VCU Discrete Mathematics Seminar

Parity Theorems about Paths, Cycles and Trees

Prof Kathie Cameron
Wilfrid Laurier University

Wednesday, Apr. 7
1:00-1:50

Zoom! @ `https://vcu.zoom.us/j/92975799914`
`password=graphs2357`

Carsten Thomassen and I proved that in any graph G, the number of cycles containing a specified edge as well as all the odd-degree vertices is odd if and only if G is eulerian. Where all vertices have even degree this is a theorem of Sunichi Toida and where all vertices have odd degree it is Andrew Thomason's generalization of Smith's Theorem.

Ken Berman extended Thomason's Theorem to trees: he used a counting argument to prove that if T is a spanning tree of a graph G where all vertices in G-E(T) have odd degree, there is an even number of spanning trees of G with the same degree as T at each vertex. I give a common generalization of these results to a parity theorem about (not necessarily spanning) trees.

Andrew Thomason proved his theorem by constructing an exchange graph X(G), his lollipop graph, in which the odd-degree vertices correspond precisely to the things he wants to show there are an even number of, namely the hamiltonian cycles in G containing the specified edge. This provides an algorithm for given one of the objects, finding another, by walking in X(G) from one odd- degree vertex to another.

I have extended Thomason's algorithm to one which, in a non-eulerian graph, finds a second cycle containing a specified edge and all the odd-degree vertices, and more generally finds a second tree with certain properties.

I will discuss some other parity theorems about paths, cycles, and trees in graphs and attempts to find exchange graph proofs of them.