Shadowed Type-2 Fuzzy Logic Systems

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Abstract— General Type-2 Fuzzy Logic Systems (GT2 FLSs) are an extension to Type-1 (T1) FLS where at least one Fuzzy Set (FS) is a GT2 FS. However, due to the high computational complexity of operations on GT2 FSs, GT2 FLSs have been rarely used in practical applications. Instead, Interval Type-2 (IT2) FLSs which employ constrained IT2 FSs, have been widely used. Despite their superior computational complexity, IT2 FLSs lack the expressive power of GT2 FSs when describing various sources of uncertainty. Further, it is unclear how to derive an IT2 FLS from a specific GT2 FLS. To alleviate these issues, this paper outlines a novel concept of Shadowed Type-2 Fuzzy Logic Systems (ST2 FLS). The ST2 FLS consists of previously proposed ST2 FSs, which are T2 FSs with secondary membership functions represented as Shadowed Sets (SSs). Because ST2 FSs are directly induced by GT2 FSs, the entire design of the ST2 FLS can be automatically derived from a specific GT2 FLS. Furthermore, the proposed ST2 FLS was shown to approximate GT2 FLS more accurately compared to IT2 FLS, while maintaining the computational efficiency of IT2 FLS.

Index Terms— General Type-2 Fuzzy Logic Systems, Interval Type-2 Fuzzy Logic Systems, Shadowed Sets, Shadowed Type-2 Fuzzy Sets, Uncertainty Modeling

I. INTRODUCTION

GENERAL Type-2 Fuzzy Logic Systems (GT2 FLSs) were originally designed as an extension to T1 FLSs [1]. While the architecture of GT2 FLS is very similar to Type-1 (T1) FLS, they differ in the nature of individual Fuzzy Sets (FSs), where GT2 FLSs use GT2 FSs to model the fuzzy rule antecedents and consequents.

The concept of GT2 FSs was originally proposed by Lotfi Zadeh [2] to address the problem of over-specification of the real-valued membership degrees of T1 FSs. GT2 FSs use membership degrees that are themselves FSs. Despite the powerful uncertainty modeling capability of GT2 FSs, the high computational complexity of computing with GT2 FSs significantly hindered their practical use. As a consequence, GT2 FLSs have been rarely applied in practice [3]. Recently, there has been a renewed interest in the area of GT2 FSs and GT2 FLSs due to the recently introduced representations of geometric T2 FSs [4], [5] or the α *-planes* [6], [7], [8] and the *zSlices* [9], [10] representations.

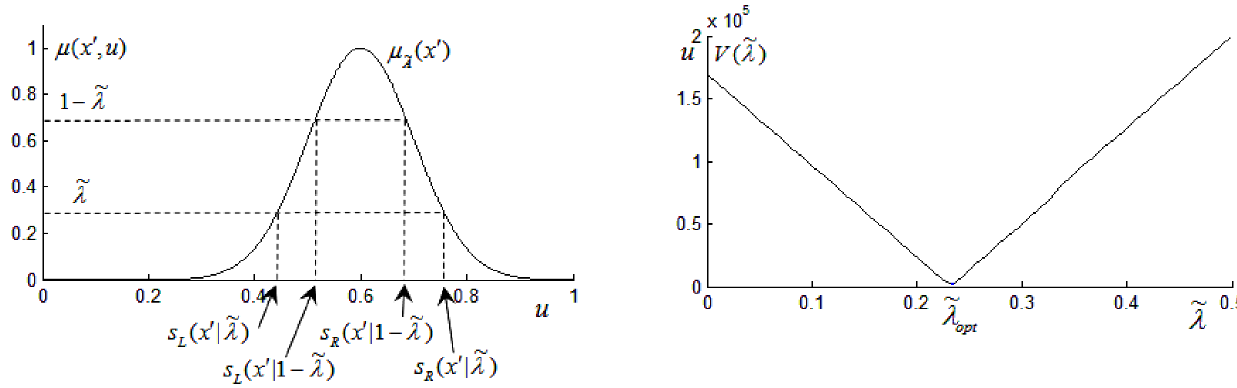
The high computational complexity of GT2 FLSs led to a wide spread of applications of their constrained version – the Interval T2 (IT2) FLSs [11], [12]. The IT2 FLSs consist of IT2 FSs which restrict the form of the secondary membership functions to intervals [13]. This simplification allows the development of efficient algorithms for fuzzy inference with IT2 FSs [14]. However, the restricted interval secondary membership functions can be seen as a significant limitation in situations where more complex representation of secondary uncertainty is required [15], [16].

Hence, on one side there are GT2 FLSs with rich uncertainty modeling capability but with unfavorable computational complexity. On the other side there are IT2 FLSs which provide efficient computational framework but at the price of significantly restricting the options for modeling various sources of uncertainty [9]. This paper proposes a new class of FLSs, the Shadowed Type-2 Fuzzy Logic Systems (ST2 FLSs), which constitute a compromise of both methods.

The proposed class of ST2 FLS is based on the previously proposed concept of ST2 FSs [17]. An ST2 FS is a GT2 FS with all secondary membership functions represented as Shadowed Sets (SSs) [18], [19], [20], [21], [22]. The computational complexity of processing ST2 FSs is significantly reduced because it is able to take advantage of the efficient fuzzy operations on IT2 FSs. However, at the same time, the ST2 FSs offer improved description of uncertainty, which is captured using the SSs rather than simple interval values for the secondary fuzzy membership functions. Similar representation to ST2 FSs, named Shadowed Fuzzy Sets was recently outlined in [23], [24].

This paper outlines the design of ST2 FLSs, which employs ST2 FSs to model the fuzzy rule antecedents and consequents. The ST2 FLS design is automatically derived from an original GT2 FLS so as to preserve the uncertainty modeled by the original GT2 FSs. ST2 FLSs can thus offer improved modeling of uncertainty when compared to IT2 FLS while also providing efficient computational framework since the secondary membership grades can only attain three values of 0, 1, or completely uncertain (shadowed) grade of [0, 1]. Similarly, as demonstrated in the experimental results section, ST2 FLSs are also computationally efficient since they apply the efficient fuzzy inference mechanisms of IT2 FLSs.

The rest of the paper is organized as follows. Section II reviews the previously published concept of ST2 FSs. The novel architecture and the design of ST2 FLS are outlined in Section III. Several examples of ST2 FLSs are presented in Section IV and the paper is concluded in Section V.



(a) (b)

Fig. 1 Secondary membership function of GT2 FS $\tilde{A}$ and its segmentation using two selected α *-planes* (b) and the optimization function $V(\tilde{\lambda})$ (b).

II. Shadowed Type-2 Fuzzy Sets

This section provides an overview of the concept of ST2 FSs which was previously proposed in [17]. The concept of Shadowed Sets (SSs) was originally developed to improve the observability and interpretability of T1 FSs and to alleviate the issues of excessive precision in describing imprecise concepts using T1 fuzzy membership functions [18]-[21]. An SS is directly induced by a T1 FS. Based on the T1 fuzzy membership grades, the SS can be divided into three regions: exclusion, core and shadow [18]-[21].

An ST2 FS is directly induced by a GT2 FS by transforming all the T1 fuzzy secondary membership functions into their SS forms [17]. In this paper all secondary membership function of the respective GT2 FSs are assumed to be convex fuzzy sets. Hence, the secondary membership functions $f_x(u)$ can be described as:

$$f_x(u)=\begin{cases} g_x(u) & u\in\left[s_L(x|0),s_L(x|1)\right], \quad g_x(u)\in[0,1] \\ 1 & u\in\left[s_L(x|1),s_R(x|1)\right] \\ h_x(u) & u\in\left[s_R(x|1),s_R(x|0)\right], \quad h_x(u)\in[0,1] \\ 0 & Otherwise \end{cases} \tag{1}$$

where $g_x(u)$ and $h_x(u)$ are monotonically non-decreasing and monotonically non-increasing functions in their respective domains.

A. Representation of ST2 FSs

An ST2 FS $\underline{\tilde{A}}$ is induced by a GT2 FS $\tilde{A}$. The process of constructing $\underline{\tilde{A}}$ constraints all the secondary membership functions of $\tilde{A}$ to be SSs. The ST2 FS $\underline{\tilde{A}}$ can be seen as functional mapping:

$$\underline{\tilde{A}}: X\times[0,1]\rightarrow\{0,1,[0,\ 1]\} \tag{2}$$

Here, the secondary membership of 1 corresponds to the core of the ST2 FSs, secondary membership of 0 corresponds to the exclusion region and the absolutely uncertain grade of [0, 1] corresponds to the shadow. The ST2 FS membership function can be expressed as follows:

$$\underline{\tilde{A}}=\{(x,u),\ \mu_{\underline{\tilde{A}}}(x,u)\ \big|\ x\in X,\ u\in[0,1],\ \mu_{\underline{\tilde{A}}}(x,u)\in\{0,1,[0,\ 1]\}\} \tag{3}$$

The process of constructing an ST2 FS $\underline{\tilde{A}}$ based on a GT2 FS $\tilde{A}$ includes elevation, reduction and balancing of the membership grades. The ST2 FS $\underline{\tilde{A}}$ is constructed using a suitable threshold $\tilde{\lambda}$. The core $core(\underline{\tilde{A}})$ of ST2 FS $\underline{\tilde{A}}$ can be described as a footprint of $\tilde{A}$ where all secondary membership degrees are greater than $1-\tilde{\lambda}$.

$$core(\underline{\tilde{A}})=\{x,u\ |\ \mu_{\tilde{A}}(x,u),x\in X,u\in[0,1],\mu_{\tilde{A}}(x,u)>(1-\tilde{\lambda})\} \tag{4}$$

The exclusion region $excl(\underline{\tilde{A}})$ of ST2 FS $\underline{\tilde{A}}$ can be defined as a footprint of $\tilde{A}$ where all secondary memberships are less than threshold $\tilde{\lambda}$:

$$excl(\underline{\tilde{A}})=\{x,u\ |\ \mu_{\tilde{A}}(x,u),x\in X,u\in[0,1],\mu_{\tilde{A}}(x,u)<\tilde{\lambda}\} \tag{5}$$

Finally, The shadow region $sh(\underline{\tilde{A}})$ of ST2 FS $\underline{\tilde{A}}$ can be constructed as a footprint of $\tilde{A}$ where all secondary memberships are between thresholds values $\tilde{\lambda}$ and $1-\tilde{\lambda}$:

$$sh(\underline{\tilde{A}})=\{x,u\ |\ \mu_{\tilde{A}}(x,u),x\in X,u\in[0,1],\tilde{\lambda}\le\mu_{\tilde{A}}(x,u)\le(1-\tilde{\lambda})\} \tag{6}$$

The process of locating the optimal value of threshold $\tilde{\lambda}$ consists of finding a pair of α -planes at levels $\tilde{\lambda}$ and $1-\tilde{\lambda}$, which optimize a fitness function $V(\tilde{\lambda})$. The objective function is composed of three components, which express the amount of uncertainty in regions that were reduced ($V^R(\tilde{\lambda})$), elevated ($V^E(\tilde{\lambda})$) or balanced ($V^B(\tilde{\lambda})$). Using the notation depicted in Fig. 1(a) the individual components can be expressed as:

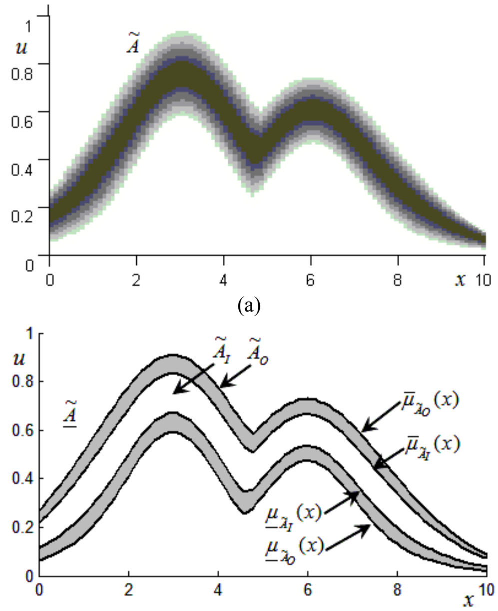


Fig. 2 GT2 FS $\tilde{A}$ (a) and its induced ST2 FS $\underline{\tilde{A}}$ (b).

$$V^{R}(\tilde{\lambda})=\int_{x\in X}\int_{0}^{s_L(x|\tilde{\lambda})}\mu_{\tilde{A}}(x,u)\,du\,dx+\int_{x\in X}\int_{s_R(x|\tilde{\lambda})}^{1}\mu_{\tilde{A}}(x,u)\,du\,dx \quad (7)$$

$$V^{E}(\tilde{\lambda})=\int_{x\in X}\int_{s_L(x|1-\tilde{\lambda})}^{s_R(x|1-\tilde{\lambda})}\mu_{\tilde{A}}(x,u)\,du\,dx \quad (8)$$

$$V^{B}(\tilde{\lambda})=\int_{x\in X}\int_{s_L(x|\tilde{\lambda})}^{s_L(x|1-\tilde{\lambda})}du\,dx+\int_{x\in X}\int_{s_R(x|1-\tilde{\lambda})}^{s_R(x|\tilde{\lambda})}du\,dx \quad (9)$$

By combining all three components the optimization function $V(\tilde{\lambda})$ can be constructed as:

$$V(\tilde{\lambda})=\left|V^{R}(\tilde{\lambda})+V^{E}(\tilde{\lambda})-V^{B}(\tilde{\lambda})\right| \quad (10)$$

In practical cases where the GT2 FS $\tilde{A}$ is represented in the α-plane framework with a finite number of α-planes, the solution can be obtained as:

$$\tilde{\lambda}_i=\tilde{\lambda}_{opt}=\arg\min_{\tilde{\lambda}_i} V(\tilde{\lambda}_i) \quad (11)$$

An example of the optimization function $V(\tilde{\lambda})$ is depicted in Fig. 1(b).

An ST2 FS $\underline{\tilde{A}}$ can be completely described using its inner and outer boundaries $\tilde{A}_I$ and $\tilde{A}_O$. Each boundary is composed of two T1 fuzzy membership functions, the lower ($\underline{\mu}_{\tilde{A}_I}(x)$, $\underline{\mu}_{\tilde{A}_O}(x)$) and the upper ($\overline{\mu}_{\tilde{A}_I}(x)$, $\overline{\mu}_{\tilde{A}_O}(x)$) membership functions. The outer boundary marks the boundary between the exclusion and the shadow region. Similarly, the inner boundary marks the transition from the shadow to the core region. This is depicted in Fig. 3, which shows a GT2 FS $\tilde{A}$ and its derived ST2 FS $\underline{\tilde{A}}$. This simplified view offers a convenient way to fully describe the ST2 FS $\underline{\tilde{A}}$ as:

$$\underline{\tilde{A}}=\{\tilde{A}_I,\tilde{A}_O\} \quad (12)$$

Here, both $\tilde{A}_I$ and $\tilde{A}_O$ are IT2 FSs.

B. Set theoretic operations with ST2 FSs

Here, the three elementary operations of intersection, union and complement on ST2 FSs are reviewed.

The intersection (meet) of two ST2 FSs $\underline{\tilde{A}}$ and $\underline{\tilde{B}}$ can be defined as follows:

$$\underline{\tilde{A}}\sqcap\underline{\tilde{B}}=\{\tilde{A}_I,\tilde{A}_O\}\sqcap\{\tilde{B}_I,\tilde{B}_O\}=\{\tilde{A}_I\sqcap\tilde{B}_I,\tilde{A}_O\sqcap\tilde{B}_O\} \quad (13)$$

The method for intersection of two IT2 FSs described in [15] can be used to calculate individual components.

The union (join) of two ST2 FSs $\underline{\tilde{A}}$ and $\underline{\tilde{B}}$ can be defined as follows:

$$\underline{\tilde{A}}\sqcup\underline{\tilde{B}}=\{\tilde{A}_I,\tilde{A}_O\}\sqcup\{\tilde{B}_I,\tilde{B}_O\}=\{\tilde{A}_I\sqcup\tilde{B}_I,\tilde{A}_O\sqcup\tilde{B}_O\} \quad (14)$$

The method for union of two IT2 FSs can be used to calculate individual components in (14).

Finally, the complement of a ST2 FSs $\overline{\underline{\tilde{A}}}$ can then be obtained as follows:

$$\overline{\underline{\tilde{A}}}=\overline{\{\tilde{A}_I,\tilde{A}_O\}}=\{\overline{\tilde{A}}_I,\overline{\tilde{A}}_O\}\quad\forall x\in X \quad (15)$$

The method for computing the complement an IT2 FS can be used to calculate individual components in (15). An example demonstrating the application of ST2 FS operations is depicted in Fig. 3.

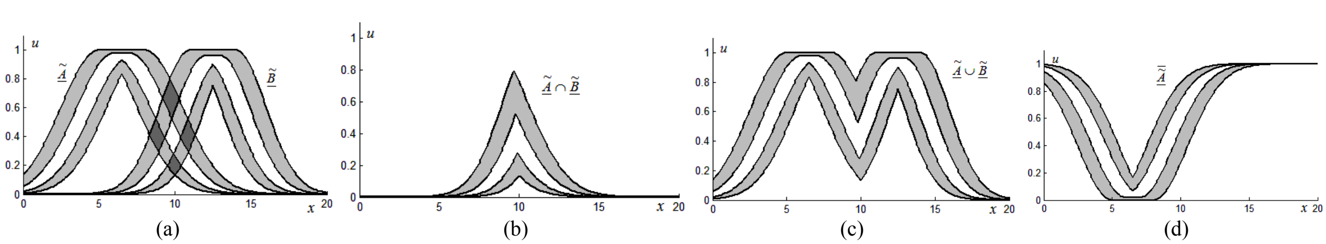


Fig. 3 Two ST2 FSs $\underline{\tilde{A}}$ and $\underline{\tilde{B}}$ (a), the results of the set theoretic operations of meet $\underline{\tilde{A}}\cap\underline{\tilde{B}}$ (b), join $\underline{\tilde{A}}\cup\underline{\tilde{B}}$ (c) and complement $\overline{\underline{\tilde{A}}}$ (d).

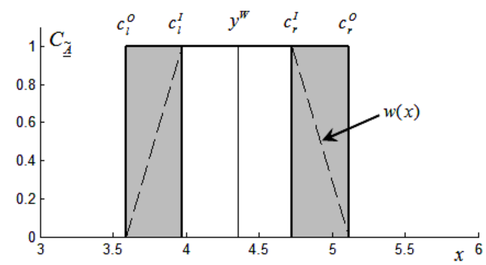


Fig. 4 Centroid of the ST2 FS $\tilde{A}$ from Fig. 3(b)

C. Type-reduction of ST2 FSs

Similar to the basic set theoretic operations, the type-reduction of ST2 FSs also takes advantage of the well-established and computationally efficient algorithms of IT2 FSs. The centroid of an ST2 FS $\tilde{A}$ denoted as $C_{\tilde{A}}$ can be described using two interval T1 FSs describing the inner and the outer centroids $C^I_{\tilde{A}}$ and $C^O_{\tilde{A}}$:

$$C_{\tilde{A}} = \{C^I_{\tilde{A}}, C^O_{\tilde{A}}\} \tag{16}$$

The inner and the outer centroid can be computed by independently type-reducing the inner and the outer boundary sets $\tilde{A}_I$ and $\tilde{A}_O$. Hence:

$$C_{\tilde{A}} = \{C_{\tilde{A}_I}, C_{\tilde{A}_O}\} = \{c^I_l, c^I_r, c^O_l, c^O_r\} \tag{17}$$

The outer centroid $C^O_{\tilde{A}}$ marks the boundary between the exclusion region and the shadowed region of the centroid. Similarly, the inner centroid $C^I_{\tilde{A}}$ creates a boundary between the shadowed boundary and the core region. An example of the centroid of ST2 FS is depicted in Fig. 4.

D. Defuzzification of ST2 FSs

Previously, three methods for defuzzification of the centroid of ST2 FSs were proposed, namely the optimistic, pessimistic and weighted defuzzification methods. The output values y^O, y^P and y^W of each method can be expressed as follows:

$$y^O = \frac{c^I_l + c^I_r}{2} \tag{18}$$

$$y^P = \frac{c^O_l + c^O_r}{2} \tag{19}$$

$$y^W = \frac{\sum_{i=1}^{N} w(x_i) x_i}{\sum_{i=1}^{N} w(x_i)} \tag{20}$$

Here, $w(x_i)$ is a specific weighting function, e.g. a trapezoidal weighting function (see Fig. 4).

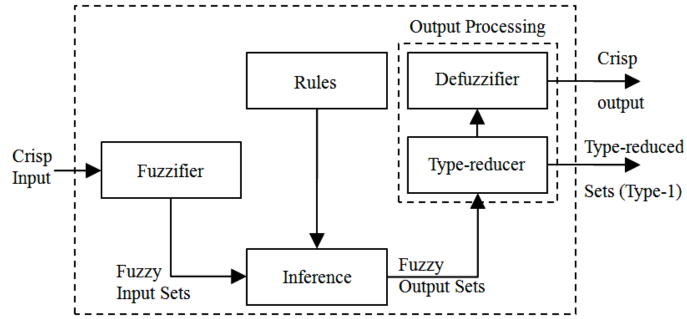


Fig. 5 Architecture of a Mamdani FLS

III. Shadowed Type-2 Fuzzy Logic Systems

This Section describes first the architecture and inference of the proposed ST2 FLSs. Next, the design of the ST2 FLS based on a specific GT2 FLS is outlined.

A. ST2 FLS Architecture and Inference

An FLS is a rule based system with individual rule antecedents and consequents represented as FSs. While many different types of FLS can be found in literature (e.g. Mamdani or Takagi-Sugeno), this paper focuses on the Mamdani type of FLS. The architecture of the Mamdani FLS can be decomposed into four major parts: input fuzzification, fuzzy inference engine, fuzzy rule base and output defuzzification or processing, as shown in Fig. 5. For the case of the proposed ST2 FLS the fuzzy rule base is populated with linguistic implicative fuzzy rules in the following form:

Rule R_k: **IF** x_1 is $\tilde{A}^k_1$ **AND** … **AND** x_n is $\tilde{A}^k_n$ **THEN** y is $\tilde{B}^k$ (21)

Each input x_i is first fuzzified by computing the membership to the respective antecedent ST2 FS. The membership grade of input x_i with respect to ST2 FSs $\tilde{A}^k_i$ is equal to the secondary membership function at of $\tilde{A}^k_i$ at coordinate x_i, which can be denoted as $\mu_{\tilde{A}^k_i}(x_i)$.

The output of rule R_k can be expressed as a ST2 fuzzy membership function $\mu_{R_k}(\vec{x}, y)$, which can be computed by applying the fuzzy meet operations to the rule antecedents and the consequents:

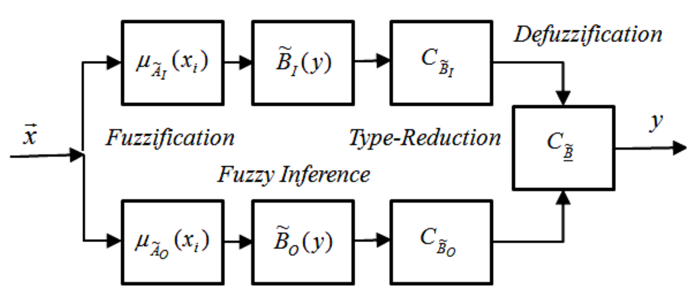


Fig. 6 Parallel processing inner and outer boundaries of the ST2 FLS

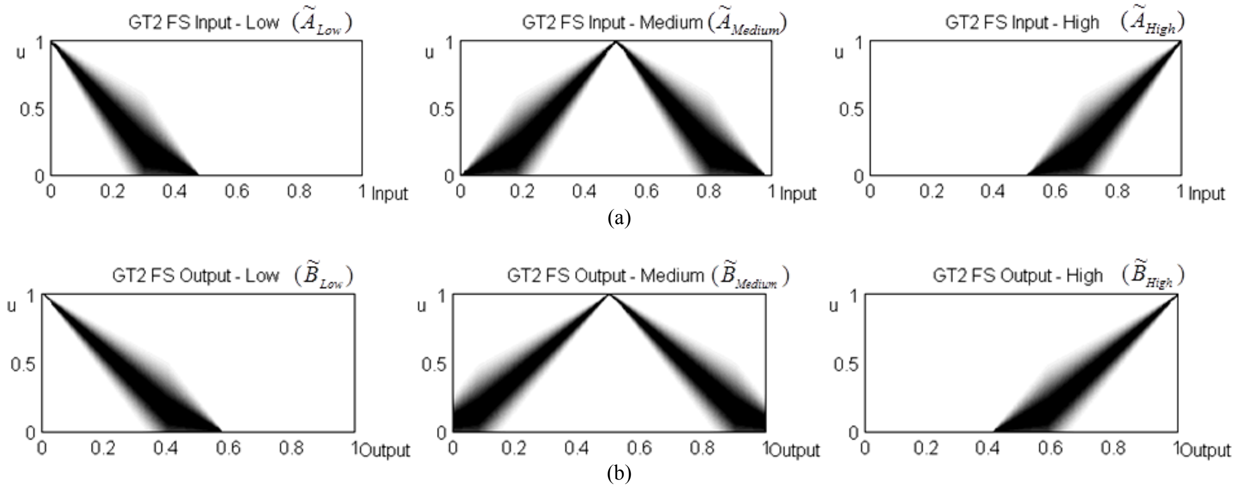


Fig. 7 GT2 FS for the inputs (a) and the output (b)

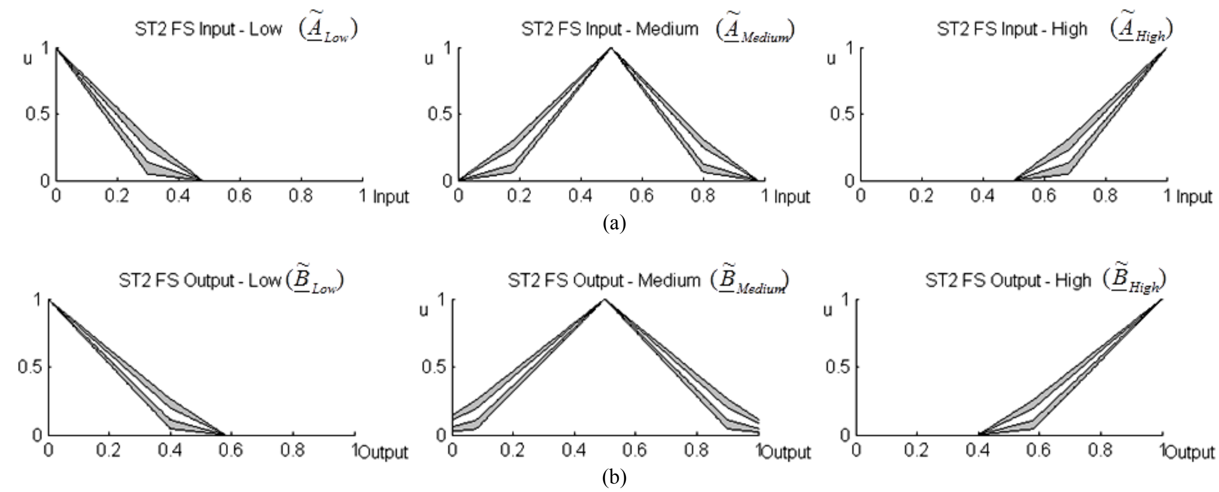


Fig. 8 ST2 FS for the inputs (a) and the output (b) (induced by the GT2 FS in Fig. 7)

$$\mu_{R_k}(\vec{x},y)=\mu_{\underline{\tilde{A}}_1^k}(x_1)\prod\ldots\prod\mu_{\underline{\tilde{A}}_n^k}(x_n)\prod\underline{\tilde{B}}^k(y) \quad (22)$$

The ST2 FSs meet operation $\prod$ can be computed according to the description provided in (13). For completeness sake, (22) can be simplified into:

$$\mu_{R_k}(\vec{x},y)=[\prod_{i=1}^{n}\mu_{\underline{\tilde{A}}_i^k}(x_i)]\prod\underline{\tilde{B}}^k(y) \qquad (23)$$

Assuming that there are K distinct fuzzy rules, the output ST2 FSs $\underline{\tilde{B}}(y)$ can be computed by aggregating the individual rule outputs via the join operation:

$$\underline{\tilde{B}}(y)=\coprod_{i=1}^{K}\mu_{R_k}(\vec{x},y) \qquad (24)$$

The ST2 FSs meet operation $\coprod$ can be computed according to the description provided in (14).

Finally, the output ST2 FS $\underline{\tilde{B}}(y)$ is first type-reduced in the first phase of output processing and subsequently defuzzified into a terminal real-valued output in the second phase. As previously discussed, by applying the IT2 FSs type-reduction algorithms such as the Enhanced Karnik Mendel algorithm [25], [26] individually to the inner and the outer boundaries of the output ST2 FSs $\underline{\tilde{B}}(y)$ its centroid $C_{\underline{\tilde{B}}}$ can be obtained. This centroid can then be defuzzified using any of the previously discussed defuzzification techniques for ST2 FSs from Section II. D.

By following the described operations of meet, join and type-reduction on ST2 FSs, the entire fuzzy inference process with ST2 FLS can be thought of as a parallel processing of two IT2 FLSs, one for the inner and one for the outer boundary IT2 FSs of the ST2 FSs. The two results are then merged during the defuzzification stage. This interpretation is depicted in Fig. 6.

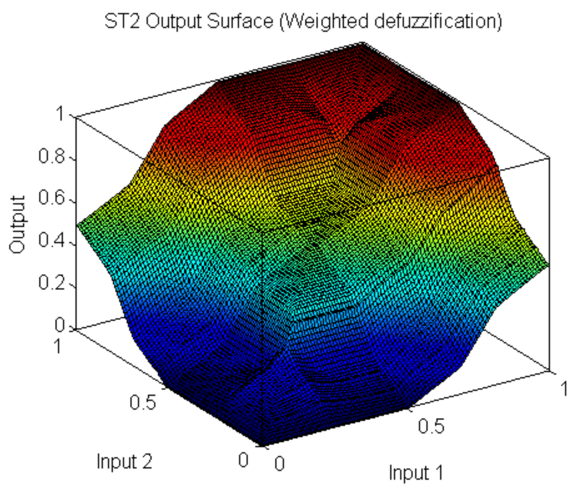


Fig. 9 Output surface of the ST2 FLS

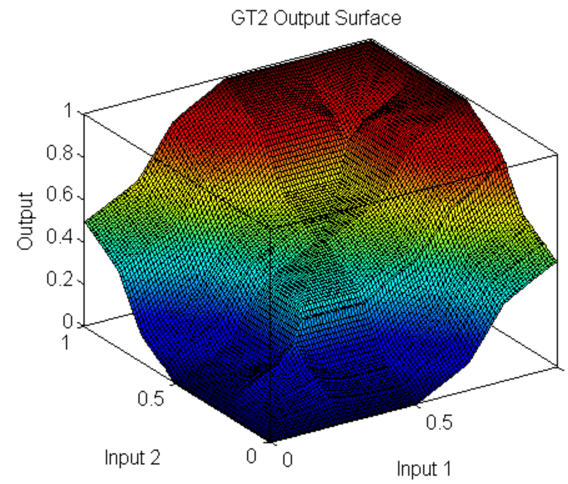


Fig. 10 Output surface of the GT2 FLS

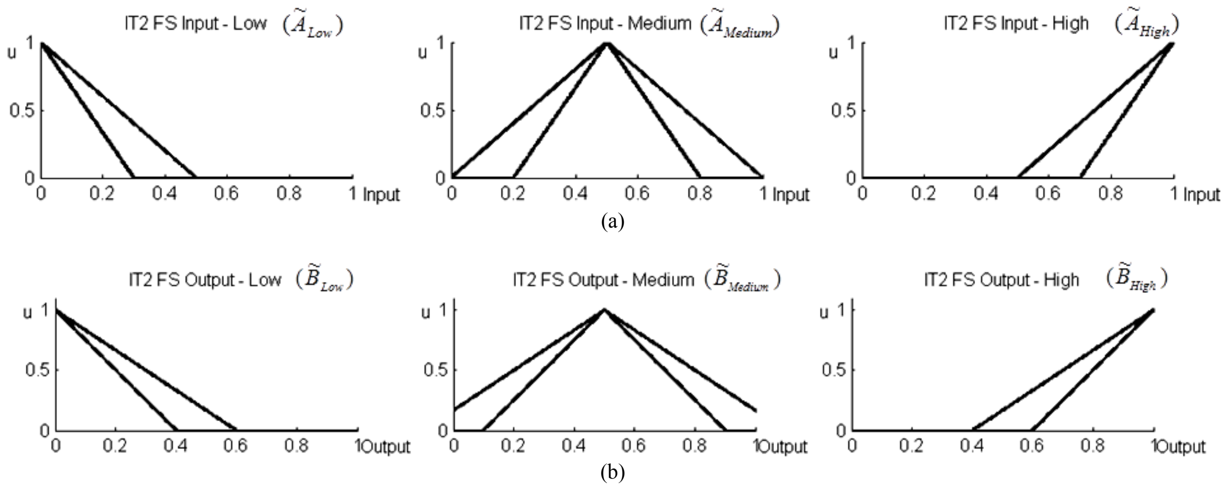


Fig. 11 IT2 FS for the inputs (a) and the output (b) (using the FOU of the GT2 FS in Fig. 7)

TABLE I. FUZZY RULE BASE

		Input 2		
		Low	Medium	High
Input 1	Low	Low	Low	Medium
	Medium	Low	Medium	High
	High	Medium	High	High

TABLE II. COMPARISON RESULTS BETWEEN IT2 FLS AND ST2 FLS

Method	MSE	Average Runtime
GT2	-	1.98×10^{-1}
IT2	3.23×10^{-4}	2.27×10^{-3}
ST2 (Optimistic)	1.70×10^{-5}	2.34×10^{-3}
ST2 (Pessimistic)	1.12×10^{-5}	2.34×10^{-3}
ST2 (Weighted)	3.10×10^{-7}	2.34×10^{-3}

B. ST2 FLS Design

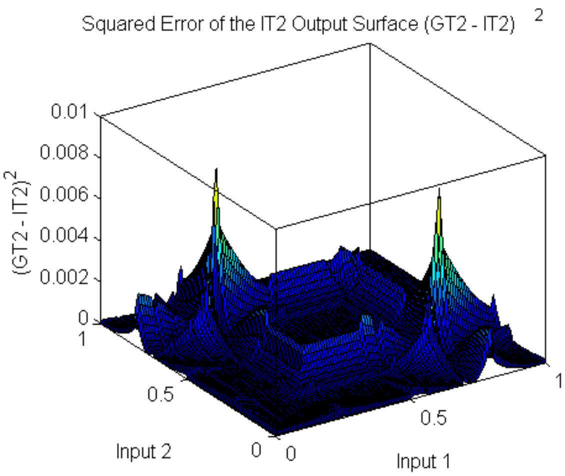
When compared to IT2 FLS, one of the major advantages of the proposed ST2 FLS is that its design is directly induced from a GT2 FLS. As a consequence of the design process the ST2 FLS preserves the uncertainty modeled by the original GT2 FLS. The design process of obtaining an ST2 FLS based on a GT2 FLS can be described in several steps as follows:

Step 1: For all antecedent and consequent GT2 FSs find the optimal splitting α -planes $\widetilde{\lambda}$ by minimizing function $V(\widetilde{\lambda})$ according to (10).

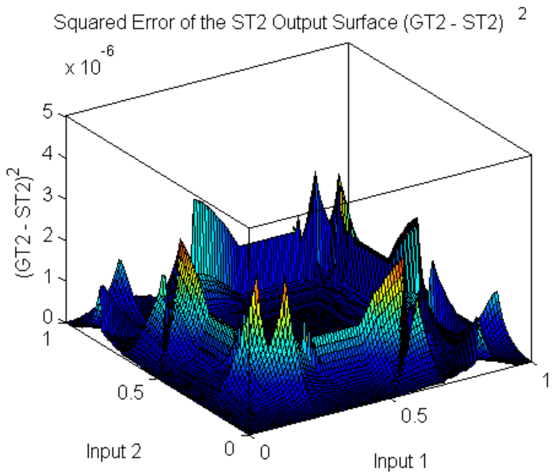
Step 2: Convert all antecedent and consequent GT2 FSs into ST2 FSs using the identified splitting α -planes $\widetilde{\lambda}$.

Step 3: Preserve the fuzzy rule base of the GT2 FLS.

Step 4: Modify the fuzzy inference process to implement the fuzzy inference operation on ST2 FSs as described in (22) and (24).



(a)



(b)

Fig. 12 Squared errors of the IT2 FLS (a) and the ST2 FLS (b). (Note the different scales used in both figures).

Step 5: Modify the output processing to implement the type-reduction and defuzzification operation on ST2 FSs as described in Sections II.C and II.D.

IV. Experimental Results

This section demonstrates the fuzzy inference process with the proposed ST2 FLS on a simple use case. The constructed FLS is composed of two inputs and a single output, all partitioned using three antecedent and consequent FSs, respectively. First, an initial GT2 FLS was created. The antecedent and consequent GT2 FSs were constructed using triangular primary membership function with symmetrically positioned Gaussian secondary membership function as shown in Fig. 7(a) and Fig. 7(b). For the inference process the GT2-FSs shown were decomposed into 200 α -planes. The fuzzy rule base used is depicted in Table I.

Second, the GT2 FLS was transformed into a ST2 FLS using the method outlined in Section III.B. The optimal values of $\tilde{\lambda}$ values were calculated and used to identify splitting α -planes of each GT2 FS (Step 2). The ST2 FSs that were directly induced by the GT2 FSs are shown Fig. 8(a) and Fig. 8(b), respectively.

Using the inference and defuzzification process detailed in Section III.A, the output surface for the ST2 FLS can be constructed. This ST2 FLS output surface computed using the weighted defuzzification method is shown in Fig. 9. For comparison, the output surface for the original GT2 FLS is depicted in Fig. 10. It can be observed, that both control surfaces are very similar.

To further investigate the modeling capability of the ST2 FLS it was compared to an IT2 FLS. The respective IT2 FLSs was constructed based on the original GT2 FLS, where individual IT2 FSs were implemented as the Footprint-Of-Uncertainty of the GT2 FSs. The IT2 FSs are shown in Fig. 11. The IT2 FLS output surface and ST2 FLS output surface were then compared to the original GT2 FLS output surface. This comparison was performed by calculating the squared error between the output of the IT2 FLS and ST2 FLS and the output of the GT2 FLS. In addition, the average computational time of the fuzzy inference process for a single input-output pair achieved by each method was also measured. Table II shows the mean squared error (MSE), and the computation time of each method. Fig. 12(a) and Fig. 12(b) show the squared error for each input value for IT2 FLS and ST2 FLS (weighted defuzzification), respectively (note the different scales used in Fig. 12).

The results show that the ST2 FLS is capable of modeling the GT2 FLS better than IT2 FLS while maintaining the computational efficiency of an IT2 FLS.

V. Conclusion

This paper presented a novel concept of Shadowed Type-2 Fuzzy Logic Systems. The ST2 FLS is an FLS using ST2 FSs for its antecedents and consequents. Since the ST2 FSs are directly induced by GT2 FSs, the entire design of the ST2 FLS can be automatically derived from a specific GT2 FLS. The ST2 FLSs can thus offer improved modeling of uncertainty when compared to IT2 FLS while also providing efficient computational framework. The experimental results demonstrated the feasibility of the proposed concept of ST2 FLS and its comparison to both IT2 FLS and GT2 FLS in terms of both modeling accuracy and computational complexity.

As future work the advantages of ST2 FLS compared to IT2 FLS will be further investigated by applying the presented methodology to real world problems.

References

[1] N. Karnik, J. M. Mendel, Q. Liang, "Type-2 Fuzzy Logic Systems," in Trans. On Fuzzy Systems, vol. 7, no. 6, pp. 643-658, Dec. 1999.

[2] L. A. Zadeh, "The Concept of a Linguistic Variable and its Approximate Reasoning - II," in *Information Sciences*, No. 8, pp. 301-357, 1975.

[3] H. Tahayori, A. G. B. Tettamanzi, G. D. Antoni, "Approximated Type-2 Fuzzy Set Operations," in *Proc. IEEE International Conference in Fuzzy Systems*, Canada, pp. 1910-1917, 2006.

[4] S. Coupland, R. John, "Geometric Type-1 and Type-2 Fuzzy Logic Systems," in *IEEE Trans. on Fuzzy Systems*, vol. 15, no. 1, pp. 3-15, February 2007.

[5] S. Coupland, R. John, "A Fast Geometric Method for Defuzzification of Type-2 Fuzzy Sets," in *IEEE Trans. on Fuzzy Systems*, vol. 16, no. 4, pp. 929-941, 2008.

[6] F. Liu, "An efficient centroid type-reduction strategy for general type-2 fuzzy logic system," in *Information Sciences*, vol. 178, pp. 2224-2236, 2008.

[7] J. M. Mendel, F. Liu, D. Zhai, "α-Plane Representation for Type-2 Fuzzy Sets: Theory and Applications," in *IEEE Transaction on Fuzzy Systems*, vol. 17, no. 5, pp. 1189-1207, Oct. 2009.

[8] J. M. Mendel, "Comments on "α-Plane Representation for Type-2 Fuzzy Sets: Theory and Applications"," in *IEEE Trans. on Fuzzy Systems*, vol. 18, no. 1, pp. 229-230, Feb. 2010.

[9] C. Wagner, H. Hagras, "Toward General Type-2 Fuzzy Logic Systems Based on zSlices," in *IEEE Transaction of Fuzzy Systems*, vol. 18, no. 4, pp. 637-660, Aug. 2010.

[10] C. Wagner, H. Hagras, "An approach for the Generation and Adaptation of zSlices based General Type-2 Fuzzy Sets from Interval Type-2 Fuzzy Sets to Model Agreement with Application to Intelligent Environments, " in *Proc. IEEE World Congress on Computational Intelligence*, Barcelona, Spain, pp. 650-657, July 18-23, 2010.

[11] H. Hagras, "Type-2 FLCs: A New Generation of Fuzzy Controllers," in *IEEE Computational Intelligence Magazine*, pp. 30-43, Feb. 2007.

[12] T. Dereli, A. Baykasoglu, K. Altun, A. Durmusoglu, B. Turksen, "Industrial applications of type-2 fuzzy sets and systems: A concise review," in *Computers in Industry*, vol. 62, issue: 2, pp. 125-137, 2011.

[13] O. Linda, M. Manic, "Uncertainty-Robust Design of Interval Type-2 Fuzzy Logic Controller for Delta Parallel Robot," in *IEEE Trans. on Industrial Informatics*, vol. 7, no. 4, pp. 661-671, Nov. 2011.

[14] O. Linda, M. Manic, "Interval Type-2 Fuzzy Voter Design for Fault Tolerant Systems," in *Information Sciences*, vol. 181, issue: 14, pp. 2933-2950, July 2011.

[15] J. M. Mendel, *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*, Prentice-Hall, Upper Saddle River, NJ, 2001.

[16] J. M. Mendel, R. John, F. Liu, "Interval Type-2 Fuzzy Logic Systems Made Simple," in *IEEE Trans. on Fuzzy Systems*, vol. 14, no. 6, pp. 808-821, 2006.

[17] O. Linda, M. Manic, "Shadowed Type-2 Fuzzy Sets –Type-2 Fuzzy Sets with Shadowed Secondary Membership Functions," in *Proc. of FUZZ-IEEE, 2012, within, IEEE World Congress On Computational Intelligence*, June 10-25, 2012.

[18] W. Pedrycz, "Shadowed Sets: Representing and Processing Fuzzy Sets," in *IEEE Trans. on Systems, Man, and Cybernetics – Part B: Cybernetics*, vol. 28, no. 1, pp. 103-109, Feb. 1998.

[19] W. Pedrycz, "Interpretation of clusters in the framework of shadowed sets," in *Pattern Recognition Letters*, vol. 26, issue: 15, pp. 2439-2449, Nov. 2005.

[20] S. Mitra, W. Pedrycz, B. Barman, "Shadowed c-means: Integrating fuzzy and rough clustering," in *Pattern Recognition*, vol. 43, issue: 4, pp. 1282-1291, April 2010.

[21] W. Pedrycz, "From fuzzy sets to shadowed sets: Interpretation and computing," in *International Journal of Intelligent Systems*, vol. 24, issue: 1, pp. 48-61, Jan. 2009.

[22] J. Zhou, W. Pedrycz, D. Miao, "Shadowed sets in the characterization of rough-fuzzy clustering," in *Pattern Recognition*, vol.: 44, issues: 8, pp. 1738-1749, Aug. 2011.

[23] H. Tahayori, A. Sadeghian, "Handling uncertainties of membership functions with Shadowed Fuzzy Sets," in *Proc. of World Automation Congress (WAC), 2012*, pp. 1-5, June 24-28, 2012.

[24] H. Tahayori, A. Sadeghian, "Shadowed Fuzzy Sets: A Framework with More Freedom Degrees for Handling Uncertainties Than Interval Type-2 Fuzzy Sets and Lower Computational Complexity Than General Type-2 Fuzzy Sets," in *New Concepts and Applications in Soft Computing Studies in Computational Intelligence*, vol. 417, pp. 97-117, 2013.

[25] D. Wu, J. M. Mendel, "Enhanced Karnik-Mendel Algorithms", in *IEEE Trans. on Fuzzy Systems*, vol. 17, No. 4, pp. 923-934, Aug. 2009.

[26] N. Karnik, J. M. Mendel, "Centroid of a type-2 fuzzy set," in *Information Sciences*, vol. 132, pp. 195-220, 2001.

[27] C. Wagner, H. Hagras, "Novel Methods for the Design on General Type-2 Fuzzy Sets based on Device Characteristics and Linguistic Labels Surveys," in *Proc. 2009 IFSA World Congress, Eusflat World Conference*, July, 2009.