Uncertainty Modeling for Interval Type-2 Fuzzy Logic Systems Based on Sensor Characteristics

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Abstract— **In the past decade Type-2 Fuzzy Logic Systems (T2 FLSs) gained increased research attention due to their potential to outperform Type-1 FLSs in applications with dynamic uncertainties. This advantage is typically attributed to the capability of T2 Fuzzy Sets (FSs) to better model the dynamic uncertainty and cope with its negative impacts. However, the accuracy, correctness and interpretation of such uncertainty modeling using the T2 FLSs have been rarely addressed or taken into account during the design of the respective fuzzy controller. The contribution of this paper is in analyzing the uncertainty modeling capabilities of the commonly used Interval T2 (IT2) FSs with uncertain parameters. In addition, a novel method for incorporating the experimentally measured input uncertainty into the design of the IT2 FLS is proposed. It is demonstrated that the novel IT2 FLS design method improves the accuracy of the input uncertainty model in the specific problem domain. As a consequence, the modeled uncertainty is then more accurately reflected in the output domain as the geometry of the type-reduced centroid.**

Keywords- Interval Type-2 Fuzzy Logic Systems; Uncertainty Modeling; Sensor Characteristics; Centroid

I. Introduction

Type-1 Fuzzy Logic Systems (T1 FLSs) become popular in many engineering areas due to their ability to cope with the linguistic uncertainty originating in the imprecise and vague meaning of words. However, dynamic uncertainties such as uncertainties about the measurements activating the system or the uncertainty about the training data used to tune the respective FLS can lead to performance deterioration [1]. This performance deterioration can be attributed to the fact that T1 FLSs use precise T1 fuzzy membership functions, parameters of which are fixed once the design process is finalized.

Type-2 Fuzzy Logic Systems (T2 FLSs), originally proposed by Zadeh [2], have recently become the scope of work of many researchers [1], [3]-[5]. T2 FLSs have been applied in many engineering areas, demonstrating their ability to perform better than T1 FLSs when facing dynamic uncertainties [6]-[8]. The major difference lays in the model of individual fuzzy sets, which use membership degrees that are themselves fuzzy sets. This new uncertainty dimension provides additional degree of freedom for modeling and coping with dynamic input uncertainties.

Despite recent advances in T2 FLSs, such as geometric, *zSlices* or the α -planes based representations [4], [5], [9], the applicability of general T2 FLS is still hindered by their high computational complexity and the lack of established design methodology. The most popular and widely used type of T2 FLSs is the Interval Type-2 (IT2) FLS [1], [10]. The IT2 FLSs constitute a compromise between the uncertainty modeling capabilities of general T2 FLSs and the computational inexpensiveness of T1 FLSs. This paper focuses on IT2 FLSs.

Many researchers argue in favor of the IT2 FLSs (and T2 FLSs in general) because of their potential to better model dynamic uncertainty and minimize its negative effects [1], [6], [5], [11]. In order to support such claims, the performance of IT2 FLSs is then compared to their T1 counterparts in various applications, typically demonstrating improvements when noise and uncertainty are introduced into the inputs of the system. This improved performance can be attributed to the Footprint of Uncertainty (FOU) of IT2 Fuzzy Sets (FSs), which can be seen as a composition of multiple T1 fuzzy membership functions.

However, a systematic methodology for designing the IT2 membership functions and constructing the FOU has not been established yet. Approaches such as gradient descent or evolving the MF via genetic algorithms typically limit the number of design degrees of freedom of the FOU to a very small number of parameters (e.g. evolving only the mean and spread of the MFs) [12], [13]. Recently, novel approaches for directly constructing the *zSlices* based general T2 FSs directly based on the training data have been presented by Wagner and Hagras [14], [15].

One of the outcomes of the output processing of an IT2 FLS is the interval centroid. Many researchers associated the geometrical properties of the output centroid (e.g. its width) with the uncertainty associated with the system's output [16]-[18]. For example in [16] it is said that: *"... the length of the type-reduced set can therefore be used to measure the extent of the output's uncertainty*". Other researchers used the centroid width to create an uncertainty bounds on the system output in problem domains such as predicting micro milling cutting forces or stock market analysis [19], [20]. However, the accuracy and correctness of such output uncertainty interpretations have not been addressed yet. This paper focuses on analysis of this uncertainty model interpretation.

In the presented paper, a more accurate modeling of input uncertainties in the FOUs of the IT2 input membership functions is shown to be a necessary condition allowing for correct interpretation of the output uncertainties. First, the notions of input and output uncertainty of an IT2 FLS are formalized. Then the uncertainty modeling capabilities of commonly used IT2 FSs (Gaussian fuzzy sets with uncertain mean and standard deviation) are analyzed. Next, a novel IT2 FLS design methodology is proposed, which first calibrates the system input sensors and then incorporates this measured uncertainty into the FOUs of the respective IT2 FSs. It is demonstrated that the proposed methodology more accurately models the input uncertainty, which is then translated into the geometry of the output centroid.

The rest of the paper is organized as follows. Section II provides background review on IT2 fuzzy sets and fuzzy logic systems. The uncertainty modeling capabilities of the ordinary IT2 FSs are demonstrated in Section III. The novel design methodology for IT2 FSs based on measured sensor uncertainty is demonstrated in Section IV. Experimental results are shown in Section V and the paper is concluded in Section VI.

II. Interval Type-2 Fuzzy Logic Systems

This section reviews fundamentals about the IT2 FLSs. The IT2 FLSs are considered in this paper, because of their computational in-expensiveness and ease of implementation. An IT2 fuzzy set $\widetilde{A}$ can be expressed as [1]:

$$\widetilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) \qquad J_x \subseteq [0,1] \tag{1}$$

Here, x and u are the primary and secondary variables, and J_x is the primary membership of x. In the case of IT2 FSs, all secondary grades of fuzzy set $\widetilde{A}$ are equal to 1. By instantiating the variable x into a specific value x', the vertical slice of the IT2 fuzzy set can be obtained:

$$\mu_{\widetilde{A}}(x = x', u) \equiv \mu_{\widetilde{A}}(x') = \int_{u \in J_{x'}} 1/u \qquad J_{x'} \subseteq [0,1] \tag{2}$$

The domain of the primary memberships J_x defines the FOU of fuzzy set $\widetilde{A}$:

$$FOU(\widetilde{A}) = \bigcup_{x \in X} J_x \tag{3}$$

Alternatively, the FOU of an IT2 fuzzy set $\widetilde{A}$ can be conveniently and completely described by its upper and lower membership functions:

$$FOU(\widetilde{A}) = \bigcup_{\forall x \in X} (\underline{\mu}_{\widetilde{A}}(x), \overline{\mu}_{\widetilde{A}}(x)) \tag{4}$$

This constitutes a substantial simplification when compared to the general T2 FLS. Here, only two T1 fuzzy membership functions (the upper and the lower boundary of the FOU) are necessary to fully describe the IT2 fuzzy set. This simplification is then transferred through a similar inference mechanism utilizing the modified T2 fuzzy join and meet operations [1]. The interval join and meet operations work exclusively with the FOU of the IT2 fuzzy sets, thus removing much of the computational burden of general T2 fuzzy sets.

In order to obtain a crisp output value, the resulting IT2 output fuzzy set $\widetilde{B}$ must be first type reduced and then defuzzified. The centroid of the IT2 fuzzy set $\widetilde{B}$ is then defined as:

$$C_{\widetilde{B}} = \int_{\theta_1 \in J_{x_1}} \cdots \int_{\theta_N \in J_{x_N}} 1 \Bigg/ \frac{\sum_{i=1}^{N} x_i \theta_i}{\sum_{i=1}^{N} \theta_i} \tag{5}$$

The centroid $C_{\widetilde{B}}$ is an interval T1 fuzzy set, which can be completely described by its left and right end points y_l and y_r. As derived by Karnik and Mendel, these boundary points can be expressed as [21]:

$$y_l = \frac{\sum_{i=1}^{L} x_i \overline{\mu}_{\widetilde{B}}(x_i) + \sum_{i=L+1}^{N} x_i \underline{\mu}_{\widetilde{B}}(x_i)}{\sum_{i=1}^{L} \overline{\mu}_{\widetilde{B}}(x_i) + \sum_{i=L+1}^{N} \underline{\mu}_{\widetilde{B}}(x_i)} \tag{6}$$

$$y_r = \frac{\sum_{i=1}^{R} x_i \underline{\mu}_{\widetilde{B}}(x_i) + \sum_{i=R+1}^{N} x_i \overline{\mu}_{\widetilde{B}}(x_i)}{\sum_{i=1}^{R} \underline{\mu}_{\widetilde{B}}(x_i) + \sum_{i=R+1}^{N} \overline{\mu}_{\widetilde{B}}(x_i)} \tag{7}$$

The L and R points are important switching points computed by the Karnik-Mendel (KM) iterative procedure [21]. Using the boundary values of the type-reduced centroid $C_{\widetilde{B}}$ the final crisp defuzzified value y can be computed as the mean of the centroid interval:

$$y = \frac{(y_l + y_r)}{2} \tag{8}$$

It is important to realize that the result of the output processing stage of the IT2 FLS is not only the crisp output value y, but also the entire centroid $C_{\widetilde{B}}$. The geometrical properties of the centroid provide additional information that can be utilized as a measure of the output uncertainty [16], [17], [19], [20]. In other words, this combined output provides additional description of the system's behavior, similar to the additional information provided by the standard deviation measure of the mean value in statistics.

III. Uncertainty of IT2 Fuzzy Sets

The fuzzy logic system can be seen as a functional mapping of the input vector X onto a set of output variables Y. Without the loss of generality the Multiple Inputs Single Output (MISO) system with output variable y is considered here. The IT2 FLS is often attributed with the potential to model and minimize the effect of dynamic uncertainty. Several authors interpreted the geometry of the output centroid as a measure of

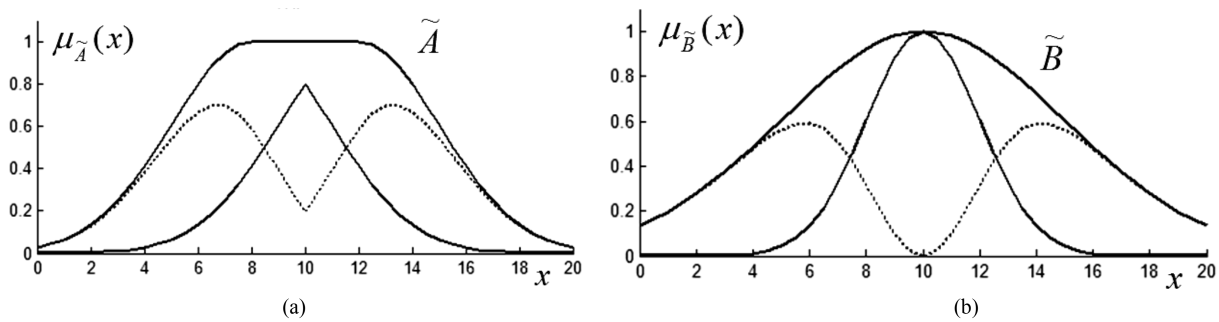


Fig. 1 Distribution of the input uncertainty (dotted line) for IT2 Gaussian fuzzy sets (solid line) with uncertain mean (a) and uncertain standard deviation (b).

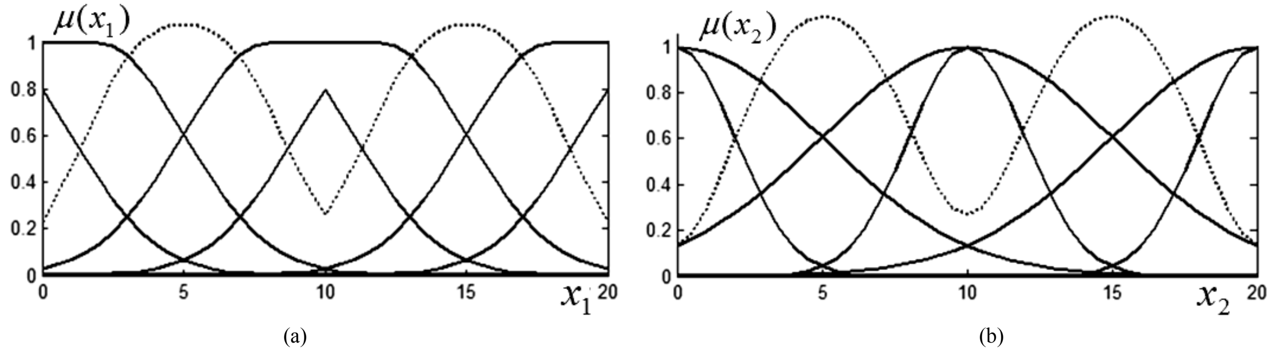


Fig. 2 Distribution of the joint input uncertainty (dotted line) for input IT2 FS (solid line) with uncertain mean (a) and uncertain standard deviation (b).

uncertainty associated with the produced output value y [16], [18]-[20], [22]. In a similar manner, the amount of input uncertainties have been associated with the width of the FOU of the input IT2 FSs. Hence, it can be assumed that the IT2 FLS also acts as a functional mapping between the system input and output uncertainty. The application of such functional mapping is the presence of a correct uncertainty measure in the output of the IT2 FLS, which constitutes additional and very valuable information.

However, to the best of authors' knowledge the questions of accuracy and interpretability of this functional uncertainty mapping have not been addressed yet. In this section, the notions of input and output uncertainties are formalized and the uncertainty modeling capabilities of typically used IT2 FSs models are investigated.

A. Input and Output Uncertainties of IT2 FLS

Consider an input IT2 fuzzy set $\tilde{A}$, defined by its FOU in terms of the upper and lower membership functions $\bar{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ as in (4). The input uncertainty $u_{\tilde{A}}(x)$ associated with the fuzzification of a crisp input value x in the fuzzy set $\tilde{A}$ can be expressed as the width of the firing interval:

$$u_{\tilde{A}}(x) = \bar{\mu}_{\tilde{A}}(x) - \underline{\mu}_{\tilde{A}}(x) \tag{9}$$

The joint input uncertainty $U^I(\vec{x})$ associated with the fuzzification of an input vector $\vec{x}$ by the IT2 FLS can be expressed as the cumulative input uncertainty of all firing intervals over all input dimensions:

$$U^I(\vec{x}) = \sum_{p=1}^{P}\sum_{j=1}^{M_p} u_{\tilde{A}_p^j}(x_p) = \sum_{p=1}^{P}\sum_{j=1}^{M_p}\left(\bar{\mu}_{\tilde{A}_p^j}(x_p) - \underline{\mu}_{\tilde{A}_p^j}(x_p)\right) \tag{10}$$

Here, P is the dimensionality of the input vector $\vec{x}$ and M_p is the number of input fuzzy sets in particular dimension p.

As previously demonstrated by other authors, the output uncertainty $U^O(\vec{x})$ (also called the *uncertainty interval* [18]) is associated with the width of the interval centroid, calculated as the type-reduced output of the IT2 FS:

$$U^O(\vec{x}) = y_r - y_l \tag{11}$$

The values of y_l and y_r denote the left and right boundaries of the interval centroid and can be obtained by the iterative Karnik-Mendel algorithm for computing the switch points, as defined in (6) and (7). Alternatively, the approximate output uncertainty can be calculated using the Wu-Mendel's uncertainty bounds method [16].

B. Uncertainty Modeling of Common IT2 FS

In a majority of available literature two types of IT2 FSs are considered: fuzzy sets with uncertain mean and fuzzy sets with uncertain standard deviation (alternatively spread for triangular MF). Here, the Guassian primary membership functions are considered. Fig. 1(a) and Fig. 1(b) depict the distribution of the input uncertainty $u_{\tilde{A}}(x)$ (dotted line) for all values of input variable x for the IT2 FS with uncertain mean and uncertain standard deviation, respectively. Quite

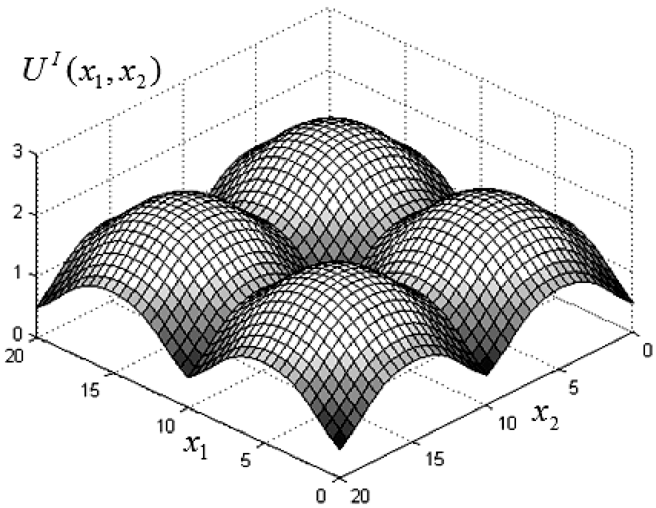


Fig. 3 Distribution of the joint input uncertainty.

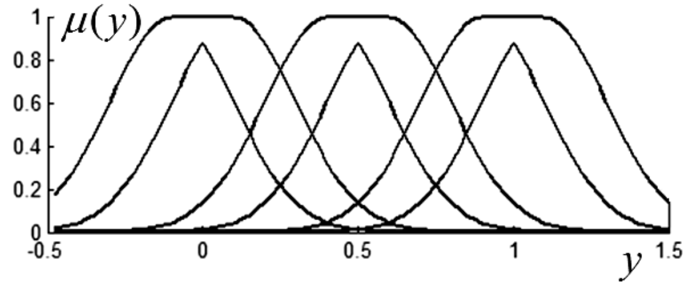


Fig. 4 The implemented IT2 output fuzzy sets.

interestingly, the distribution of the input uncertainty $u_{\tilde{A}}(x)$ along the input domain follows a specific pattern defined by the geometry of the IT2 FS.

As Fig. 1 clearly demonstrates, the modeling of the input uncertainty distribution in majority of designed IT2 FLSs is defined by the used architecture of the IT2 FSs. Hence, it does not reflect the uncertainty distribution contained in the input data along the specific input dimension (e.g. the different amplitudes of noise associated with different values of sensor readings).

In addition, consider a simple IT2 FLS with two inputs x_1 and x_2, each fuzzified by three IT2 Gaussian fuzzy sets with uncertain mean and uncertain standard deviation as shown in Fig. 2. This figure also depicts the joint input uncertainty distribution in both input domains (dotted line). It is clearly visible that the model assumes maximum joint input uncertainty around values 5 and 15 in both input dimensions. The joint input uncertainty surface of this system can be constructed by plotting the amplitude of the joint input uncertainty $U^I(x_1, x_2)$ for all combinations of input values x_1 and x_2 as depicted in Fig.3. It can be observed that the distribution of the joint input uncertainty is dependent on the input values. However, this dependency has not been derived from the input data and their statistical properties. Rather it is biased by the typical geometry of the used IT2 fuzzy membership functions.

Finally, assume that the IT2 FLS is described by a set of 9 fuzzy rules, defined in Table I, which use three output IT2 fuzzy sets, depicted in Fig. 4. Fig. 5(a) and Fig. 5(b) depict the output control surface and the associated output uncertainty $U^O(x_1, x_2)$, which is calculated as the width of the interval

TABLE I
FUZZY RULE TABLE

&		x_2		
		Small	Medium	Large
x_1	Small	Straight	Right	Right
	Medium	Left	Straight	Right
	Large	Left	Left	Straight

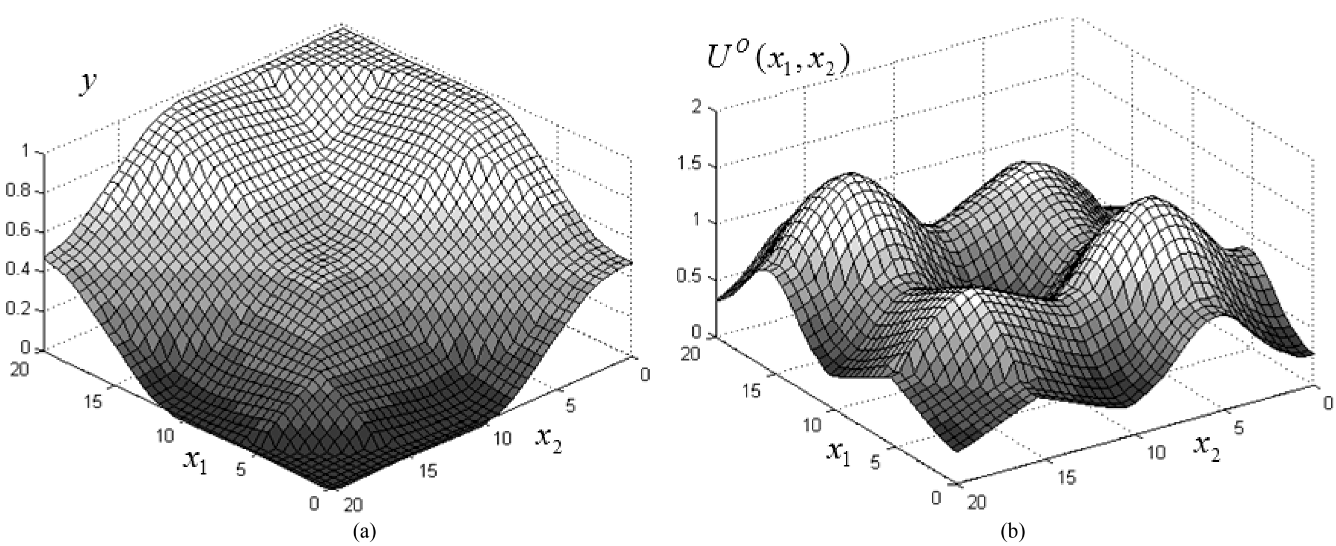
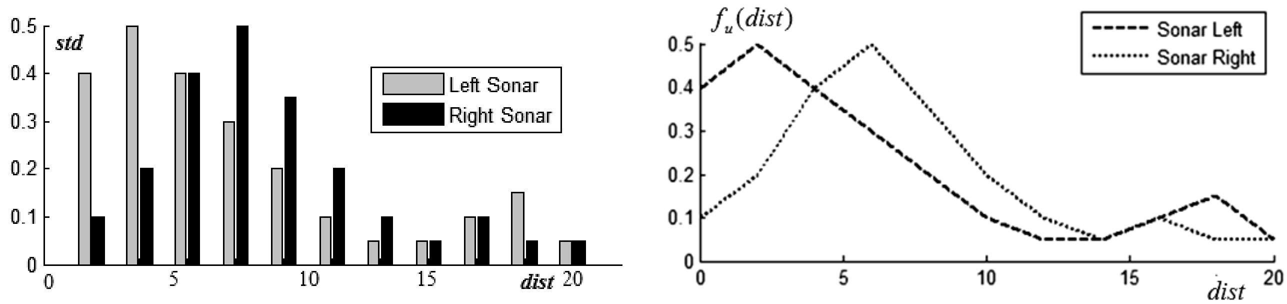


Fig. 5 The output control surface (a) and the distribution of the output uncertainty (b) for the considered IT2 FLS.



(a) (b)

Fig. 6 The measured sensor uncertainties (a) and the interpolated continuous input uncertainty distribution (b).

centroid of the IT2 FLS as in (11).

Two important observations can be made. Firstly, the uncertainty associated with the output decision is highly non-linear and variable in the problem domain. Secondly, there is a strong correlation between the joint input uncertainty distribution and the output uncertainty distribution (compare Fig. 3 and Fig. 5(b)). This second observation leads to the following conclusion. In order for the output uncertainty $U^O(\vec{x})$ to be correctly interpretable in the problem domain, the IT2 FLS must accurately model the distribution of the uncertainty in the system inputs. In other words, if the model of the input uncertainty does not accurately reflect the actual uncertainty distribution in the input domains then the interpretation of the geometry of the output centroid as a measure of output uncertainty does not reflect the input uncertainties correctly.

IV. FOU Construction Based on Measured Input Uncertainty

In order to obtain a more accurate model of output uncertainty $U^O(\vec{x})$, the input uncertainty distribution must be first accurately modeled by the FOUs of the input fuzzy sets. This section proposes a design method for incorporating the experimentally measured information about the input uncertainty into the design of the IT2 FLS.

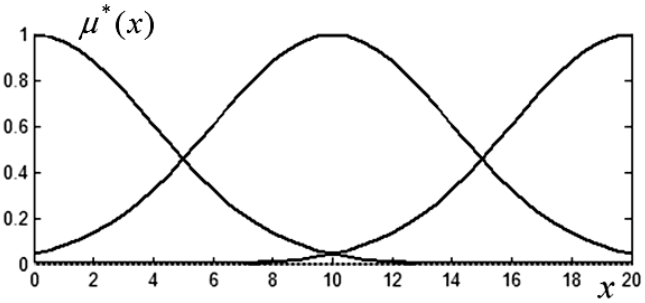


Fig. 7 The input principle membership functions.

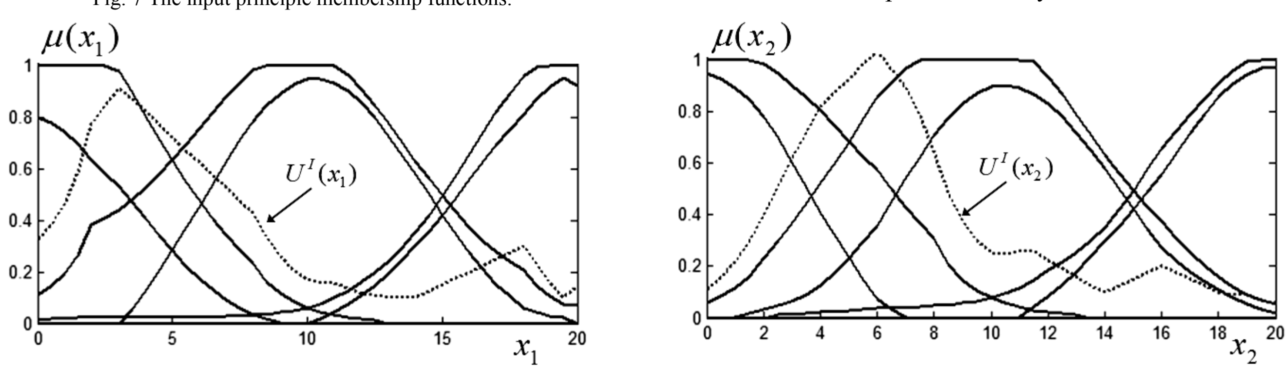
A. Sensor Calibration

In many control applications the inputs of an FLS constitute physical devices. As an example, consider the sonar range finders mounted on a mobile robot. In such case, the input uncertainty distribution of each sensor can be approximated by calibrating the device's response against known ground truth values. Many sources of uncertainties are commonly present at the same time. For instance, the sonar measurements are subject to uncertainties due to the beam width, signal attenuation, variable reflectivity of surrounding materials or manufacturing defects in the sonic emitter or receiver. For the ease of explanation, a mobile robot with two sonar range finders is considered in the following text.

First, the available range of input values is discretized into *M* samples. The sonar sensors are placed at the calibrated distance from the reflective surface (e.g. wall) and a set of input measurements are obtained for each of them. The standard deviation of the signal values at particular sample is computed and stored as the amount of uncertainty associated with that particular calibration input value. The set of *M* measured values defines the sampled input uncertainty of each sensor, as shown in Fig. 6(a). The continuous input uncertainty distribution $f_u(x)$ can be obtained by applying the linear interpolation between the sampled values. The interpolated uncertainty distribution for both sonar sensors is depicted in Fig. 6(b).

B. FOU Construction

The measured input uncertainty distribution is used during



(a) (b)

Fig. 8 The FOUs of the input fuzzy sets (solid line) with the embedded measured input uncertainty (dotted line) for the left (a) and right (b) sonar sensors.

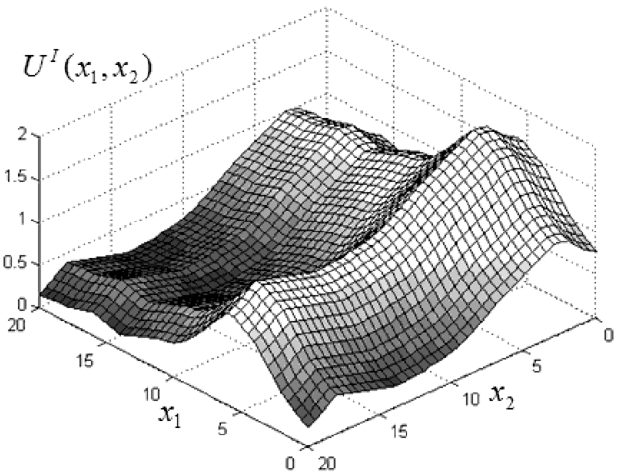


Fig. 9 Distribution of the joint input uncertainty.

the design of the FOU of the input IT2 fuzzy sets. First, the principle T1 membership functions for all input FSs are defined. In this work, the Gaussian principle membership functions are considered. The major criteria for selecting the parameters mean (m^i) and the standard deviation (σ^i) of principal membership function $\mu^*_{A^i}(x)$ is the sufficient overlap between neighboring fuzzy sets and a continuous coverage of the entire input domain. In this example, three equidistantly spaced Gaussian principal membership functions with equal standard deviations are considered as depicted in Fig. 7. The design of these fuzzy sets together with the construction of the fuzzy rule base is performed by an application expert.

Next, the FOU of the input IT2 FSs is constructed by fusing the interpolated uncertainty distribution $f_u(x)$ with the principal membership functions. Firstly, a mapping between the sensor input domain and the uncertainty domain of the IT2 FS must be defined. This is achieved by scaling the amplitude of the uncertainty distribution by the maximum required input uncertainty $u_{\tilde{A}}(x)$. The lower and the upper membership functions $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ of the input IT2 FS $\tilde{A}$ are then obtained as follows:

$$\overline{\mu}_{\tilde{A}}(x) = \mu^*_A(x) + \frac{f_u(x)}{2} \tag{12}$$

$$\underline{\mu}_{\tilde{A}}(x) = \mu^*_A(x) - \frac{f_u(x)}{2} \tag{13}$$

In order to achieve admissible design of the input IT2 FSs, the maximum and the minimum values of both upper and lower membership functions are bounded in the interval [0, 1]. The sphere of influence in the primary domain of each fuzzy set is also bounded in the interval [$m^i - 2\sigma^i$, $m^i + 2\sigma^i$]. The amplitudes of both lower and upper membership functions are smoothly forced to zero outside this interval.

The resulting FOU of the input IT2 FS together with plotted distributions of the joint input uncertainties $U^I(\vec{x})$ for each sonar sensor are depicted in Fig. 8(a) and Fig. 8(b). Here, the left and right sonar inputs are denoted as inputs x_1 and x_2. It can be seen that the model of the input uncertainty distribution now reflects the real uncertainty distribution in the calibrated input sensors (compare to Fig. 6(b)).

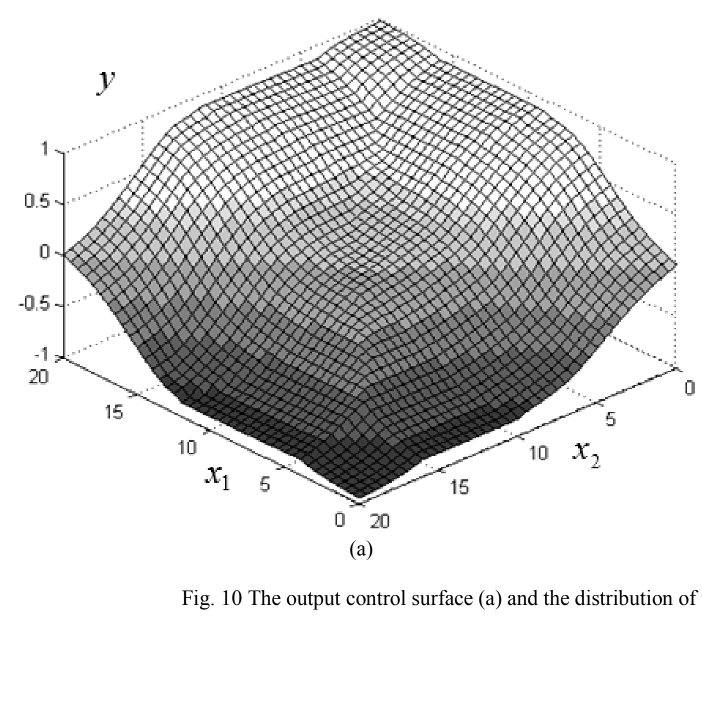
A similar approach to construction of IT2 FSs was recently proposed in [23] for modeling linguistic label perception.

V. Experimental Results

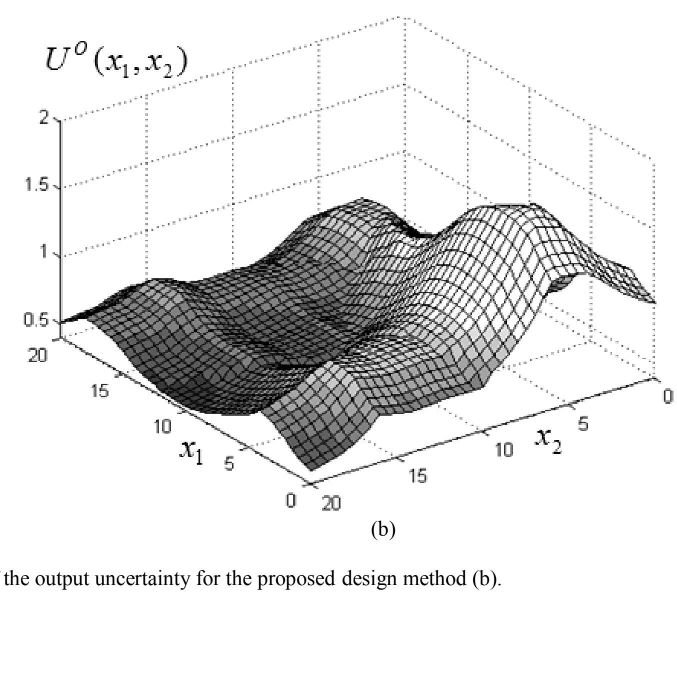
This section demonstrates the uncertainty modeling of the IT2 FLS designed by means of the proposed method for incorporating the measured input uncertainty. The considered system uses input fuzzy sets as depicted in Fig. 8 and the output fuzzy sets and the rule base as shown in Fig. 4 and Table I, respectively.

Fig.9 shows the joint input uncertainty distribution $U^I(x_1, x_2)$ for the designed system. Comparing Fig. 3 and Fig. 9 it can be seen the proposed design method achieves more accurate modeling of the input uncertainty distributions based on the measured sensor characteristics. This uncertainty distribution resembles the measured sensory uncertainties embedded into the FOU design.

Next, Fig. 10(a) and Fig. 10(b) depict the output control surface and the output uncertainty distribution for the designed IT2 FLS. When compared to Fig. 5, it can be seen that while



(a)



(b)

Fig. 10 The output control surface (a) and the distribution of the output uncertainty for the proposed design method (b).

the control performance of the controller does not change significantly, the distribution of the calculated output uncertainty is substantially different. Hence, the more accurate input uncertainty modeling by the proposed FOU design method results in more accurate output uncertainty model. This output uncertainty model can then be correctly interpreted as the uncertainty associated with the system output.

Nevertheless, the interpretation of the actual output uncertainty amplitude is still an open question. Rather, its relative distribution within the input space can provide useful information (e.g. identification of sub-regions with associated increased uncertainty). However, the authors believe that this additional information about the relative output uncertainty distribution can be very important and beneficial in many applications. For instance, in mobile robotics application when the robot is aware of the uncertainty associated with its inputs, which then propagates to the output decision accordingly. This awareness can be transferred into an additional decision making process. For instance such "uncertainty-aware" robot can slow down or average several input samples to reduce the negative impacts of the increased input uncertainty. Actual implementation and testing of the proposed design methodology with real robotic hardware is a scope of future work.

VI. CONCLUSION

This paper analyzed the input and output uncertainty modeling capabilities of commonly used IT2 FSs. The concepts of input, joint input and output uncertainty have been formalized. It was shown that the commonly used IT2 FSs, such as Gaussian fuzzy sets with uncertain mean or uncertain standard deviation, do not correctly capture and model the input uncertainty distribution. Rather the uncertainty model is biased by the chosen geometry of the IT2 FS.

A novel approach for input uncertainty driven design of IT2 FLSs based on calibrated input sensors' characteristics was presented. It was shown that this approach results in an increased accuracy of the input uncertainty model, which is then translated via the fuzzy inference process into the output uncertainty. The proposed method allows for a meaningful interpretation of the geometry of the interval output centroid as a measure of uncertainty associated with system output.

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