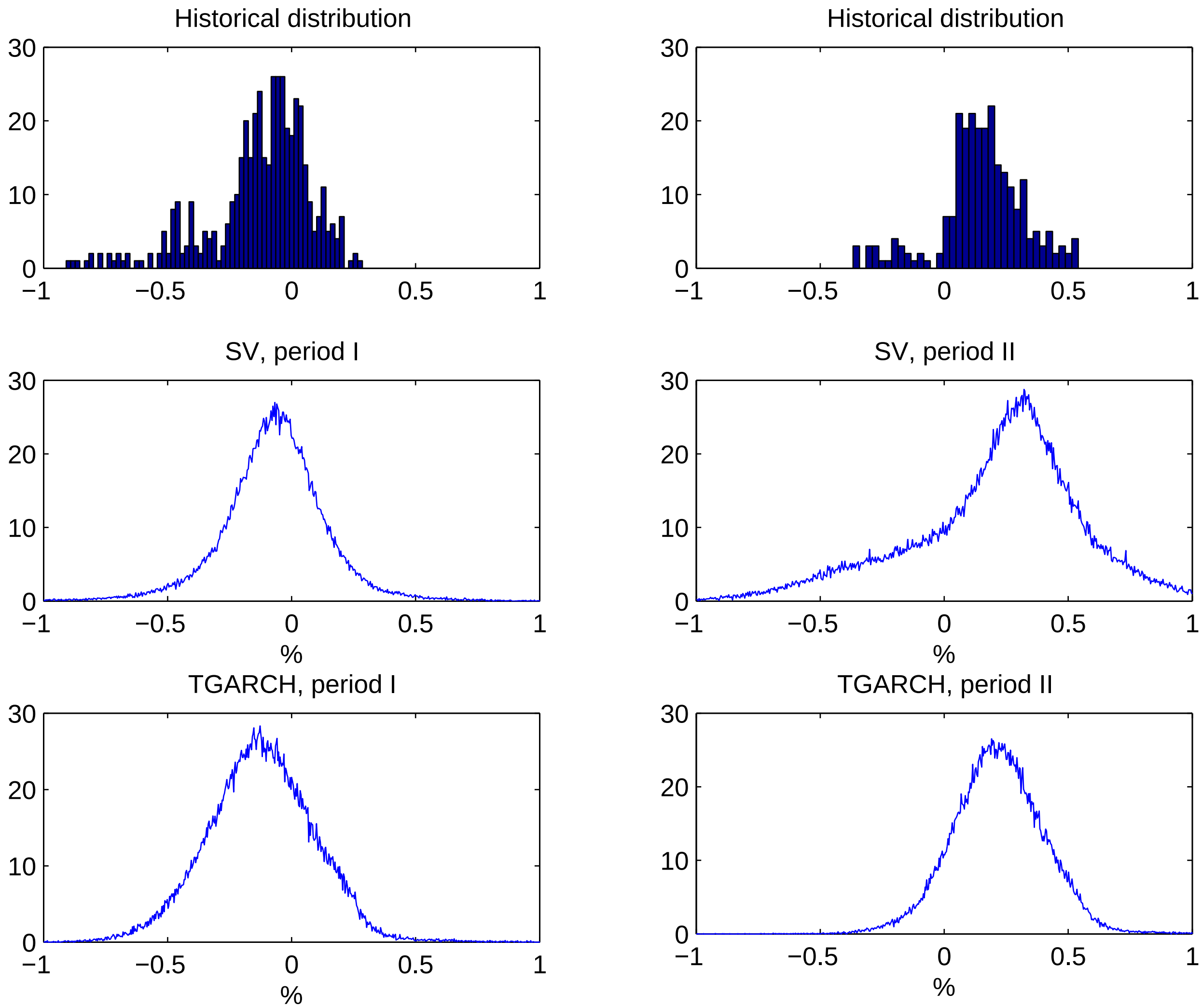

Figure 7: Historical and simulated marginal density of crude oil price. A histogram of the marginal density based on 100000 generated data points is plotted in the figure.