Uncertainty-Robust Design of Interval Type-2 Fuzzy Logic Controller for Delta Parallel Robot

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Abstract— Type-2 Fuzzy Logic Controllers (T2 FLCs) have been recently applied in many engineering areas. While understanding the control potentials of T2 FLCs can still be considered an open question, researchers commonly claim superiority of T2 FLCs based on a limited exploration of the space of design parameters. The contribution of this work is based on a problem-driven design of uncertainty-robust Interval T2 (IT2) FLCs. The presented methodology starts with a baseline optimized T1 FLC. Next, a group of IT2 FLCs is designed using partially-dependent approach by symmetrically blurring the membership functions around the original T1 fuzzy sets. This constrained design space allows for its systematic exploration and analysis. The performance of the designed controllers was evaluated on delta parallel robot hardware under two kinds of commonly encountered uncertainties: i) sensory noise and ii) uncertain system parameters. The experimental results showed that IT2 FLCs provide improved control performance against T1 FLCs when appropriate design of IT2 fuzzy sets is performed. In addition, it was demonstrated that excessive amount of "type-2 fuzziness" in the IT2 FLC design leads in rapid performance degradation.

***Index Terms*— Delta Parallel Robot, Interval Type-2 Fuzzy Logic Control, Robustness, Uncertainty Handling**

I. Introduction

Type-1 Fuzzy Logic Controllers (T1 FLCs) have been successfully applied in various engineering areas over the last 40 years [1]-[6]. This fact can be attributed to their ability to cope with the linguistic uncertainty originating in the imprecise and vague meaning of words. However, when various kinds of dynamic uncertainties are encountered, the control performance of T1 FLCs can significantly deteriorate [7]. The various sources of uncertainty were identified as follows: i) uncertainty in the linguistic knowledge used to construct the FLC (different designers have different opinions about the optimal behavior), ii) uncertainty about the correct outputs of the system, iii) uncertainty associated with noisy inputs, and iv) uncertainty about the data that were used to tune the parameters of the control system. These sources of uncertainty can lead to significant performance degradation of the T1 FLC. This degradation is primarily due to the T1 fuzzy membership functions themselves, which become fixed once the design process is finalized.

As an extension to T1 fuzzy logic, the Type-2 (T2) fuzzy logic was originally proposed by Zadeh [8]. T2 FLCs have experienced a widespread of research interest from many researchers in the past decade [7], [9]-[13]. T2 FLCs found successful application in many engineering areas, demonstrating their ability to outperform T1 FLCs in presence of dynamic uncertainties [14]-[18]. The fundamental difference between T1 and T2 FLCs is in the model of individual fuzzy sets. T2 fuzzy sets employ membership degrees that are themselves fuzzy sets. This additional uncertainty dimension provides new degrees of freedom for modeling dynamic uncertainties.

However, the understanding and correctness of the design process of T2 FLCs can still be considered an open question. In addition, a qualitative comparison of T1 and T2 FLCs and a complex assessment of the real potentials of T2 fuzzy logic is currently an active area of research [19]. Nevertheless, it is very common that researchers claim superiority in terms of both performance and uncertainty robustness of T2 FLCs based on a limited exploration of the space of design parameters. However, as previously demonstrated, T2 FLCs might exhibit slower responsiveness and excessive dumping of the output signal in specific scenarios (e.g. autonomous robot navigation) [11], [17]. It is natural to expect that with the increased amount of "type-2 fuzziness" in the controller's design, such negative effects will be further accentuated.

This manuscript provides a systematic analysis of the performance and uncertainty robustness of T2 FLCs [20]. The investigated fuzzy controllers were used for position control of parallel delta robot [21]. The robustness of a system was defined in the work of Biglarbegian et al as the maximum deviation of the output as a result of the deviations of the inputs [22]. In this manuscript, the uncertainty robustness of a system is understood as its ability to retain satisfactory performance despite perturbations of its input or parameters.

In this work, the Interval T2 (IT2) FLCs were considered [13]. The IT2 FLC assumes only interval membership grades for each T2 fuzzy set. This constitutes a significant simplification of the controller's design process. In order to allow for a systematic analysis, the space of design parameters was further constrained by using the partially-dependent design of IT2 FLC [7]. The partially-dependent design can be broken into two steps as follows: i) construct an optimal T1 FLC using one of the well-established optimization techniques (e.g. genetic algorithms), ii) blur the T1 fuzzy membership functions inserting "type-2 fuzziness" into the controller design. While this design methodology of IT2 FLC is significantly constrained and clearly not optimal, it allows for systematic exploration and understanding of the space of available design parameters. The authors believe that the presented study of this constrained set of IT2 FLCs provides important conclusions about the design process, which are widely applicable to fuzzy logic control.

In this paper, an experimental setup of three IT2 FLCs was used for position control of the end-effector of a 3DOF parallel delta robot. Such systems are applicable in many engineering areas, including robotic tele-operation, remote welding, and industrial control [23], [24]. This specific robotic architecture was chosen for its suitability for the systematic uncertainty robustness evaluation. The controllers' performance was tested on different levels of uncertainties consisting of injected noise and variable system parameters.

The rest of the paper is organized as follows. Section II provides background review of T1 and IT2 fuzzy logic control. The utilized 3DOF delta parallel robot is introduced in Section III. Section IV discusses the design of the fuzzy logic controllers. Finally, the results are presented in Section V and the paper is concluded in Section VI.

II. T1 and IT2 Fuzzy Logic Controllers

This section provides a review of T1 and IT2 FLCs.

A. Type-1 Fuzzy Logic Control

T1 FLCs have been successfully applied to many engineering problems [1], [7]. The main advantage of T1 FLCs is the ability to encode knowledge via linguistic fuzzy rules, which can be easily understood and constructed by humans. Furthermore, T1 FLCs can cope with ambiguity, imprecision and uncertainty in linguistic expressions.

In general, a T1 FLC is composed of four major parts – input fuzzification, fuzzy inference engine, fuzzy rule base and output defuzzification [7]. In this paper, the Mamdani-type FLC is considered [25]. This type of controller maintains a fuzzy rule base populated with fuzzy linguistic rules in an implicative form. As an example, consider rule R_k:

$$\textbf{IF } x_1 \text{ is } A_1^k \textbf{ AND } \ldots \textbf{ AND } x_n \text{ is } A_n^k \textbf{ THEN } y_k \text{ is } B^k \qquad (1)$$

Here, symbol A_j^k and B^k denote the j^{th} input fuzzy set and the output fuzzy set, n is the dimensionality of the input vector $\vec{x}$, and y_k is the associated output variable. System's inputs are first fuzzified using the fuzzy membership function (e.g. Gaussian, triangular, trapezoidal, etc.). The fuzzification of input x_i into fuzzy set A_i^k results in a fuzzy membership grade $\mu_{A_i^k}(x_i)$. Using the minimum t-norm, the degree of firing of rule R_k can be computed as follows:

$$\mu_{R_k}(\vec{x}) = \arg\min_{i=1..n}\{\mu_{A_i^k}(x_i)\} \qquad (2)$$

The output of each fuzzy rule is computed by applying the rule firing strength via the t-norm operator (e.g. minimum or product) to the associated rule consequent. Next, the output fuzzy sets from all rules are aggregated using the t-conorm operator (e.g. the maximum operator), resulting in an output

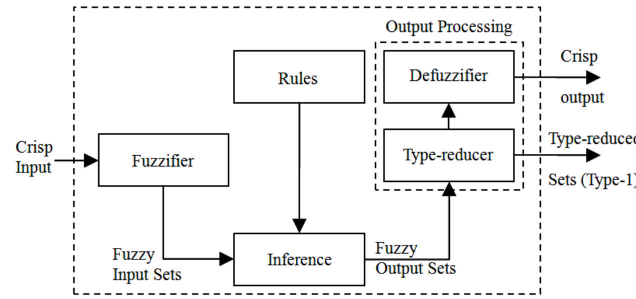


Fig. 1 Type-2 fuzzy logic control system [7].

fuzzy set B. Detailed description of the fuzzy inference process can be found in [7].

Finally, the defuzzification of the output fuzzy set B yields a crisp output value y. Several defuzzification techniques can be found in literature [7]. The centroid defuzzifier method was selected in this work. Assuming that the output domain is discretized into N samples the crisp output value y is obtained as:

$$y = \frac{\sum_{i=1}^{N} y_i \mu_B(y_i)}{\sum_{i=1}^{N} \mu_B(y_i)} \qquad (3)$$

B. Interval Type-2 Fuzzy Logic Control

The IT2 FLCs are considered in this paper because of their computational inexpensiveness and ease of implementation [13]. A block diagram of T2 FLC is depicted in Fig. 1. The major difference of T2 FLC when compared to T1 FLC is that at least one implemented fuzzy sets must be of type 2. The fuzzy inference engine then uses the T2 fuzzy inference methods to compute the T2 fuzzy output. Finally, the T2 FLC contains the output processor, which first type-reduces the fuzzy output before obtaining the crisp output through the process of defuzzification.

The number of design parameters of IT2 FLCs is substantially smaller when compared to the full-blown general T2 FLCs. An IT2 fuzzy set $\widetilde{A}$ can be expressed as follows:

$$\widetilde{A} = \int_{x\in X}\int_{u\in J_x} 1/(x,u) \qquad J_x \subseteq [0,1] \qquad (4)$$

Here, x and u are the primary and secondary variables, and J_x is the primary membership of x. All secondary membership grades of the IT2 fuzzy set $\widetilde{A}$ are equal to 1. By instantiating the variable x into a specific value x', the vertical slice of the IT2 fuzzy set can be obtained as:

$$\mu_{\widetilde{A}}(x=x',u) = \mu_{\widetilde{A}}(x') = \int_{u\in J_{x'}} 1/u \quad J_{x'} \subseteq [0,1] \qquad (5)$$

The domain of the primary memberships J_x defines the Footprint-Of-Uncertainty (FOU) of fuzzy set $\widetilde{A}$:

$$FOU(\widetilde{A}) = \bigcup_{x\in X} J_x \qquad (6)$$

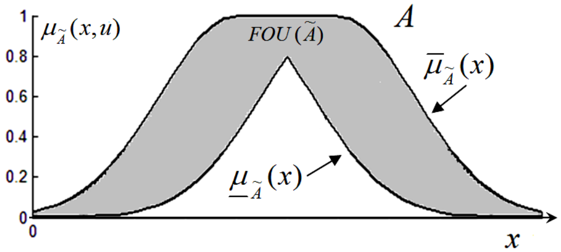


Fig. 2 Interval type-2 fuzzy set $\tilde{A}$.

The FOU of an IT2 fuzzy set is schematically depicted in Fig. 2. Alternatively, the FOU of an IT2 fuzzy set $\tilde{A}$ can be completely described by its upper and lower membership functions:

$$FOU(\tilde{A}) = \bigcup_{\forall x \in X} (\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)) \tag{7}$$

This constitutes a substantial simplification when compared to the general T2 fuzzy sets. Here, only two T1 fuzzy membership functions (the upper and the lower boundary of the FOU) are needed to fully describe the IT2 fuzzy set. This simplification is then transferred through the inference mechanism of the IT2 FLC taking advantage of the modified interval T2 fuzzy join and meet operations [7]. The interval join and meet operations work exclusively with the FOU of the IT2 fuzzy sets, thus removing much of the computational burden associated with processing of general T2 fuzzy sets.

In order to obtain a crisp output value, the resulting IT2 output fuzzy set $\tilde{B}$ is first type reduced and then defuzzified. The centroid of the IT2 fuzzy set $\tilde{B}$ can be defined as [26]:

$$C_{\tilde{B}} = \int_{\theta_1 \in J_{y_1}} \cdots \int_{\theta_N \in J_{y_N}} 1 \Bigg/ \frac{\sum_{i=1}^{N} y_i \theta_i}{\sum_{i=1}^{N} \theta_i} \tag{8}$$

Here, every combination $\theta_1,\ldots,\theta_N$ forms an embedded fuzzy set. The centroid $C_{\tilde{B}}$ is an interval T1 fuzzy set, which can be completely described by its left and right end points y_l and y_r:

$$y_l(\tilde{B}) = \frac{\sum_{i=1}^{L} y_i\, \overline{\mu}_{\tilde{B}}(y_i) + \sum_{i=L+1}^{N} y_i\, \underline{\mu}_{\tilde{B}}(y_i)}{\sum_{i=1}^{L} \overline{\mu}_{\tilde{B}}(y_i) + \sum_{i=L+1}^{N} \underline{\mu}_{\tilde{B}}(y_i)} \tag{9}$$

$$y_r(\tilde{B}) = \frac{\sum_{i=1}^{R} y_i\, \underline{\mu}_{\tilde{B}}(y_i) + \sum_{i=R+1}^{N} y_i\, \overline{\mu}_{\tilde{B}}(y_i)}{\sum_{i=1}^{R} \underline{\mu}_{\tilde{B}}(y_i) + \sum_{i=R+1}^{N} \overline{\mu}_{\tilde{B}}(y_i)} \tag{10}$$

The important switching points L and R can be calculated using the Karnik-Mendel iterative procedure [27]. Using the boundary values of the centroid $C_{\tilde{B}}$, the final crisp defuzzified value y can be computed as the mean of the centroid interval:

$$y = \frac{(y_l + y_r)}{2} \tag{11}$$

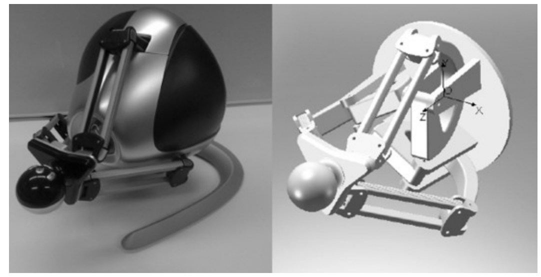


Fig. 3 Novint Falcon haptic device [21].

III. 3DOF Delta Parallel Robot

In this paper, the IT2 FLC is applied to the problem of precise and uncertainty robust position control of a 3DOF delta parallel robot [28]-[30]. In general, parallel robotic architectures feature increased structural rigidity and available workload when compared to their sequential counterparts. However, parallel robots also suffer from constrained workspace and increased complexity of the kinematic and dynamic mechanism [21], [28].

The utilized robotic platform consists of the Novint Falcon haptic device [31]. This force-controlled delta parallel robot configuration has three degrees of freedom that are force-feedback and tactile sensation enabled. Working frequency of 1kHz and position resolution of 400 dpi within the robot's workspace allows for smooth control of the multi-robot system as well as fluent perception of the generated haptic force. A detailed specification of the robot was reported in [21]. The Novint Falcon haptic device is depicted in Fig. 3.

The Falcon haptic device typically serves as a 3DOF input device. However, due to the presence of force-feedback, the flow of the control signal can be reversed creating a 3DOF robotic manipulator. The Falcon device can then be viewed as a 3DOF delta parallel robot. From a robotic control point of view the Novint Falcon constitutes a suitable experimental platform as its sampling frequency of 1kHz and smooth actuation allow for very precise position sensing and high fidelity motion control. The application of the Falcon device for research as a robotic manipulator was investigated in [21]. The Falcon's programmable interface abstracts the user from the inverse kinematics of the robot, which provides convenient control of robot's motion on the three motion axis – x, y, z.

IV. IT2 FLC Design For 3DOF Delta Parallel Robot

This section describes the initial baseline design of the T1 FLC for position control of the delta parallel robot and its subsequent extension to IT2 FLC. The robustness of the constructed IT2 FLC is then evaluated with respect to the baseline T1 FLC. The design methodology can be summarized in three steps as follows:

Step 1: Manually design an initial T1 FLC.

Step 2: Tune the parameters of the initial T1 FLC using one of the available optimization techniques (here the Particle Swarm Optimization (PSO) algorithm was used).

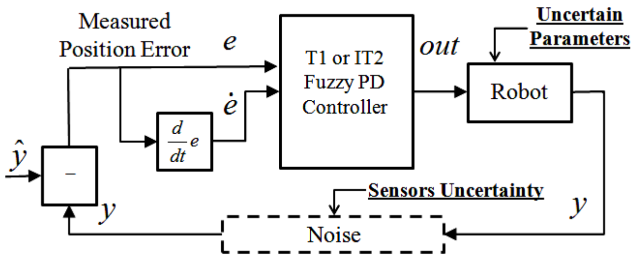


Fig. 4 The fuzzy PD control system of the delta parallel robot.

TABLE I
FUZZY RULE TABLE

Rule 1:	IF *error* IS *negative* AND *error derivative* IS *negative* THEN *out* IS *negative*
Rule 2:	IF *error* IS *negative* AND *error derivative* IS *positive* THEN *out* IS *zero*
Rule 3:	IF *error* IS *positive* AND *error derivative* IS *negative* THEN *out* IS *zero*
Rule 4:	IF *error* IS *positive* AND *error derivative* IS *positive* THEN *out* IS *positive*

Step 3: Extend the T1 FLC into an IT2 FLC using the partially dependent design.

Individual steps are described below.

A. Initial T1 FLC Design

For the position control of the delta parallel robot, the fuzzy Proportional-Derivative (PD) controller was considered. Here, the control signal $out(t)$ at time t is proportional to the measured error signal $e(t)$ and its time derivative $\dot{e}(t)$. The error is calculated as the difference between the end-effector's actual and desired position $y(t)$ and $\hat{y}(t)$. A schematic view of the fuzzy PD control system connected to the robotic platform is presented in Fig. 4. The figure also depicts the two considered sources of uncertainty, the uncertain parameters of the robotic system and the uncertainty in the robot's position sensing.

The fuzzy PD controller is commonly adopted in engineering applications for its several advantages: i) it can be constructed using linguistic knowledge about the controlled system, ii) it features more design degrees of freedom, and iii) it was shown to produce smoother and more robust control behaviors [32].

In this paper, the fuzzy controller was designed using two trapezoidal input fuzzy sets {*negative*, *positive*} for describing the input signals error e and error derivative $\dot{e}$. The controller's output signal *out* was modeled using three triangular output fuzzy sets {*negative*, *zero*, *positive*}. The fuzzy rule base presented in Table I lists the four linguistic fuzzy rules for the position control. This rule base constitutes a commonly adopted set of rules for fuzzy PD controllers. As derived in [32], the fuzzy PD controller can be understood as a composition of multiple classical PD controllers with variable gains.

In this specific application, three fuzzy PD controllers were implemented for each motion axis x, y, z of the 3DOF delta parallel robot. The error of the controller was calculated as the difference between the desired and the actual position of the robot's end-effector. The initial parameters of the T1 fuzzy sets were manually adjusted to produce stable and continuous initial robot's performance [33]. The initial design of the input and output fuzzy sets for position control on the robot's x-axis is depicted in Fig. 5(a)-(c). Similarly, the T1 FLCs were

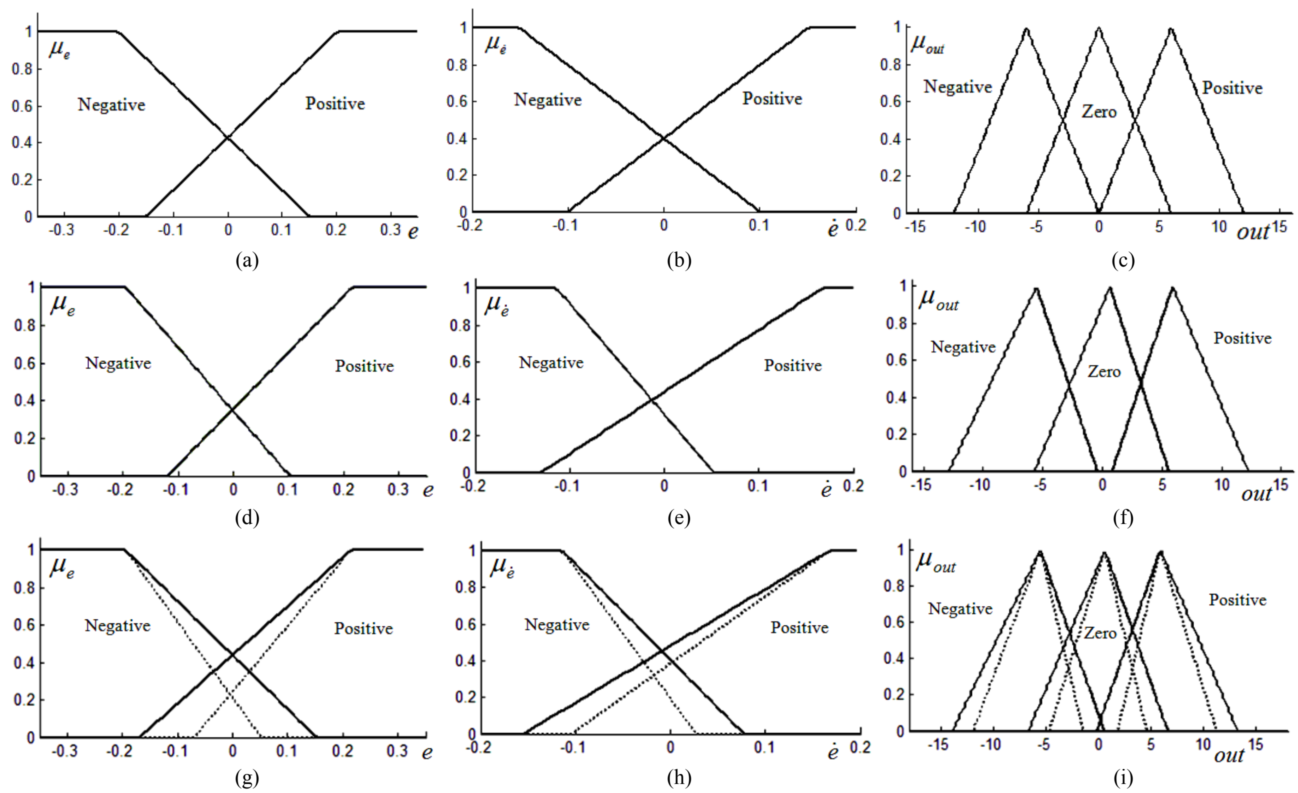
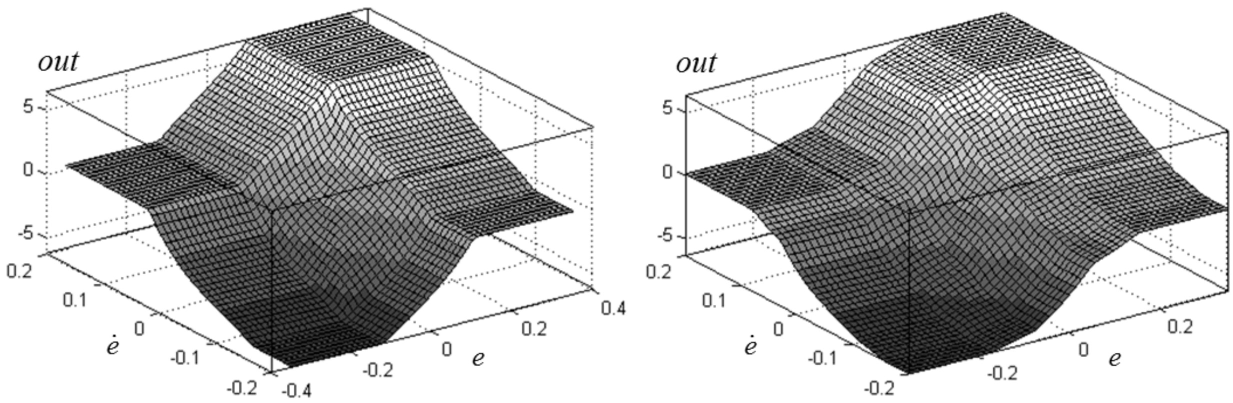
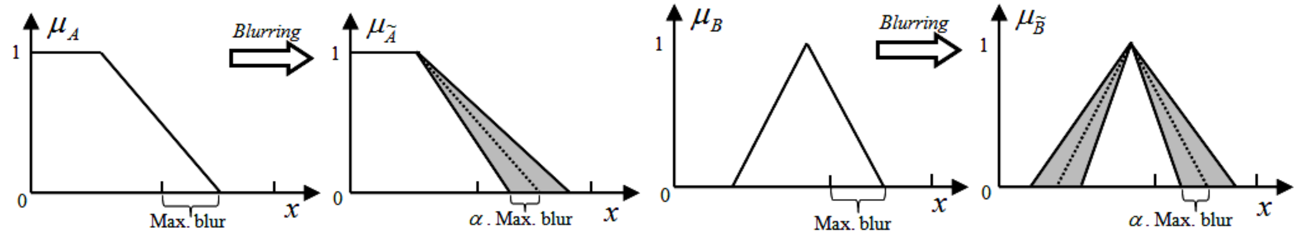


Fig. 5 Fuzzy control for the robot's x-axis: initial design of T1 FLC (a)-(c), optimized design of the T1 FLC (d)-(f) and the IT2 FLC constructed via the partially-dependent approach (g)-(i)



(a) (b)

Fig. 6 The output surface of the optimized T1 FLC (a) and the IT2 FLC constructed via the partially-dependent approach (b).



(a) (b)

Fig. 7 Illustration of the partially-dependent symmetrical blurring of T1 fuzzy membership functions with blurring parameter α = 0.5 for trapezoidal (a) and for triangular (b) fuzzy membership functions.

designed for robot's *y* and *z* axes.

Next, the tools of evolutionary computation were used to further optimize the performance of the T1 FLC. Here, the PSO algorithm was used to tune the parameters of the fuzzy membership functions of all three T1 FLCs. The linguistic rules of the controllers remained fixed during the parameter tuning process. The PSO algorithm is a biologically and physically inspired paradigm, which has been successfully applied to many optimization problems [34]. Fig. 5(d)-(f) depict the architecture of the optimized T1 FLC for the robot's *x*-axis. Fig. 6(a) shows the associated output control surface. Note that other optimization techniques could be applied in this step (e.g. gradient descent [35]).

B. IT2 FLC Design

In general, there are two available strategies for designing IT2 FLCs [7]. First, the controller can be designed via fully-independent approach, where the entire IT2 FLC is designed from a scratch. Secondly, the partially-dependent approach can be used when an initial T1 FLC is constructed first. This initial T1 FLC is then extended into the IT2 FLC via a blurring process. Hence, the IT2 FLC builds on the architecture of the original T1 FLC providing additional performance improvements.

The partially-dependent IT2 FLC design was favored in this work for two primary reasons. First, this approach reduces the number of design parameters. Second, the constructed IT2 FLC can be objectively compared to the initial baseline T1 FLC. In order to allow for systematic exploration of the design space, the simplified version of the partially-dependent design was used, where triangular and trapezoidal fuzzy sets with identical symmetrical spreads were implemented.

The partially-dependent design approach can be described in two steps as follows. First, for each fuzzy set of the optimized T1 FLC the maximum allowable spread is determined ensuring continuous control behavior [33]. Next, a blurring parameter $\alpha \in [0.0, 1.0]$, specifying the portion of the maximum spread, is used to construct the fuzzy sets of the IT2 FLC. Note that blurring parameter $\alpha = 0.0$ reduces the IT2 FLC to the original T1 FLC, whereas blurring parameter $\alpha = 1.0$ implements IT2 fuzzy set with the maximum amount of blur and the widest FOUs of individual fuzzy sets. An illustrative example is depicted in Fig. 7 for the case of trapezoidal and triangular membership functions blurred with the blurring parameter $\alpha = 0.5$.

Identical value of the blurring parameter was used for all fuzzy membership function of the entire IT2 FLC. In this manner, the number of design parameters of the partially-dependent approach was reduced to a single parameter – the blurring parameter α. Fig. 5(g)-(i) depict the resulting IT2 FLC designed with the blurring parameter $\alpha = 0.5$ for the *x*-axis. Fig. 6(b) plots the associated output control surface. It can be observed that the IT2 FLC offers substantially smoother control performance.

V. Experimental Results

This Section presents the experimental analysis of the performance and uncertainty robustness of the designed fuzzy position controllers for the delta parallel robot. First, the performance of a group of IT2 FLCs under varying levels of sensory noise was studied. Next, the robustness of the FLCs was evaluated under uncertain system parameter. Finally, the performance of the located uncertainty robust IT2 FLCs was verified by an experimental study. At the end of this Section, a discussion of the results is given.

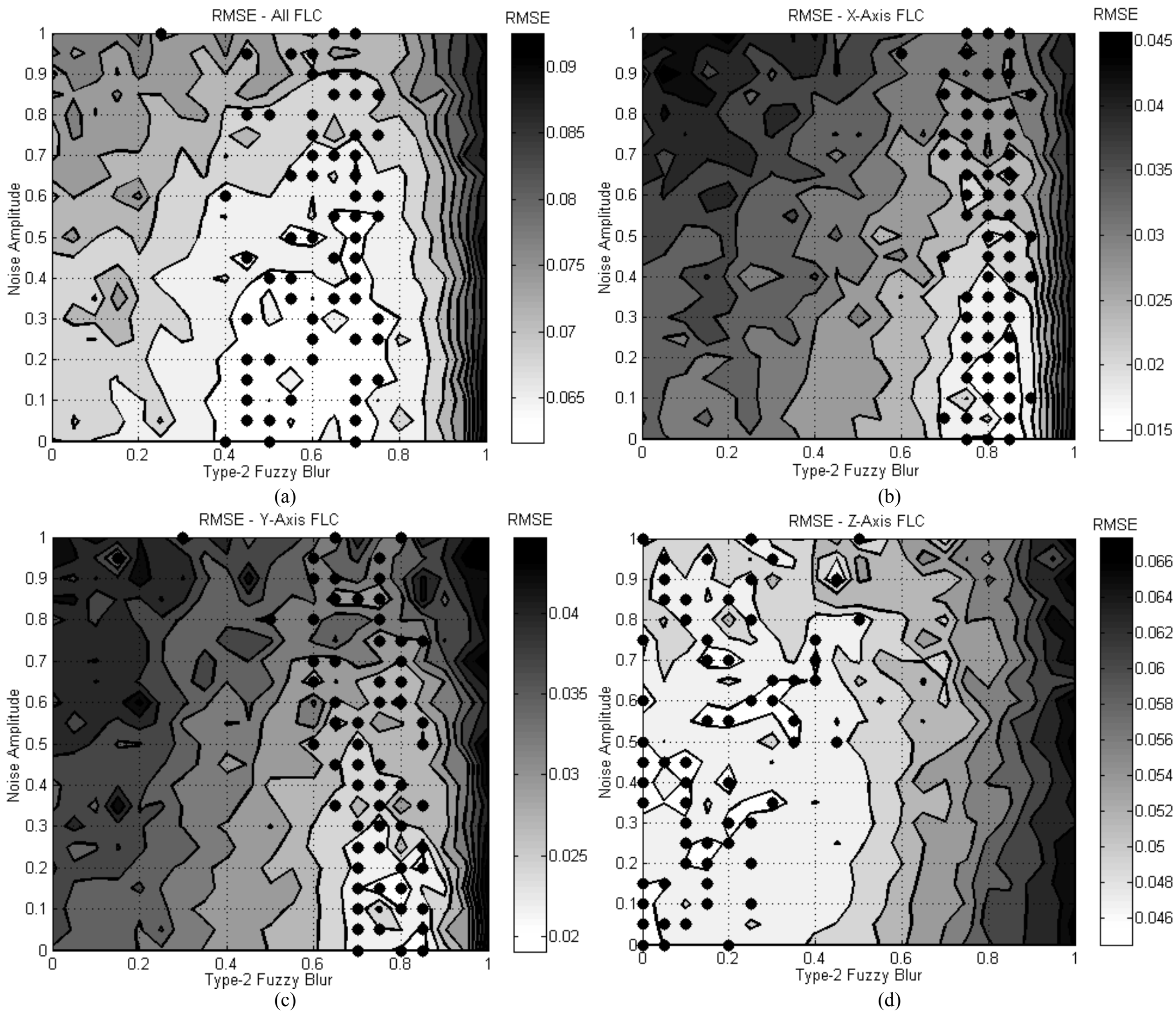


Fig. 8 The RMSE values in the Noise/Blur space for the 3DOF delta parallel robot position control (a), the *x*-axis control (b), the *y*-axis control (c), and the *z*-axis control (d) (the black points depict 3 levels of blur with the lowest RMSE for the each noise level).

A. Sensory Uncertainty

In this experiment, the performance of a set of IT2 FLCs under varying levels of sensory uncertainty was studied. A group of twenty-one IT2 FLCs was constructed by applying an increasing blurring parameter α to the original optimized T1 FLC covering the entire range of [0.0, 1.0]. Each designed controller was tested under dynamically changing level of sensory noise with amplitude ranging from 0% to 10% of the measured input signal. This range of noise was considered to appropriately cover the range of potential disturbances. This experimental methodology allowed for a systematic analysis of the constrained Noise/Blur (N/B) space. As such, an important insight into the performance quality and the uncertainty robustness of different IT2 FLCs for the delta parallel robot was obtained.

The sensory noise was implemented as white noise injected to the input measurements. Each constructed IT2 FLC was tested five times for each level of uncertainty in order to reduce the statistical variance of the results. The RMSE of the entire testing run was recorded for all controllers on all three robot's control axis. The testing runs consisted of applying a sinusoidal signal to all 3DOF of the robot.

The value of the blurring parameter and the noise amplitude correspond to a unique position in the N/B space. The experimental testing assigns the achieved RMSE of the robot's position controllers to each such point. The discretization of the intervals of possible values blurring parameter α and possible noise amplitudes allowed for systematic reconstruction of the distribution of RMSE values in N/B space. This experimentally constructed RMSE distribution provides a clear picture about the performance quality and uncertainty robustness of each IT2 FLC architecture for the specific uncertainty level.

Fig. 8 shows the RMSE values in the N/B space for all three FLCs on individual control axis of the robot as well as the accumulated RMSE for the 3D position control. The RMSE values are color-encoded with white/black tones representing the smallest/largest achieved RMSE values. For an ease of understanding, the best three IT2 FLC designs are depicted with black dots for each noise amplitude level. The value of the noise amplitude was normalized into a unit interval. By observing the location of the black points, the most robust IT2 FLC designs with respect to the sensory uncertainty levels can be found.

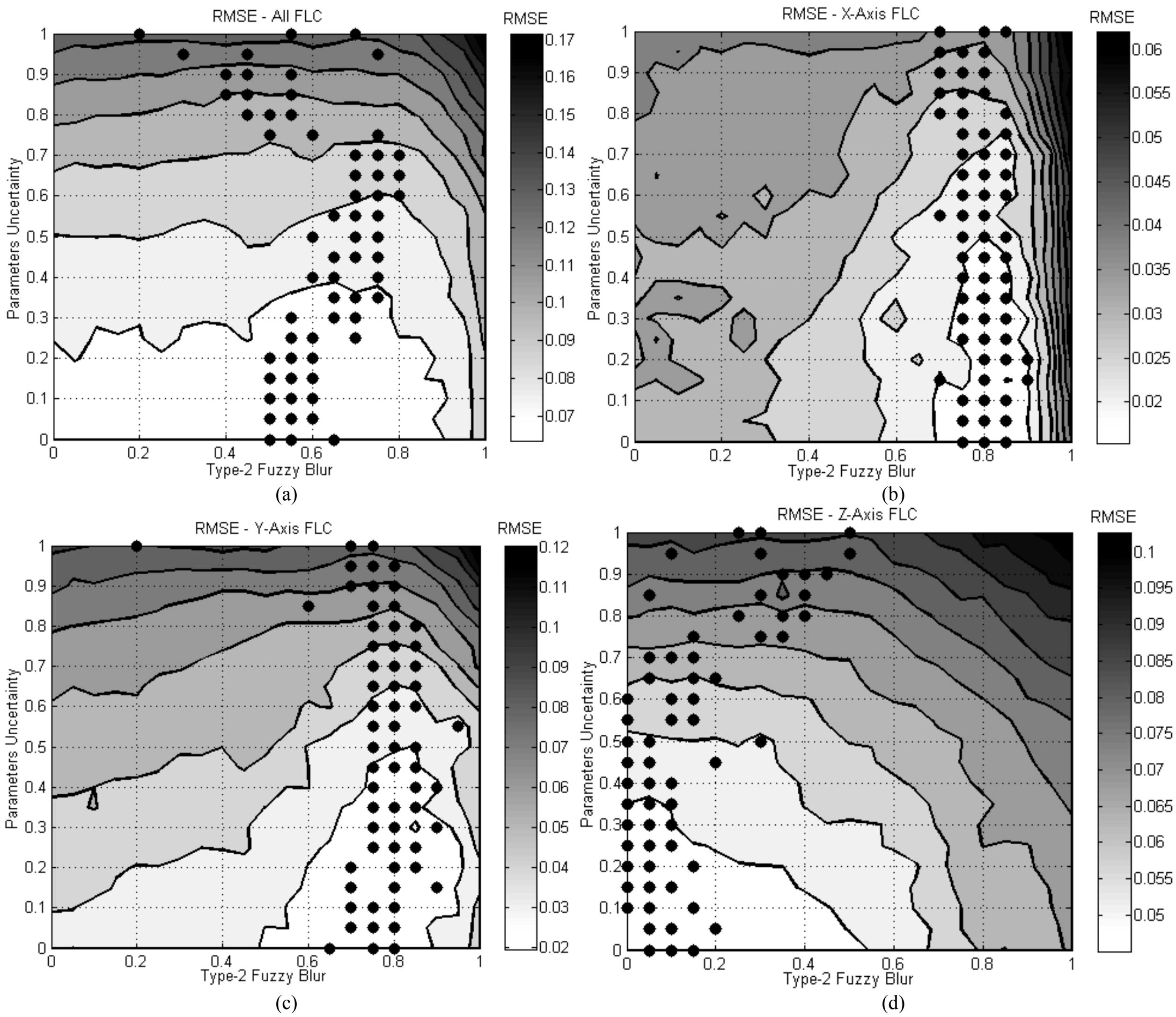


Fig. 9 The RMSE values in the uncertain Parameters/Blur pace for the 3DOF delta parallel robot position control (a), the *x*-axis control (b), the *y*-axis control (c), and the *z*-axis control (d) (the black points depict 3 levels of blur with the lowest error for the each parameters' uncertainty level).

B. System Parameters Uncertainty

In this next experiment, the performance of a set of IT2 FLCs under varying levels of system parameters' uncertainty was studied. Identical group of twenty-one IT2 FLCs was constructed as in the previous experiment. Each designed controller was tested under different levels of system parameters' uncertainty. This experiment methodology allowed for a systematic analysis of the entire uncertain Parameter/Blur (P/B) space. The uncertain system parameters' were implemented as a simulated variable friction on robot's motor drives. In this implementation the control performance of the IT2 FLC was deteriorating due to its inability to produce appropriate control signal to position the robot's end-effector accordingly to the set-point. Each constructed IT2 FLC was tested five times for each level of parameter uncertainty. Again the RMSE of the test runs was recorded for all controllers on all three robot's control axis.

Fig. 9 shows the RMSE values in the P/B space for all three FLCs on individual control axis of the robot together with the accumulated RMSE surface of the 3D trajectory tracking performance. The RMSE value is also color-encoded as in the previous experiment. The value of the amplitude of the uncertainty in system parameters was normalized into a unit interval. Similarly, the best three controller designs were depicted with black dots for each system parameters' uncertainty level. By observing the location of the black points, the most robust IT2 FLC designs with respect to uncertain system parameters can be determined.

C. IT2 FLC Robustness Testing

In this experiment, the constructed RMSE distributions in the N/B and P/B spaces were utilized for selecting the most uncertainty robust IT2 FLCs. The performance of the selected controllers was then verified.

Firstly, the performance of the T1 and IT2 FLC for robot's *x*-axis under the normalized noise level of 0.8 was studied. Following the RMSE distribution depicted in Fig. 8(b), the optimal blurring parameter value was set as 0.85. Both T1 and IT2 FLCs were applied to position control of the delta parallel robot tracking the input sinusoidal signal. The recorded and the desired trajectories are depicted in Fig. 10(a)-(b). Fig. 10(c) compares the absolute position error generated by both controllers.

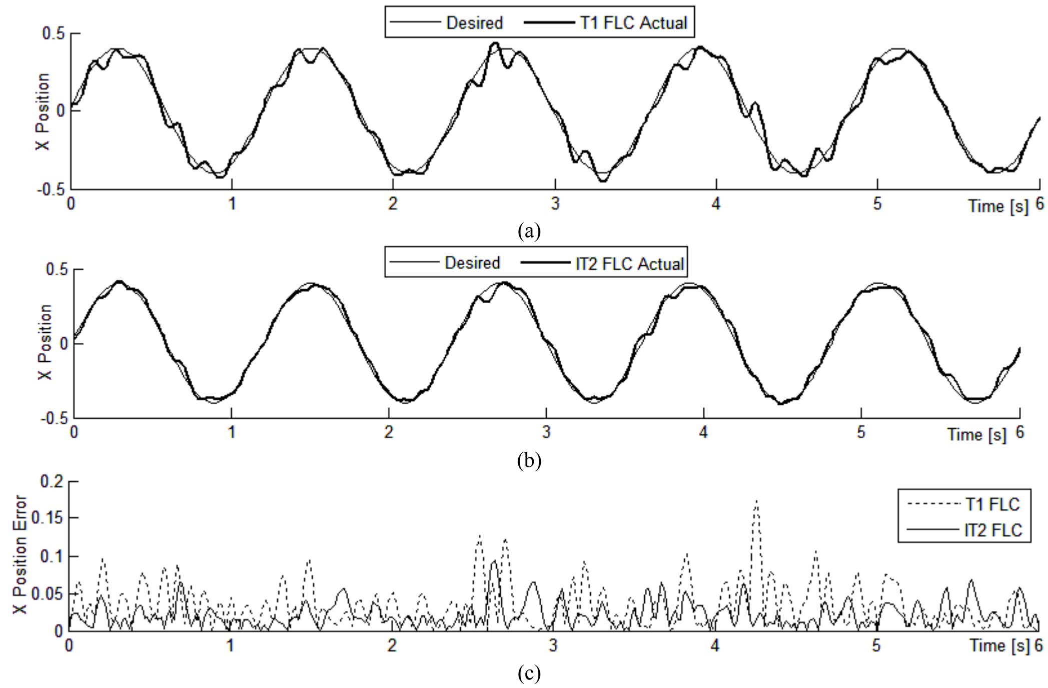


Fig. 10 The *x*-axis position control performance for the T1 FLC (a) and the IT2 FLC with amount of blur 0.85 (b) for a normalized noise level of 0.8. The position error of both controllers is depicted in (c).

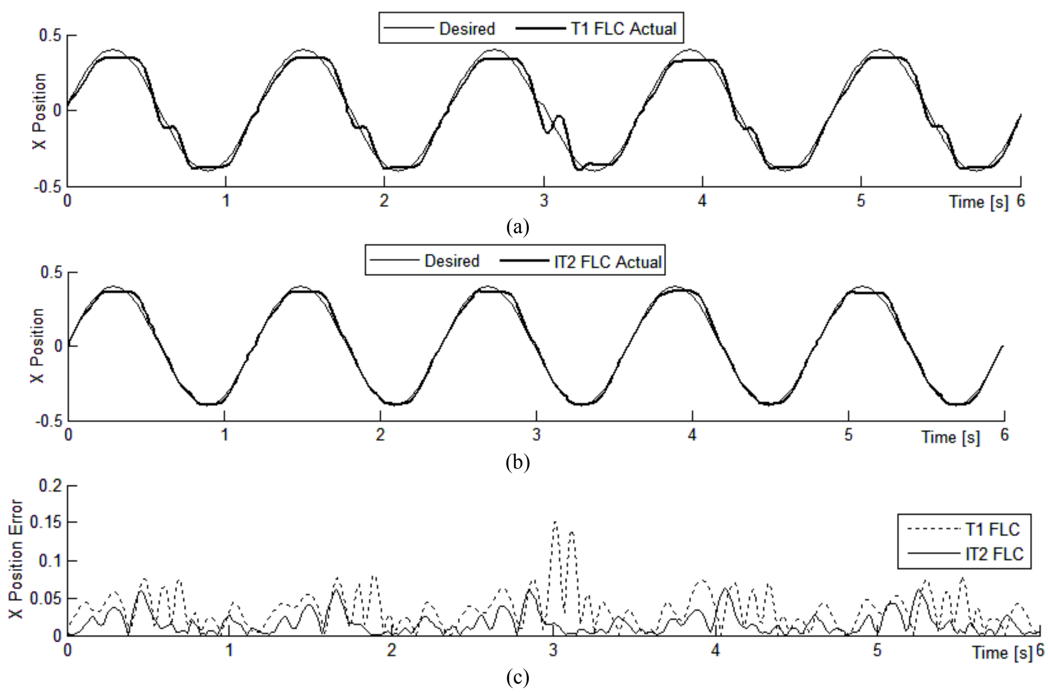


Fig. 11 The *x*-axis position control performance for the T1 FLC (a) and the IT2 FLC with amount of blur 0.85 (b) for a normalized parameters' uncertainty level of 0.7. The position error of both controllers is depicted in (c).

Secondly, the performance of the T1 and IT2 FLC for position control of the *x*-axis under the normalized system parameters' uncertainty of 0.7 was studied. Based on the RMSE distribution in P/B space shown in Fig. 9(b), the IT2 FLC was constructed with the blurring parameter of 0.85. This value provided the most uncertainty-robust design. The recorded and the desired trajectories are depicted in Fig. 11(a)-(b). Fig. 11(c) compares the absolute position error generated by both controllers. Similar results can be obtained for the *y* and *z* control axes but are omitted due to limited space.

Table II provides statistical summary of the performance evaluation of the constructed T1 and IT2 FLCs. The mean values of the position control RMSE averaged over 20 runs together with the standard deviations for all control axes *x*, *y* and *z* are provided, denoted as Err_x, Err_y and Err_z respectively. In addition, the combined mean and the standard deviation of the position tracking error $Err_{x,y,z}$ in the combined *x*, *y*, *z* 3D space are reported.

TABLE II
SUMMARY OF EXPERIMENTAL COMPARISON OF T1 AND IT2 FLCS – ERROR MEAN (LARGE FONT) AND ERROR STD (SMALL FONT).

FLC	Noise Robustness				Parameters' Uncertainty Robustness			
	$Err_{x,y,z}$	Err_x	Err_y	Err_z	$Err_{x,y,z}$	Err_x	Err_y	Err_z
T1 FLC	0.07126 ± 0.00448	0.04172 ± 0.00590	0.03717 ± 0.00281	**0.04386** ± 0.00307	0.09530 ± 0.00101	0.03829 ± 0.00087	0.06507 ± 0.00053	**0.05815** ± 0.00073
IT2 FLC	**0.07011** ± 0.00389	**0.02692** ± 0.00443	**0.03416** ± 0.00499	0.05464 ± 0.00288	**0.08760** ± 0.00349	**0.02265** ± 0.00205	**0.04194** ± 0.00480	0.07338 ± 0.00110

D. Discussion

The presented experiments analyzed the performance quality and uncertainty robustness of the IT2 FLCs applied to the position control of the delta parallel robot. Several observations can be drawn from the presented results.

The distribution of the RMSE values in the N/B space (Fig. 8) demonstrated the robustness of the IT2 FLCs when dealing with dynamic sensory noise. For both *x* and *y* control axes the increasing value of blurring parameter led to performance improvements all the way to reaching its peak point of 0.85. Quite interestingly, exceeding this peak point resulted in rapid performance deterioration, worsening in many cases below the performance of the original T1 FLC. This behavior can be attributed to the excessively wide FOUs and overly strong dumping of the controllers' response [17]. In addition, it can be observed that incorporating IT2 fuzzy logic into the design of the *z*-axis motion controller did not provide performance improvement. Instead, performance deterioration was experienced. This observation can be explained by the difference between individual robot's control axes, where the control motion in the *z*-axis (front-back) must overcome the combined friction of all three motor drives. Hence, additional output dumping by the IT2 FLC lead to performance deterioration. Clearly, the benefits in terms of performance quality and robustness provided by the IT2 FLC are application dependent, which is consistent with the recent study published by Biglarbegian et al [22]. Similar conclusions can be drawn from the distribution of the RMSE values in the P/B space depicted in Fig. 9.

Finally, Fig. 10 and Fig. 11 validated the increased robustness of the selected IT2 FLCs. In both cases, the control performance of the IT2 FLC can be found to feature increased precision and robustness in presence of different sources of dynamic uncertainties. In addition, the plotted position error in Fig. 10(c) and Fig. 11(c) illustrates significantly smaller error amplitudes and reduced position overshooting for the IT2 FLC. The summarized average performance in Table II validates the previous observations.

VI. CONCLUSION

This paper provided a problem-driven design methodology together with a systematic assessment of the performance quality and uncertainty robustness of IT2 FLCs. The method was evaluated on the problem of position control of delta parallel robots. In order to allow for systematic exploration of the problem domain, the constrained partially-dependent design of IT2 FLC was used. Two primary sources of system uncertainty were considered: sensory noise and the uncertain system parameters.

The systematic analysis of the space of design parameters and uncertainty levels provided important insights into the real-world performance quality and uncertainty robustness of IT2 FLCs. The three main conclusion points of the presented work can be summarized as follows: i) it was concluded that the IT2 FLC provides improvements in terms of both performance and robustness compared to T1 FLC, when appropriate design of IT2 fuzzy sets is performed, ii) increasing the amount of "type-2 fuzziness" monotonically improves the uncertainty robustness of the IT2 FLC up to its peak point, and iii) exceeding the optimal amount of "type-2 fuzziness" and excessively blurring the fuzzy membership functions can lead to rapid performance degradation.

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