Multi-Criteria Based Staging of Optimal PMU Placement using Fuzzy Weighted Average

Ondrej Linda, Milos Manic
University of Idaho
Idaho Falls, ID, USA
olindaczech@gmail.com, misko@uidaho.edu

Annarita Giani
Los Alamos National Laboratory
Los Alamos, NM, USA
annarita@lanl.gov

Miles McQueen
Idaho National Laboratory
Idaho Falls, ID, USa
Miles.McQueen@inl.gov

Abstract— **In this paper, a multi-criteria based two step method for Optimal PMU Placement (OPP) using Fuzzy Weighted Average (FWA) is proposed. In the first step, a Genetic Algorithm is used to compute the OPP solution based on the requirement of full system observability and maximum measurement redundancy. In the second step, PMU installation criteria are modeled as Fuzzy Sets (FSs) and the FWA is applied to rank the selected PMU installation sites. The criteria of observability, cost, importance, and security are used here for the multi-criteria decision making. It is shown that the proposed method using the FWA can handle a mixture of criteria types (real values, intervals and fuzzy sets) and produce a suitable staging strategy for the PMU installation.**

Keywords— Fuzzy Sets, Fuzzy Weighted Average, Genetic Algorithm, Optimal PMU Placement, Power Grid, Phasor Measurement Units, Staging

I. INTRODUCTION

Phasor Measurement Units (PMUs) have recently become the focus of work for many researchers primarily due to their potential to become one of the major enablers of Wide Area Monitoring, Protection, And Control (WAMPAC) for power network systems [1]. WAMPAC can be seen as one of the fundamental components of the envisioned smart grid concept. WAMPAC technology offers improvements in stability, reliability, and security of power production, transmission, and distribution systems [1], [2].

In general, PMUs can be considered the most advanced synchronized measurement technology for the power grid. When compared to traditional power measurements, the PMUs offer the following major capabilities: 1) location independent measurement synchronization using the Global Positioning System (GPS), 2) direct measurements of voltage and current phase angles and 3) increased accuracy, frequency, reliability, and security of the state measurements [3]. The installation of PMUs in the power grid can be seen as a major contribution to the overall resiliency of this critical infrastructure [4], [5].

A PMU is capable of observing the voltage and current phasors from all power network branches incident to the given bus on which it is installed [6]. The possibility of using a relatively small number of PMUs, combined with the high cost of both PMUs and their associated communication infrastructure, is the main reason for recent significant research effort in designing methods for optimal PMU placement.

One of the most widely used approaches is the Integer Linear Programming framework where the topology of the network can be modeled and solved using linear constraints [7], [8]. A probabilistic approach to the OPP was suggested in [9]. Various computational intelligence approaches such as Particle Swarm Optimization (PSO) [10], Binary PSO [11], Genetic Algorithms (GAs) [12], Nondominated Sorting GA [13], Immunity GA [14], Bacterial Foraging Algorithm [15], Adaptive Clonal Algorithm [16], Tabu Search [17] or Simulated Annealing (SA) [6] have also been tried. A distinctly different method of using an exhaustive binary search and sequential adding or removing of PMUs was proposed in [18], [19]. Several authors also considered the task of placing PMUs with a limited number of measurements channels [20], or combining the PMU measurements with standard power flow measurements [21].

It is unrealistic to expect that the selected optimal set of PMUs will be installed all at once. Rather, the PMU placement will have to be scheduled into multiple stages possibly over several years to cope with year-to-year financial constraints. The prioritization of PMU placement is a function of various criteria, each with different weight. In real world applications, some criteria and weights are difficult to model using discrete numerical values (e.g. it is difficult to precisely express the weight of different criteria or the relative importance of a power bus using single real values) [22]. Previously, the prioritization of different PMU placement configurations based on multi-criteria decision making schemes such as analytic hierarchy processing or a simple weighted averages was discussed in [23], [24] and several authors also considered the task of staging in the PMU placement [7], [9], [24], [25]. However, all of the previous method considered only real valued and precisely known PMU placement criteria.

To alleviate this issue a multi-criteria based staging method for optimal PMU placement using Fuzzy Weighted Average (FWA) is proposed in this paper. The method is composed of two steps. In the first step, a Genetic Algorithm (GA) is used to compute the OPP solution based on the requirement of full system observability and a desire for maximum measurement redundancy. In the second step, PMU installation criteria and their weights are modeled as Fuzzy Sets (FSs) and the FWA is applied to rank the selected PMU installation sites. The criteria of observability, cost, importance, and security are used here for the multi-criteria decision making. The FWA provides a convenient and flexible framework where various decision making criteria and their weights can be modeled as a mixture of real values, interval values, or fuzzy values. The proposed method was applied to the IEEE 30-bus data set.

The rest of the paper is organized as follows. Section II reviews the optimal PMU placement problem. The OPP solution using GA is outlined in Section III. Section IV discusses the FWA and its application to multi-criteria decision making. The FWA application to staging of PMU placement is outlined in Section V. Finally, the experimental results are demonstrated in Section VI and the paper is concluded in Section VII.

II. Optimal PMU Placement Problem

This Section provides an overview of the PMU placement problem.

A. Problem Definition

A power grid is composed of power buses and power lines between individual buses. An example of a power grid, the IEEE 14-bus test data set, is depicted in Fig. 1. The topological representation can be encoded using a connectivity matrix **A** as:

$$\mathbf{A}(i,j)=\begin{cases}1 \text{ if } i=j\\ 1 \text{ if bus } i \text{ and } j \text{ are connected}\\ 0 \text{ otherwise}\end{cases} \tag{1}$$

The PMU placement configuration in the power grid is expressed in a vector x defined as:

$$x(i)=\begin{cases}1 \text{ if PMU is installed in bus } i\\ 0 \text{ otherwise}\end{cases} \tag{2}$$

Using the introduced notation, the task of optimal PMU placement can be defined as:

$$\min\, w^T x \tag{3}$$

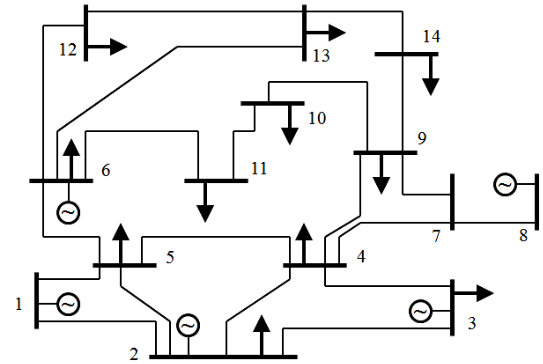


Fig. 1 IEEE 14-bus test data set (arrows and circles represent loads and generators).

Subject to: $$\mathbf{A}x \geq b \tag{4}$$

Here, w is a vector, which expresses the relative cost of installing a PMU at particular bus and $b=[111...11]^T$ is an observability constraint vector, which ensures that all buses are directly or indirectly covered by PMU measurements. For simplicity sake, and without loss of generality, in this paper the relative cost of all buses is considered equal for the OPP, hence $w=[111...11]^T$.

Certain problem domain knowledge can be utilized to simplify the solution to the OPP problem. A radial bus is a power bus, which is connected to the rest of the grid via a single power line. An example of a radial bus is bus 8 in Fig. 1. The set of radial buses can be excluded from the set of candidate buses for PMU placement since placing a PMU at radial bus will always lead to requiring at least as many PMUs than when the PMU is placed on the single neighboring power bus of the radial bus.

Some power buses are only used as transfer buses and do not contain any power injection (e.g. load or generator) into the grid. Such buses are called Zero-Injection (ZI) buses and they can potentially be used to further reduce the minimal set of installed PMUs in order to ensure full system observability. This reduction can be accomplished by using Kirchhoff's Current Law (KCL) to indirectly infer the electrical measurements in specific configurations. Consider a zero-injection bus with n connected power lines (e.g. bus 7 in Fig. 1). When the current measurements are known on n-1 power lines, the current on the remaining power line can be computed.

III. Solving OPP Using Genetic Algorithm

This Section describes the use of Genetic Algorithms for solving the OPP problem.

A. Genetic Algorithm

Genetic Algorithms are part of the field of evolutionary algorithms that use the paradigm of simulated evolution. This paradigm is based on Darwin's theory of evolution that is translated into an effective tool for global optimization [26]. The common underlying idea is that the algorithm maintains a

```
Genetic Algorithm
1:     Initialize the population with random solutions
2:     Evaluate population
3:     Repeat until population converged
  3.1:  Select parents
  3.2:  Recombine pairs of parents
  3.3:  Mutate offspring
  3.4:  Evaluate new population
  3.5:  Select individuals for new population
```

Fig. 2 Pseudo-code of a Genetic Algorithm.

population of individuals that are forced to compete for limited resources. The goodness of each individual can be evaluated using a fitness function. Parents for the next generation are selected using selection operators. New offspring are produced by recombination operators and randomly altered by mutation operators. The main cycle is repeated for a specified number of iterations or until another convergence criterion is met, such as the desired level of the best fitness value or the standard deviation of the fitness value within the population. The general pseudo-code of GA is given in Fig. 2. Further details on GA can be found in [26].

B. Genetic Algorithm for OPP

A GA was applied to the problem of optimal placement of PMUs in the power grid. GA and its modifications have been previously used for OPP in [12]-[14].

The gene of the GA individual is represented as a binary vector similar to the vector x described in (2). A value of 1 means that a PMU will be installed at the power bus and a value of 0 represents a power bus without an installed PMU. The knowledge of radial buses is used to reduce the dimensionality of the search space. Here, the radial buses are excluded from the gene of the GA. The GA uses tournament offspring selection, two-point cross-over operation, and random bit flip mutation.

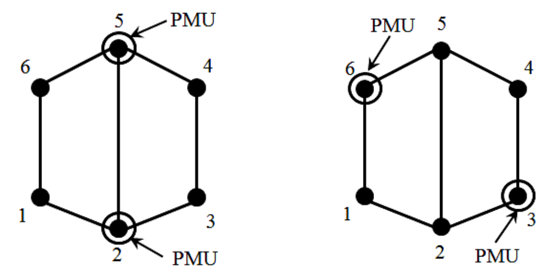
The fitness of each candidate solution, which is to be minimized, is evaluated with respect to ensuring the desired full observability of the power grid and with respect to minimizing the number of required PMUs. The fitness value $F(x)$ of each particular solution x is calculated as follows:

$$F(x) = \begin{cases} N_{PMU} + N_{Bus} + (N_{Bus} - N_{Observ}) & \text{if unobservable} \\ N_{PMU} + (1 - RI) & \text{if observable} \end{cases} \tag{5}$$

Here, N_{PMU} stands for the number of installed PMUs, N_{BUS} is the number of power buses in the grid, N_{Observ} expresses the number of power buses that are currently observed, and RI is the measurement redundancy index of the current PMU placement configuration. The RI value can be computed as follows:

$$RI = \frac{|\mathbf{A}x|}{N_{Bus}} \tag{6}$$

Here, the operator | | denotes a vector norm computing the sum of vector elements. As an example, consider a simple



(a) (b)

Fig. 3 Demonstration of measurements redundancy index.

power grid depicted in Fig. 3 with two PMU placement configurations. The RI for these two configurations is 1.33 and 1.00, respectively. Hence, despite using 2 PMUs and providing full-network observability in both cases, the configuration in Fig. 3(a) should be preferred during the design process since it offers increased measurement redundancy. It is important to note that the value of RI will always be greater than 0 and for a majority of tested data sets it was also found to be less than 1.

The calculation of the fitness function can be explained as follows. When a solution provides full network observability the fitness value is governed by the number of PMUs installed. When two solutions with identical number of PMUs are obtained, the solution providing a maximum degree of measurement redundancy is preferred. The higher the RI value the more information about the state of the power grid can be retained should a PMU malfunction.

Failure to provide full network observability is penalized by adding the number of power buses to the fitness value. Hence, any solution which does not guarantee full grid observability will be worse than any solution that does provide full observability. To further guide the search algorithm towards the desired solution, the PMU configurations that do not provide full observability but cover larger portions of the power grid are preferred.

IV. Fuzzy Weighted Average for Multi-Criteria Decision Making

This Section reviews the fundamentals of fuzzy logic and Fuzzy Weighted Average.

Fuzzy Sets

Fuzzy Set (FS) theory can be seen as a generalization of crisp set theory. The degree of belonging of element x to a particular FS A is determined by a membership grade $\mu_A(x)$ taking on a value from the unit interval [0, 1]. The fuzzy set A in the universe of discourse X can be defined as a set of ordered pairs of element x and its degree of membership $\mu_A(x)$ [22]:

$$A = \{(x, \mu_A(x)) | x \in X\} \tag{7}$$

For the special case when the universe of discourse X is a continuous space of real numbers the membership function μ_A for the fuzzy set A can be described as:

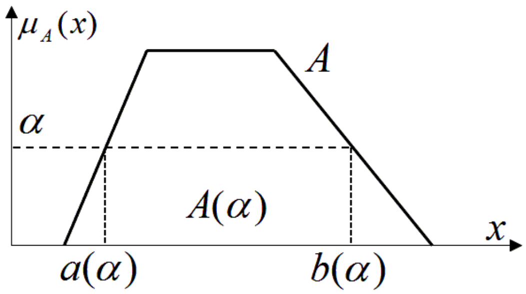


Fig. 4 Fuzzy Set A and its α *-cut*.

$$A = \int_{x \in X} \mu_A(x)/x \tag{8}$$

An important concept in the FS theory is the concept of an α *-cut*. An α *-cut* $A(\alpha)$ can be expressed as an interval:

$$A(\alpha) = \{x \,|\, \mu_A(x) \geq \alpha\} = [a(\alpha), b(\alpha)] \tag{9}$$

An example of FS A and its α *-cut* $A(\alpha)$ is illustrated in Fig. 4.

A. Fuzzy Weighted Average

As denoted in [27] the Arithmetical Weighted Average (AWA) is probably the most common type of aggregation operator. Recall that for a decision making problem with n criteria x_i and associated weights w_i the output of AWA can be computed as:

$$y_{AWA} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \tag{10}$$

The task of the FWA is to compute the weighted average in situation when criteria and weights are modeled as FSs X_i and W_i, hence [27], [28]:

$$y_{FWA} \equiv \frac{\sum_{i=1}^{n} W_i X_i}{\sum_{i=1}^{n} W_i} \tag{11}$$

The first step in computing the FWA is to construct k α *-cuts* of the criteria and the fuzzy valued weights expressed as follows:

$$X_i(\alpha) = [a_i(\alpha), b_i(\alpha)] \quad i = 1..n \tag{12}$$

$$W_i(\alpha) = [c_i(\alpha), d_i(\alpha)] \quad i = 1..n \tag{13}$$

Next, since each α *-cut* is composed of a set of intervals, the Interval Weighted Average can be used to compute an α *-cut* of the result y_{FWA} where:

$$y_{FWA}(\alpha) = [y_L(\alpha), y_R(\alpha)] \tag{14}$$

Using the α *-cut* obtained in (12) and (13) the solution to (14) can be expressed as shown in [27]:

$$y_L(\alpha) = \min_{\forall w_i(\alpha) \in [c_i(\alpha), d_i(\alpha)]} \frac{\sum_{i=1}^{n} a_i(\alpha) w_i(\alpha)}{\sum_{i=1}^{n} w_i(\alpha)} \tag{15}$$

$$y_R(\alpha) = \max_{\forall w_i(\alpha) \in [c_i(\alpha), d_i(\alpha)]} \frac{\sum_{i=1}^{n} b_i(\alpha) w_i(\alpha)}{\sum_{i=1}^{n} w_i(\alpha)} \tag{16}$$

The values of $y_L(\alpha)$ and $y_R(\alpha)$ can be computed for example using the Enhanced Karnik Mendel algorithm [29]. The FWA method is demonstrated in Fig. 5 for three criteria C_1, C_2, C_3 and their weights W_1, W_2 and W_3. Further details on the computation of FWA can be found in [27], [28].

V. Staging of PMU Placement Using FWA

This section applies the FWA to the problem of staging PMU placements. First the PMU placement criteria used in this paper are described. Next, the application of the FWA to ranking the selected power buses for PMU placement is outlined.

A. PMU Placement Criteria

When staging the placement of individual PMUs on the selected power buses, multiple diverse criteria can come into play. These criteria are typically defined by the different stake holders involved, and their respective incentives and constraints. It should be noted here that the presented paper does not attempt to identify or enumerate all of the important criteria for PMU placement. Rather, it attempts to present a general methodology for fusing an arbitrary set of criteria irrespective of whether the criteria are represented as real-values, interval values, or linguistic terms modeled as FSs. For the specific implementation presented in this paper the criteria of *Observability*, *Cost*, *Importance,* and *Security* have been considered.

The *Observability* criterion C^O is related to the fact that installing a PMU on a specific bus also allows an indirect measurement of additional parameters in the local neighborhood of the bus. Despite the fact that full system

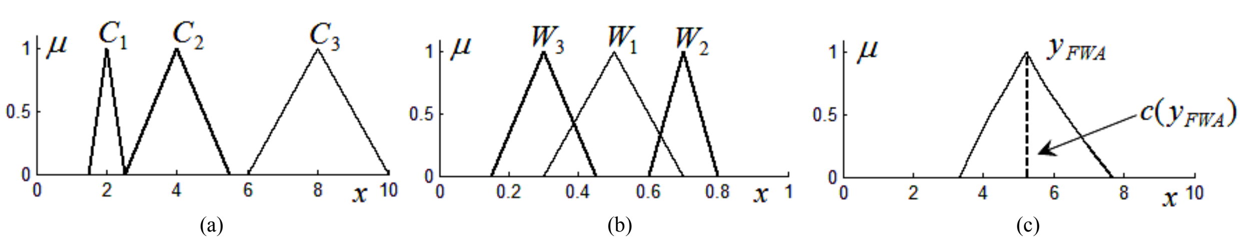


Fig. 5 Criteria values (a), weights (b) and the product of the FWA (c).

observability will be achieved only after all PMUs in the optimal set are installed, it is still desirable to install PMUs with more incident buses first. Installation of these PMUs upfront will ensure a higher level of system observability in the initial stages of the placement. The actual value of the observability criterion C^O for a bus can be computed as the number of incident power lines to that bus. Typically, the value of C^O can be expressed as a discrete integral value.

The *Cost* criterion C^C expresses the different cost associated with installation of each PMU. This cost might constitute an aggregate of individual costs required on the communication infrastructure, the PMU hardware, or the actual installation (e.g. existence of PMU-ready equipment [23]). The placement of a PMU with associated lower cost should be preferable at the early stages of the placement, since more PMUs could be installed with the constrained budget. Typically, a precise cost value might be difficult to estimate a priori. Instead, it might be more suitable to express the cost of PMU installment as an interval value.

The *Importance* criterion C^I expresses the subjective importance of each power bus in the given context. For example, this criterion might express the spatial closeness of important assets (e.g. military base, critical infrastructure, critical cyber assets), existence of critical regional corridors or the existence of major generators and loads. The notion of relative importance is difficult to express using precise numerical values. Rather, a linguistic description of importance using terms such as "low" or "medium" modeled as a FS may be more appropriate.

The *Security* criterion C^S captures the vulnerability of each power bus to potential cyber attack. Since the resiliency and cyber-security of the power distribution network is becoming a major concern for the smart grid concept, the PMUs can also be utilized as counter-measures for possible cyber attacks on the power distribution grid. For this paper the assessment of the vulnerability of each power bus is based on the potential presence of sparse data integrity attacks on the given bus or its incident power lines. The notion of these sparse attacks and the algorithm for their detection can be found in the work of Giani et al. [2].

Finally, each criterion might have different significance for various stakeholders, and this can be expressed using different criteria weights. Again, it seems counter-intuitive to express these subjective weights using precise real values. Instead, the method proposed in this paper allows for using linguistic terms such "very low" or "high" modeled as FSs.

B. Placement Staging using FWA

It is assumed that the GA algorithm computes the solution to the OPP problem, which consists of installing exactly K PMUs into the power grid. The desired full system observability will be achieved only after all K PMUs have been installed. The task of the proposed placement staging method is to rank the selected K PMU placement locations based on a specified utility function. Given the PMU installation constraints such as financial or time constraints, the K PMU placement locations can then be divided into consecutive stages based on the computed ranks.

The utility function must account for multiple diverse criteria that need to be simultaneously optimized. The proposed method utilizes the FWA to compute the utility value. The result of the FWA for the i^{th} candidate power bus can be denoted as y^i_{FWA} , which itself is a FS. The proposed ranking method is based on the defuzzified value $c(y^i_{FWA})$ of the FWA result. Hence:

$$y^1_{FWA} \underset{>}{\le} y^2_{FWA} \text{ if } c(y^1_{FWA}) \underset{>}{\le} c(y^2_{FWA}) \qquad (17)$$

For completeness, the defuzzified value of the fuzzy set y^i_{FWA} can be computed as:

$$c(y^i_{FWA}) = \frac{\sum_{j=1}^{M} y_j \, \mu_{y^i_{FWA}}(y_j)}{\sum_{j=1}^{M} \mu_{y^i_{FWA}}(y_j)} \qquad (18)$$

Here, M denotes the number of samples in the output domain and y_j is the j^{th} sample in the domain of output variable y.

VI. Experimental Results

This section first describes the experimental test case. Next, the result of the optimal PMU placement and the FWA based staging of the placement are demonstrated.

A. Test Data Set

The proposed method for multi-criteria based staging of OPP was applied to the IEEE 30-bus data set. For solving the OPP the Zero-Injection (ZI) buses were considered. The location of the ZI buses was adapted from [30] and for clarity it is summarized in Table I.

As described above the four criteria of observability, cost, importance, and security were considered. The observability criterion was calculated as the number of incident power lines. Due to the lack of available information, the values of the cost

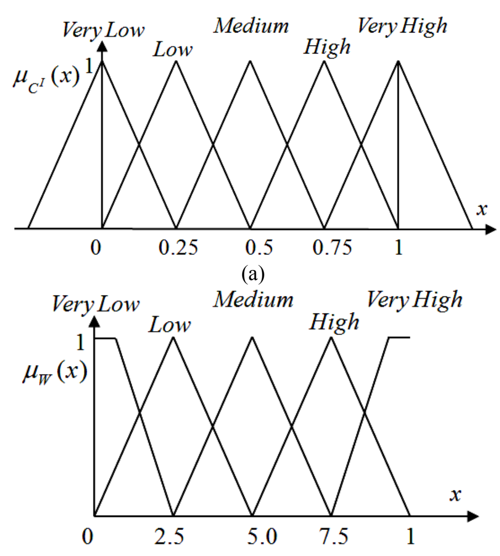


Fig. 6 Fuzzy membership of the importance criteria (a) and of the criteria weights (b).

criterion were randomly initialized as interval values. Note, that since smaller cost is more desirable, the actual value of the cost criterion is inversely proportional to the cost. The importance of each bus for PMU placement was also randomly initialized for this experimental demonstration of the proposed methodology. Five triangular FSs were used to express the importance value as depicted in Fig. 6(a)

The security measure is proportional to the number of sparse data integrity attacks that involve a particular power bus or its incident line. Lists of possible 3 and 5 sparse attacks (attacks including 3 or 5 buses and power lines) is provided in Table II [2]. The more attacks which could be executed using a specific bus the higher the value of the security criterion for that bus.

The criteria selection weights have been modeled using 5 trapezoidal and triangular fuzzy sets as depicted in Fig. 6(b). It is important to note here that all criteria values have been normalized into a unit interval between 0 and 1. Table III provides a list of the used criteria for the IEEE 30-bus system.

In order to visualize the distribution of different criteria values in the power grid, a novel method using the Voronoi Diagram (VD) was used. The VD constitutes a space decomposition, where each individual cell constitutes a set of points for which the specific cell object is the nearest neighbor. In the implemented VD, the power buses were used as the cell objects. The color of each cell then represents the value of specific criteria. It should be noted that the size of different cells is not important and it is determined by the spatial topology of the grid. The VD allows simple visual assessment of the distribution of different criteria values for each bus in the network. Fig. 7 shows the values of the four criteria used for the IEEE 30-bus data set visualized using the VD method. Note that for the interval and the fuzzy values the defuzzified value was used in the figure. The darker the color the higher the value is.

TABLE I
DATA SET DESCRIPTION FOR THE IEEE 30-BUS TEST CASE

Test Case	Number of Lines	Number of ZI buses	Location of ZI buses
30-bus	41	5	6, 9, 25, 27, 28

TABLE II
LIST OF SPARSE ATTACKS PRESENT IN THE IEEE 30-BUS DATA SET

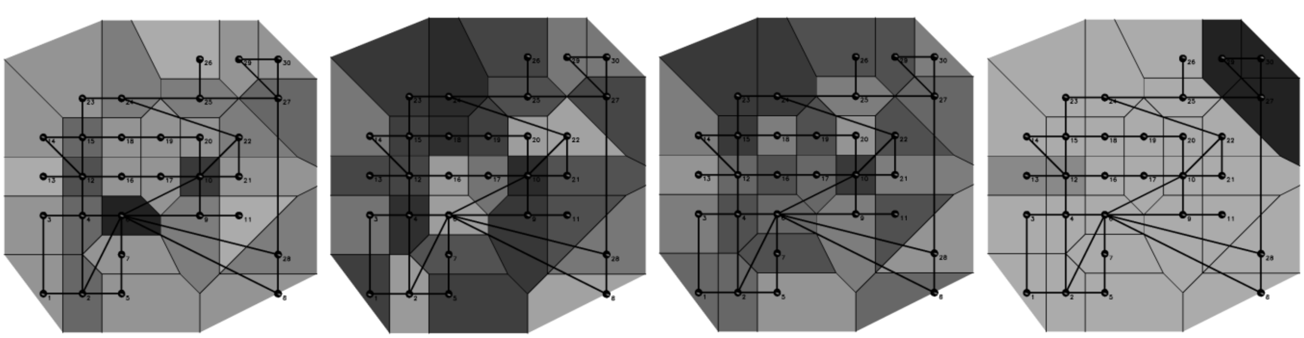
Attack Number	Attack Type	Attacked Buses	Attacked Lines
1	3-Sparse	12, 13	12-13
2	5-Sparse	27, 29	27-29, 27-30, 29-30
3	5-Sparse	27, 30	27-29, 27-30, 29-30
4	5-Sparse	29, 30	27-29, 27-30, 29-30

Optimum PMU Placement Using Genetic Algorithm

First, the GA was used to calculate the optimum placement of PMUs in the power grid. The implementation details of the GA are as follows. The population consisted of 100 individuals and the optimization was terminated after 100 iterations. The two-point cross-over and a random bit-flip mutation operators were used with a mutation rate set at 0.2. Tournament selection was used for parent selection with tournament size of 4. The above mentioned algorithm parameters were empirically selected based on extensive

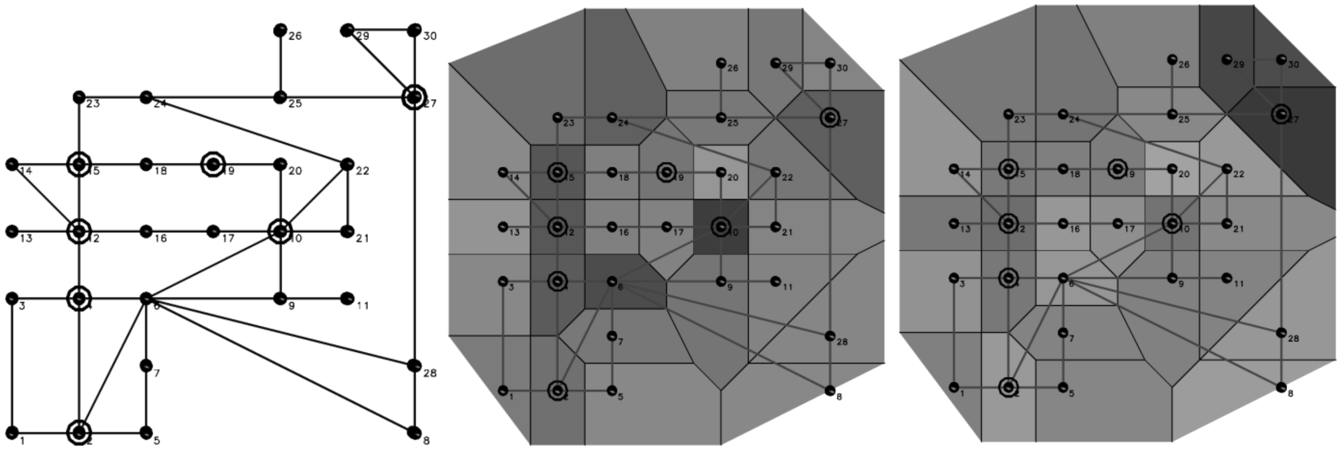
TABLE III
LIST OF CRITERIA VALUES FOR THE IEEE 30-BUS DATA SET

Bus	C^O	C^C	C^I	C^S		Bus	C^O	C^C	C^I	C^S
1	0.17	[0.70 0.90]	Medium	0.00		16	0.17	[0.09, 0.19]	High	0.00
2	0.50	[0.09, 0.19]	Medium	0.00		17	0.17	[0.32, 0.52]	Medium	0.00
3	0.17	[0.32, 0.52]	Low	0.00		18	0.17	[0.82, 1.00]	Low	0.00
4	0.50	[0.82, 1.00]	High	0.00		19	0.17	[0.69, 0.89]	High	0.00
5	0.17	[0.69, 0.89]	Very Low	0.00		20	0.17	[0.02, 0.22]	Very Low	0.00
6	1.0	[0.02, 0.22]	Medium	0.00		21	0.17	[0.61, 0.71]	Low	0.00
7	0.17	[0.61, 0.71]	High	0.00		22	0.33	[0.04, 0.14]	High	0.00
8	0.17	[0.04, 0.14]	Medium	0.00		23	0.17	[0.75, 0.95]	Very High	0.00
9	0.33	[0.75, 0.95]	Low	0.00		24	0.33	[0.88, 0.98]	Very High	0.00
10	0.83	[0.28, 0.48]	Very High	0.00		25	0.33	[0.58, 0.78]	Low	0.00
11	0.00	[0.58, 0.78]	High	0.00		26	0.00	[0.66, 0.86]	High	0.00
12	0.66	[0.66, 0.86]	Medium	0.25		27	0.50	[0.64, 0.84]	Medium	1.00
13	0.00	[0.64, 0.84]	Very Low	0.25		28	0.33	[0.34, 0.44]	Very Low	0.00
14	0.17	[0.34, 0.44]	Low	0.00		29	0.17	[0.34, 0.44]	High	1.00
15	0.50	[0.70, 0.90]	Very High	0.00		30	0.17	[0.34, 0.44]	Low	1.00



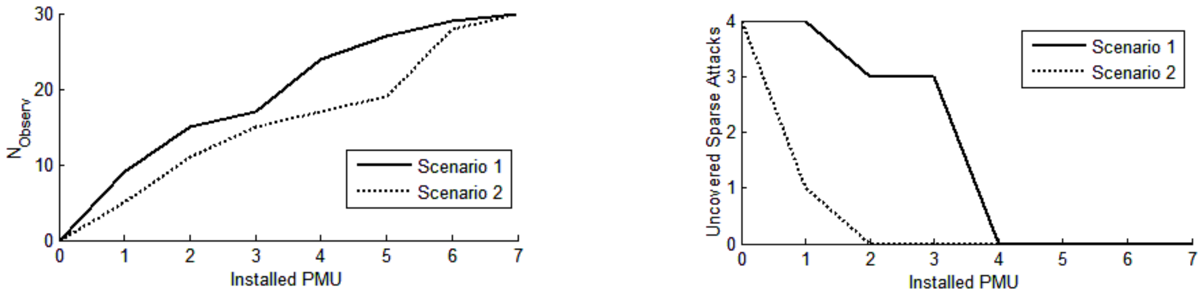
(a) (b) (c) (d)

Fig. 7 Voronoi Diagram visualization of the observability (a), cost (b), importance (c) and the security criteria (d).



(a) (b) (c)

Fig. 8 Computed OPP solution (a) and the utility function for scenario 1 (b) and for scenario 2 (c).



(a) (b)

Fig. 9 The number of observed buses (a) and the number of uncovered sparse attacks (b) as a function of installed PMUs in two test scenarios.

experimental testing. The initial population was randomly initialized in the solution space.

The selected buses for PMU placement are buses: 2, 4, 10, 12, 15, 19 and 27. The buses are circled in the grid topology visualized in Fig. 8(a).

Staging PMU Placement Using FWA

In order to demonstrate the staging of the PMU placement, two scenarios with different criteria weights were implemented. The criteria weight assignment is listed in Table IV. It can be seen that in the first scenario the maximum weight has been assigned to observability followed by importance. In the second scenario, security was the most important criterion followed by the cost. The result of the PMU staging in both scenarios is listed in Table V. The VD views of the grid with aggregated utility values computed using the FWA are shown in Fig. 8(b) and Fig 8(c).

The results demonstrate the proposed method reasonably ranks the candidate PMU buses according to the utility function based on the mixture of provided criteria values. For example, buses 27 and 12 have the highest values of the security criterion. However, since the weight of this criterion was selected as very low in the first scenario it takes 4 PMU installations before both buses 12 and 27 (and all their possible sparse data integrity attacks) are covered. On the other hand, in the second scenario where the security criterion was assigned very high weight the PMUs on buses 27 and 12 will be installed as the first two PMUs in the power grid.

TABLE IV
FUZZY CRITERIA WEIGHTS FOR THE TESTING SCENARIOS

Scenario	W^O	W^C	W^I	W^S
1	Very High	Very Low	Low	Very Low
2	Very Low	Medium	Low	Very High

TABLE V
PHASING OF THE PMU INSTALLATION FOR THE TESTING SCENARIOS

Bus Rank	Scenario 1		Scenario 2	
	Bus No.	Utility	Bus No.	Utility
1	10	0.75	27	0.83
2	12	0.61	12	0.45
3	15	0.56	4	0.37
4	27	0.54	15	0.36
5	4	0.53	19	0.34
6	2	0.45	10	0.26
7	19	0.30	2	0.13

Fig. 9 shows the number of observed buses and the number of uncovered sparse data integrity attacks in the grid as a function of the installed PMUs. It can be verified that the number of observed buses increases faster in the first scenario where observation is highly important, while the number of uncovered sparse attacks drops quickly in the second scenario where security is considered more important.

CONCLUSION

This paper proposed a multi-criteria based two step method for optimal PMU placement using a GA followed by Fuzzy

Weighted Average. In the first step of the method, the Genetic Algorithm was used to compute the OPP solution. In the second step, the PMU installation criteria were modeled as Fuzzy Sets (FSs) and the FWA was applied to rank the selected PMU installation sites for use in partitioning the PMU installments into multiple stages. The criteria of observability, cost, importance, and security were used here for the multi-criteria decision making. The proposed method was applied to the IEEE 30-bus data sets. It was demonstrated that the proposed method can handle a mixture of criteria types (real values, intervals and fuzzy sets) and produce appropriate staging strategy for the PMU installation.

REFERENCES

[1] V. Terzija, G. Valverde, D. Cai, P. Regulski, V. Madani, J. Fitch, S. Skok, M. M. Begovic, A. Phadke, "Wide-Area Monitoring Protection, and Control of Future Electric Power Networks," in *Proc. of the IEEE*, vol. 99, issue: 1, pp. 80-93, Jan. 2011.

[2] A. Giani, E. Bitar, M. McQueen, P. Khargonekar, K. Poolla, "Smart Grid Data Integrity Attacks: Characterization and Countermeasures," in *Proc. Of IEEE SmartGridComm*, Oct. 2011.

[3] S. Chakrabarti, G. K. Venayagamoorthy, E. Kyriakides, "PMU Placement for Power System Observability using Binary Particle Swarm Optimization," in *Proc. of Australasian Universities Power Engineering Conference,* pp. 1-5, Dec. 2008.

[4] C. G. Rieger, D. I. Gertman, M. A. McQueen, "Resilient Control Systems: Next Generation Design Research," in *Proc. 2nd IEEE Conf. on Human System Interactions*, pp. 632-636, May 2009.

[5] M. McQueen, A. Giani, "'Known Secure Sensor Measurements' for Critical Infrastructure Systems: Detecting Falsification of Systems State," in *Proc. of SERENE*, 2011.

[6] T. L. Baldwin, L. Mili, M. B. Boisen Jr., R. Adapa, "Power System Observability With Minimal Phasor Measurement Placement," in *IEEE Trans. on Power Systems*, vol. 8, no.2, pp. 707-715, May 1993.

[7] D. Dua, S. Dambhare, R. K. Gajbhiye, S. A. Soman, "Optimal Multistage Scheduling of PMU Placement: An ILP Approach," in *IEEE Trans. on Power Delivery*, vol. 23, no. 4, pp. 1812-1820, Oct. 2008.

[8] B. Gou, "Generalized Integer Linear Programming Formulation for Optimal PMU Placement, " in *IEEE Trans. on Power Systems*, vol. 23, no. 3, pp. 1099-1104, Aug. 2008.

[9] F. Aminifar, M. Fotuhi-Firuzabad, M. Shahidehpour, A. Khodaei, "Probabilistic Multistage PMU Placement in Electric Power Systems," in *IEEE Trans. on Power Delivery*, vol. 26, no. 2, pp. 841-849, April 2011.

[10] A. Sadu, R. Kumar, R. G. Kavasseri, "Optimal placement of Phasor Measurement Units using Particle swarm Optimization," in *Proc. of World Congress on Nature & Biology Inspired Computing*, pp. 1708-1713, 2009.

[11] M. Hajian, A. M. Ranjbar, T. Amraee, B. Mozafari, "Optimal placement of PMUs to maintain network observability using a modified BPSO algorithm," in *Intl. Journal of Electrical Power & Energy Systems*, vol. 33, issue: 1, pp. 28-34, Jan. 2011.

[12] F. J. Marin, F. Garcia-Lagos, G. Joya, F. Sandoval, "Genetic algorithms for optimal placement of phasor measurement units in electrical networks," in *Electronics Letters*, vol. 39, issue: 19, pp. 1403-1405, Sep. 2003.

[13] B. Milosevic, M. Begovic, "Nondominated Sorting Genetic Algorithm for Optimal Phasor Measurement Placement," in *IEEE Trans. on Power Systems*, vol. 18, issue: 1, pp. 69-75, Feb. 2003.

[14] F. Aminifar, C. Lucas, A. Khodaei, M. Fotuhi-Firuzabad, "Optimal Placement of Phasor Measurement Units Using Immunity Genetic Algorithm, " in *IEEE Trans. on Power Delivery*, vol. 24, no. 3, pp. 1014-1020, July 2009.

[15] K. Mazlumi, H. Vahedi, "Optimal Placement of PMUs in Power Systems Based on Bacterial Foraging Algorithm," in *Proc. of 18th Iranian Conference on Electrical Engineering*, pp. 885-888, May 2010.

[16] X. Bian, J. Qiu, "Adaptive Clonal Algorithm and Its Application for Optimal PMU Placement," in *Proc. of Intl. Conf. on Communications, Circuits and Systems*, pp. 2102-2106, June 2006.

[17] J. Peng, Y. Sun, H. F. Wang, "Optimal PMU placement for full network observability using Tabu search algorithm," in *International Journal of Electrical Power & Energy Systems*, vol. 28, issue: 4, pp. 223-231, May 2006.

[18] S. Chakrabarti, E. Kyriakides, "Optimal Placement of Phasor Measurement Units for Power System Observability," in *IEEE Trans. on Power Systems*, vol. 23, no. 3, pp. 1433-1440, Aug. 2008.

[19] Ch. Rakpenthai, S. Premrudeepreechacharn, S. Uatronngjit, N. R. Watson, "An Optimal PMU Placement Method Against Measurement Loss and Branch Outage," in *IEEE Trans. on Power Delivery*, vol. 22, no. 1, pp. 101-107, Jan. 2007.

[20] M. Korkali, A. Abur, "Placement of PMUs with Channel Limits," in *Proc. of IEEE Power & Energy Society General Meeting*, pp. 1-4, July 2009.

[21] R. Kavesseri, S. K. Srinivasan, "Joint Placement of Phasor and Power Flow Measurements for Observability of Power Systems," in *IEEE Trans. on Power Systems*, vol. 26, no. 4, pp. 1929-1936, Nov. 2011.

[22] J. M. Mendel, *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*, Upper Saddle River, NJ: Prentice-Hall, 2001.

[23] V. Madani, M. Parashar, J. Giri, S. Durbha, F. Rahmatian, D. Day, M. Adamiak, G. Sheble, "PMU Placement Considerations – A Roadmap for Optimal PMU Placement, " in *Proc. of Power Systems Conference and Exposition*, pp. 1-7, March 2011.

[24] R. Sodhi, S. C. Srivastava, S. N. Singh, "Multi-criteria decision-making approach for multi-stage optimal placement of phasor measurement units," in *IET Generation, Transmission & Distribution*, vol. 5, issue: 2, pp. 181-190, 2011.

[25] R. F. Nuqui, A. G. Phadke, "Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability," in *IEEE Trans. on Power Delivery*, vol. 20, no. 4, pp. 2381-2388, Oct. 2005.

[26] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley Professional, 1989.

[27] J. M. Mendel, D. Wu, *Perceptual Computing: Aiding People in Making Subjective Judgments*, Hoboken, NJ: Wiley-IEEE Press, 2010.

[28] F. Liu, J. M. Mendel, "Aggregation using the fuzzy weighted average, as computed using the Karnik-Mendel Algorithms," in *IEEE Trans. on Fuzzy Systems*, vol. 12, no. 1, pp. 1-12, 2008.

[29] D. Wu, J. M. Mendel, "Enhanced Karnik-Mendel Algorithms," in *IEEE Trans. on Fuzzy Systems*, vol. 17, no. 4, pp. 9123-934, Dec. 2008.

[30] B. Mohammadi-Ivatloo, S. H. Hosseini, "Optimal PMU Placement for Power System Observability Considering Secondary Voltage," in *Proc. of Canadian Conference on Electrical and Computer Engineering*, pp. 365-368, May 2008.