Uncertainty Modeling with Interval Type-2 Fuzzy Logic Systems in Mobile Robotics

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Abstract—Interval Type-2 Fuzzy Logic Systems (IT2 FLSs) have been commonly attributed with the capability to model and cope with dynamic uncertainties. However, the interpretation of this uncertainty modeling using the IT2 FLSs have been rarely addressed or taken into consideration during the design of the respective fuzzy controller. This paper extends the previously proposed method for incorporating the experimentally measured input uncertainty into the design of the IT2 FLS. Two novel uncertainty quantifiers are proposed to track the uncertainty modeling throughout the inference process: the antecedent uncertainty and the consequent uncertainty quantifiers. Further, the new IT2 FLS design method was used to design a wall-following navigation controller for an autonomous mobile robot. It is demonstrated that the new IT2 FLS design offers improved uncertainty modeling, when compared to classical design methodologies. It was shown that the modeled input uncertainty is more accurately reflected in the system output as the geometry of the type-reduced interval centroid. This uncertainty model provides valuable information about the uncertainty associated with the output decision and can be used for more informer decision making.

***Index Terms*— Interval Type-2 Fuzzy Logic Systems, Uncertainty Modeling, Centroid, Type-Reduction.**

I. Introduction

Type-2 Fuzzy Logic Systems (T2 FLSs), proposed by Zadeh [1], have received increased attention of many researchers in the past decade [2]-[4]. T2 FLSs have been applied in many engineering areas, demonstrating their ability to outperform Type-1 (T1) FLSs mainly in the presence of dynamic uncertainties [5]-[7]. The major difference between the T1 and T2 FLSs is in the model of individual Fuzzy Sets (FSs), which use membership degrees that are themselves FSs.

The most commonly used kind of T2 FLS is the Interval T2 (IT2) FLS, which uses interval membership degrees. Many researchers argue in favor of IT2 FLSs because of their potential to model and minimize the effects of dynamic uncertainties [4], [6], [8]. Typically, the performance of IT2 FLSs in various applications is compared to their T1 counterparts demonstrating improvements when noise and uncertainty are introduced into the system. The improved performance can be attributed to the Footprint of Uncertainty (FOU) of the IT2 FSs, which provide additional dimension for designing the fuzzy membership functions.

The inference process of IT2 FLSs results in an output IT2 FS. This IT2 FS must be first type-reduced into its interval T1 centroid, which can then be defuzzified into the final output value [9]. Many researchers associate the geometrical properties of the interval centroid with the uncertainty about the system's output [10]-[14]. For instance, Wu and Mendel state in [13] that: *"…the length of the type-reduced set can therefore be used to measure the extent of the output's uncertainty…"*. Other researchers used the width of the interval centroid to create an uncertainty bounds on the system's output for predicting micro milling cutting forces [12]. However, the correctness of such output uncertainty interpretation has not been previously addressed. This paper focuses on the analyzing the interpretation of uncertainty modeling with IT2 FLSs.

Previously, a method for incorporating the experimentally measured input uncertainty into the design of the IT2 FLS was proposed [15]. This design method first extracts the input uncertainty distribution from the input data and then incorporates the measured uncertainty into the FOU of the IT2 FSs. This presented paper extends the previous work in two major ways: i) two additional uncertainty quantifiers for tracking the uncertainty modeling throughout the inference process are proposed, and ii) the novel design method is used to implement an IT2 FLS for wall-following behavior of an autonomous mobile robot. The implemented wall-following behavior on real mobile robot validates the presented conclusions in real operational settings. It is shown that the modeled input uncertainty is more accurately reflected in the system output as the geometry of the type-reduced interval centroid.

The rest of the paper is organized as follows. Section II provides background review on IT2 FSs and FLSs. The novel uncertainty quantifiers for IT2 FLSs are introduced in Section III. The design of the IT2 FLSs for autonomous wall-following mobile behavior is explained in Section IV. Experimental results are shown in Section V and the paper is concluded in Section VI.

II. Interval Type-2 Fuzzy Logic Systems

This section provides brief overview of IT2 FLSs. An IT2 FLS employs one or more IT2 FSs. An IT2 FS $\tilde{A}$ can be expressed as [4]:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) \qquad J_x \subseteq [0,1] \tag{1}$$

Here, x and u are the primary and secondary variables, X is the domain of variable x and J_x is the primary membership of x. In the special case of IT2 FSs, all secondary grades of fuzzy set $\widetilde{A}$ are equal to 1. By instantiating the variable x into a specific value x', the vertical slice of the IT2 FS is obtained:

$$\mu_{\widetilde{A}}(x=x',u) \equiv \mu_{\widetilde{A}}(x') = \int_{u\in J_{x'}} 1/u \quad J_{x'} \subseteq [0,1] \qquad (2)$$

This vertical slice is an interval T1 fuzzy set. The domain of the primary memberships J_x defines the FOU of $\widetilde{A}$. The FOU of an IT2 FS $\widetilde{A}$ can be bounded by its upper and lower membership functions (see Fig. 1):

$$FOU(\widetilde{A}) = \bigcup_{\forall x\in X} (\underline{\mu}_{\widetilde{A}}(x), \overline{\mu}_{\widetilde{A}}(x)) \qquad (3)$$

This representation constitutes a substantial simplification when compared to the general T2 FSs. Here, only two T1 fuzzy membership functions (the upper ($\overline{\mu}_{\widetilde{A}}(x)$) and the lower ($\underline{\mu}_{\widetilde{A}}(x)$) boundary of the FOU) are sufficient to completely describe the IT2 FS. The simplification removes much of the computational burden of general T2 FLSs. The fuzzy inference process then uses interval join and meet operations to calculate the output fuzzy sets based on the set of provided linguistic rules [4].

In order to obtain a crisp output value y, the resulting output IT2 FS $\widetilde{B}$ must be first type reduced and then defuzzified. The interval centroid is an interval T1 FS that can be described by its left and right end points y_l and y_r. These endpoints can be computed for instance by the Karnik-Mendel (KM) iterative procedure [9]. Using the boundary values of the type-reduced interval centroid the final crisp defuzzified value y can be obtained as the mean of the centroid interval:

$$y = \frac{(y_l + y_r)}{2} \qquad (4)$$

The combined result of the output processing stage of the IT2 FLS is the crisp output value y accompanied by the interval centroid. The presence of the centroid constitutes one of the main advantages of IT2 FLSs when compared to T1 FLSs. The geometrical properties of the interval centroid (e.g. its width) can provide additional information commonly interpreted as a measure uncertainty associated with the output value.

III. Uncertainty Modeling with IT2 FLSs

In order to analyze the uncertainty modeling of IT2 FLSs, the joint input uncertainty quantifier has been previously proposed in [15]. In this Section, two novel uncertainty quantifiers are introduced. The uncertainty quantifiers are intended to track the uncertainty in the IT2 FLS through the individual steps of the fuzzy inference process: fuzzification, rule firing, rule aggregation and type-reduction.

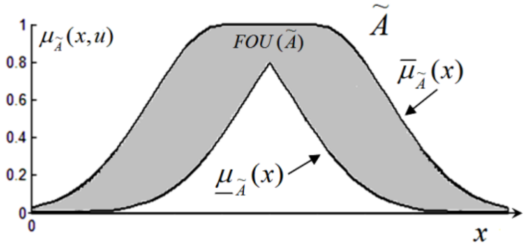


Fig. 1 Interval Type-2 fuzzy set.

The IT2 FLS is commonly attributed with the capability to model and minimize the effects of dynamic uncertainties [13]. The uncertainty is modeled by the FOUs of the IT2 FSs and it is translated into the interval centroid via the fuzzy inference process. Therefore, the IT2 FLS performs a functional mapping between the modeled input uncertainties (i.e. the FOUs) and the output uncertainty (i.e. the interval centroid). Analyzing the interpretation of this uncertainty mapping is important since the presence of reliable uncertainty measure associated with the output variable constitutes valuable information.

A. Joint Input Uncertainty Quantifier

The joint input uncertainty quantifier was previously proposed in [15]. Consider an IT2 FS $\widetilde{A}$ described by its FOU using the upper and the lower membership functions $\overline{\mu}_{\widetilde{A}}(x)$ and $\underline{\mu}_{\widetilde{A}}(x)$ as in (4). The input uncertainty $u_{\widetilde{A}}(x)$ associated with the fuzzification of an input value x by fuzzy set $\widetilde{A}$ can be expressed as the width of the firing interval:

$$u_{\widetilde{A}}(x) = \overline{\mu}_{\widetilde{A}}(x) - \underline{\mu}_{\widetilde{A}}(x) \qquad (5)$$

In the first step of the IT2 fuzzy inference process the input variables are fuzzified using the input IT2 FSs. The proposed joint input uncertainty quantifier $U^I(\vec{x})$ associated with the fuzzification of an input vector $\vec{x}$ can be expressed as the aggregated input uncertainty from all firing intervals over all input variables:

$$U^I(\vec{x}) = \sum_{p=1}^{P}\sum_{j=1}^{M_p} u_{\widetilde{A}_p^j}(x_p) = \sum_{p=1}^{P}\sum_{j=1}^{M_p}\left(\overline{\mu}_{\widetilde{A}_p^j}(x_p) - \underline{\mu}_{\widetilde{A}_p^j}(x_p)\right) \qquad (6)$$

Here, P is the dimensionality of the input vector $\vec{x}$ and M_p is the number of IT2 FSs for the specific dimension p.

B. Antecedent Uncertainty Quantifier

The next step in the fuzzy inference process is calculating the rule firing strength of each fuzzy rule by combining the membership degrees of all rule antecedents. As the fuzzified

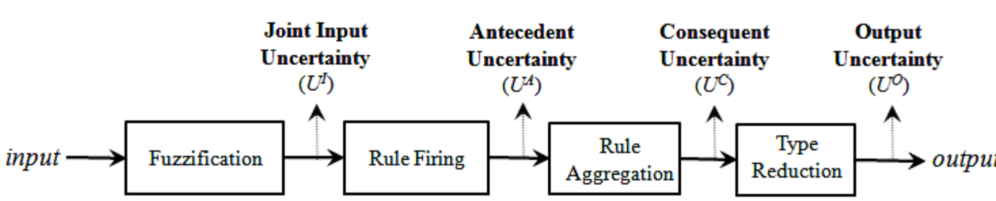


Fig. 2 The IT2 FLS and the uncertainty quantifiers.

input values are transformed into the rule antecedents, the joint input uncertainty $U^I(\vec{x})$ is transformed into the proposed antecedent uncertainty quantifier $U^A(\vec{x})$. This proposed measure of uncertainty is calculated as the cumulative uncertainty associated with the rule firing strengths:

$$U^A(\vec{x}) = \sum_{k=1}^{K} \left(\prod_{p=1}^{P} \overline{\mu}_{A_p^k}(x_p) - \prod_{p=1}^{P} \underline{\mu}_{A_p^k}(x_p) \right) \quad (7)$$

Here, K is the number of fuzzy rules in the rule base and symbol $\prod$ denotes the t-norm operator (e.g. minimum).

C. Consequent Uncertainty Quantifier

The fuzzy inference process follows with the rule aggregation step. Typically, multiple fuzzy rules share common consequents. The proposed consequent uncertainty quantifier U^C provides an insight into how much aggregated uncertainty from all fuzzy rules will be applied to the rule consequents. The consequent uncertainty $U^C(\vec{x})$ can be expressed as follows:

$$U^C(\vec{x}) = \sum_{m=1}^{C} \left(\coprod_{j=1}^{K_m} \left(\prod_{p=1}^{P} \overline{\mu}_{A_p^j}(x_p) \right) - \coprod_{j=1}^{K_m} \left(\prod_{p=1}^{P} \underline{\mu}_{A_p^j}(x_p) \right) \right) \quad (8)$$

Here, C denotes the number of distinct rule consequents, K_m is the number of fuzzy rules contributing to the m^{th} consequent and symbol $\coprod$ denotes the specific t-conorm operator (e.g. maximum).

D. Output Uncertainty Quantifier

Finally, the output processing stage transforms the output fuzzy set into the output value y. As previously demonstrated by other authors, the output uncertainty $U^O(\vec{x})$ (also called the *uncertainty interval* [13]) is computed as the width of the interval centroid:

$$U^O(\vec{x}) = (y_r - y_l) \quad (9)$$

The values of y_l and y_r denote the left and right boundaries of the interval centroid. These values can be obtained using the iterative Karnik-Mendel algorithm. Alternatively, an approximate output uncertainty can be calculated using the Wu-Mendel's uncertainty bounds method [13].

The calculation of the proposed uncertainty quantifiers is summarized in Fig. 2. It is trivial to show, that in accordance with the fundamental design principles of T2 FLS, when all sources of uncertainty disappear, the uncertainty quantifiers all reduce to zero.

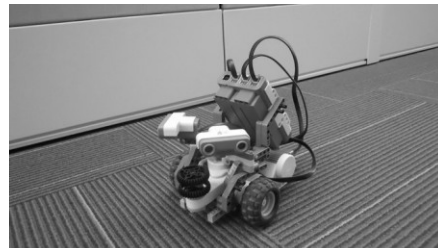


Fig. 3 Lego NXT autonomous mobile robot equipped with two sonar sensors.

IV. Design Of Wall-Following IT2 FLS Based On Measured Input Uncertainty

A novel design methodology for incorporating the measured input uncertainties into the design of IT2 FLS was previously proposed in [15]. In this paper, the proposed design method is used to implement a wall-following navigation behavior for an autonomous mobile robot.

The robotic platform consists of an autonomous Lego NXT mobile robot equipped with two sonar range finders. The task of the mobile robot was to autonomously navigate in an unstructured indoor environment while negotiating obstacles. The navigation is controlled by an IT2 FLS that is implemented on a laptop CPU remotely communicating with the mobile robot over a Bluetooth connection. The inputs of the IT2 FLS are connected to the sonar sensors. The calculated output signal is proportional to the power applied to the robot's differential motor drives. The Lego NXT robotic platform is depicted in Fig. 3.

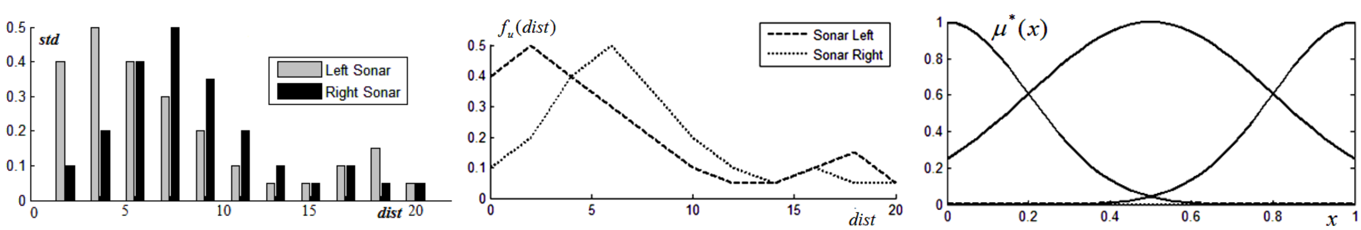
The design method can be summarized in three steps as follows:

Step 1: Measure the uncertainty distribution of the inputs by calibrating the input sensors.

Step 2: Construct a T1 FLS with the desired control behavior.

Step 3: Extend the T1 FLS into the IT2 FLS by creating the FOUs of the IT2 FSs via fusing the measured input uncertainty with the T1 fuzzy membership functions.

The first step of the proposed design methodology is to measure the input uncertainty of the actual input data. The input data are sampled from a pair of sonar range finders mounted on the mobile robot. In this application, the distribution of input uncertainty can be experimentally measured by calibrating the sensors' response against known ground truth values. The received sonar measurements can be considered subject to many kinds of uncertainties, e.g. due to



(a) (b) (c)

Fig. 4 The measured sensor uncertainties (a), the interpolated continuous input uncertainty distribution (b), and the input T1 fuzzy sets (c).

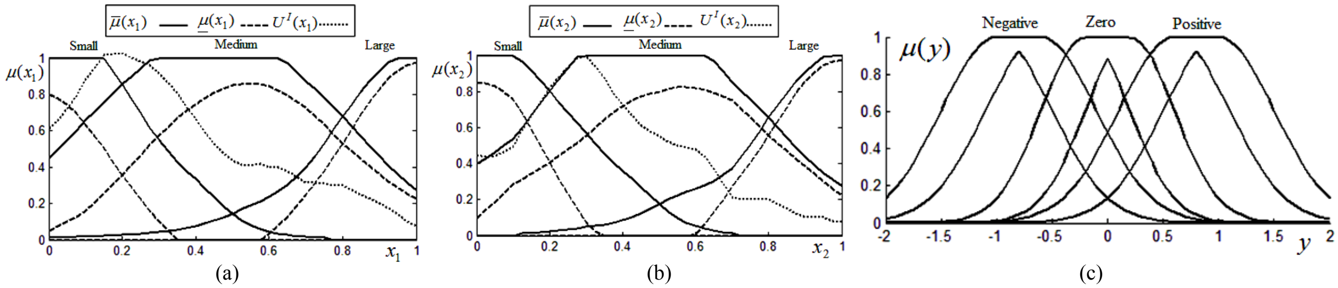


Fig. 5 The FOUs of the input fuzzy sets with the embedded joint input uncertainty for the left (a) and right (b) sonar sensors and the output fuzzy sets (c).

TABLE I
FUZZY RULE TABLE

&	x_2	Small	Medium	Large
x_1	Small	Zero	Positive	Positive
	Medium	Negative	Zero	Positive
	Large	Negative	Negative	Zero

the beam width, signal attenuation, variable reflexivity of surrounding materials or manufacturing defects in the sonic emitter or receiver [16].

First, the available range of input values is discretized into M samples. The sonar sensors are placed at the calibrated distance from the reflective surface (e.g. wall). A set of input measurements is obtained for each calibrated distance. The standard deviation of the sensory readings at particular distance sample is computed and stored as the amount of uncertainty associated with that particular input value.

The set of M measured standard deviation values defines the sampled input uncertainty for each robot's sonar sensor. An illustrative example is shown in Fig. 4(a). The continuous distribution of input uncertainty $f_u(x)$ for variable x can be obtained by applying linear interpolation between the calibration points. The interpolated uncertainty distribution for both robot's sonar sensors is depicted in Fig. 4(b).

The second step in the proposed design methodology is the construction the initial T1 FLS. Here, it is assumed that the user has already constructed the initial T1 FLS, using one of the well-established design methodologies [4]. In this work, the Gaussian principal membership functions are considered. The major criteria for selecting the mean (m^i) and the standard deviation (σ^i) of principal membership function $\mu^*_{A^i}(x)$ is sufficient overlap between neighboring fuzzy sets ensuring continuity of the output variable [17]. In the presented case study three equidistantly spaced Gaussian principal membership functions with identical standard deviations were considered. The respective T1 FSs are depicted in Fig. 4(c). The fuzzy rules of the T1 FLS are listed in Table I.

In the third and final step, the T1 FLS is extended into the IT2 FLS. The FOU of the input IT2 FSs is constructed by fusing the experimentally measured distribution of input uncertainty $f_u(x)$ with the corresponding principal membership functions. Firstly, a mapping between the range of measured standard deviations and the normalized uncertainty domain of the IT2 FSs must be established. This is achieved by scaling the amplitude of the measured uncertainty distribution by the maximum required input uncertainty $u_{\tilde{A}}(x)$. The lower and the upper membership functions $\overline{\mu}_{\tilde{A}}(x)$ and $\underline{\mu}_{\tilde{A}}(x)$ of the input IT2 FS $\tilde{A}$ can then obtained as follows:

$$\overline{\mu}_{\tilde{A}}(x) = \min\left[\mu^*_A(x) + \frac{f_u(x)}{2},\ 1.0\right] \quad (10)$$

$$\underline{\mu}_{\tilde{A}}(x) = \max\left[\mu^*_A(x) - \frac{f_u(x)}{2}, 0.0\right] \quad (11)$$

In order to achieve admissible design of the input IT2 FSs, the values of both upper and lower membership functions are bounded in the interval [0, 1].

The resulting FOU of the input IT2 FSs together with the plotted distributions of the joint input uncertainty quantifier $U^I(x)$ for both sonar sensors are depicted in Fig. 5(a)-(b). Here, the left and the right sonar inputs are denoted as inputs x_1 and x_2. It can be observed that the distribution of the joint input uncertainty quantifier $U^I(x)$ accurately reflects the actual distribution of uncertainty in the input data firing the IT2 FLS. The outputs of the IT2 FLS are designed as IT2 FS with uncertain mean to account for output uncertainties in the system as depicted in Fig. 5(c). Figures 6(a)-6(d) depict the distribution of the uncertainty quantifiers U^I, U^A, U^C and U^O in the input domain. Furthermore, the output control surface is shown in Fig. 6(e).

V. EXPERIMENTAL RESULTS

This section presents experimental testing of the implemented IT2 FLS for autonomous mobile robot wall-

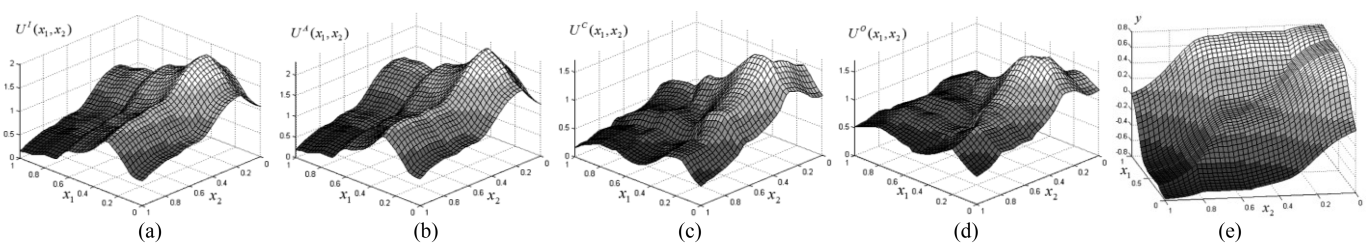


Fig. 6 Distribution of the joint input uncertainty (a), the antecedent uncertainty (b), the consequent uncertainty (c), the output uncertainty (d) and the output control surface (e) for the proposed wall-following IT2 FLS.

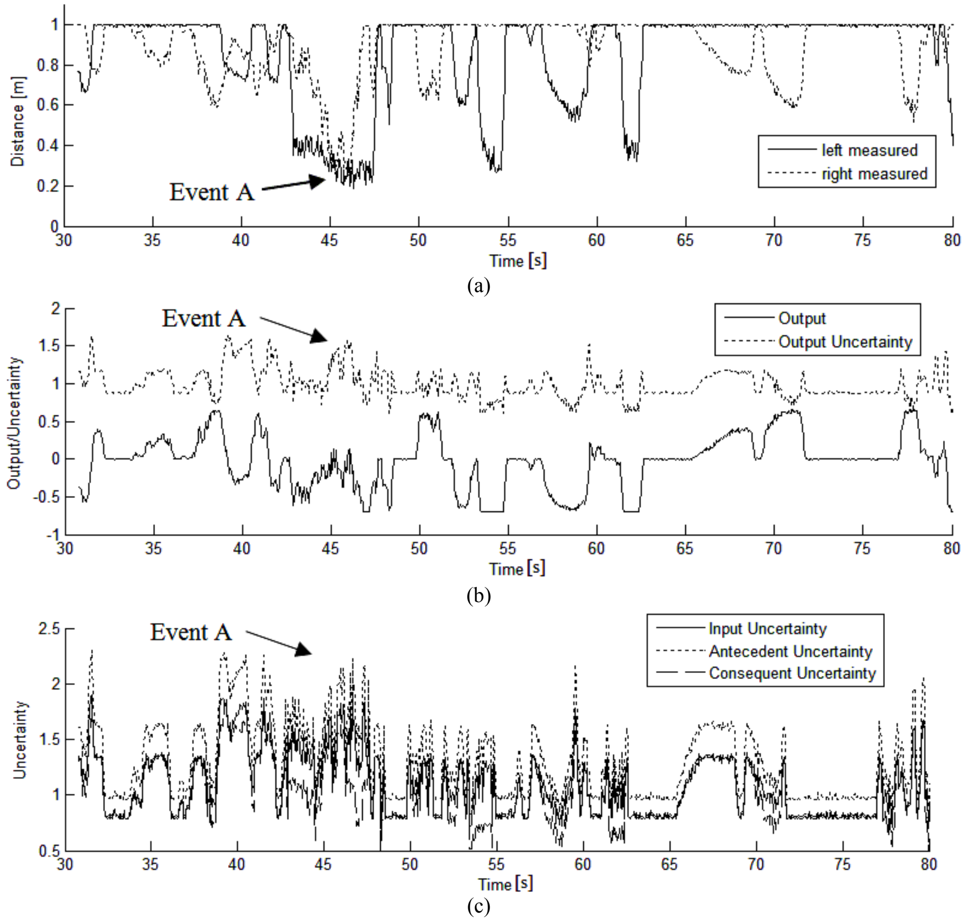


Fig. 7 The input signals (a), the output signal and its uncertainty (b), and the uncertainty quantifiers (c) during an autonomous navigation of a mobile robot using the common IT2 FLS.

following behavior. Both the common and the proposed IT2 FLS design methodology were implemented and experimentally compared. The common IT2 FLS used IT2 FSs with uncertain mean. The proposed IT2 FLS design was implemented according to the description given in the previous Section with the input uncertainty distribution as in Fig. 4(a). The robot's sonar sensors were also artificially augmented with random noise signal following the distribution on Fig. 4(a). It should be noted here that the subject of this experimental study is mainly to analyze the uncertainty modeling performance rather than measuring minor differences in the robot's trajectory. Both FLCs succeeded in satisfactory robot navigation but featured significantly different uncertainty modeling performance.

First, the recorded signals from robot with the common IT2 FLS are plotted in Fig. 7. It is easy to see when the robot approached an obstacle as well as when it negotiated a narrow corridor (i.e. Event A in Fig. 7(a)).

The output of the common IT2 FLS with the calculated output uncertainty U^O is depicted in Fig. 7(b). Negative and positive values of the output signal show robot's steering to the right and to the left while negotiating the sensed obstacles. The proposed uncertainty quantifiers U^I, U^A, U^C are plotted in Fig 7(c). A main observation can be made as follows. The amplitude of the calculated output uncertainty does not reflect the uncertainty in the sensory inputs (noise amplitude). In the labeled Event A, high input uncertainty was present in both robot's inputs (negotiating a narrow corridor). However, because the distribution of the input uncertainty was not taken into account during the design of the common IT2 FLS, the output uncertainty U^O does not reflect the increased input uncertainty accordingly.

Fig. 8 plots the measured sonar readings, the robot's control signal with the associated output uncertainty U^O and the calculated uncertainty quantifiers U^I, U^A, U^C for the proposed IT2 FLS. Two cases with increased input uncertainty are labeled as Event B and Event C in Fig 8(a), where the robot was negotiating narrow corridors.

The following two observations can be made. First, the output uncertainty U^O now more accurately reflects the increased uncertainty in the input sensory readings (manifested as increased noise amplitude). For example, the output uncertainty signal plotted in Fig. 8(b) clearly marks Event B and Event C as events with increased uncertainty associated with the calculated output. Second, as it was already demonstrated before, the proposed uncertainty quantifiers U^I, U^A, U^C feature strong positive correlation with the output uncertainty U^O and provide a good approximation of the output uncertainty.

VI. Conclusion

This paper extended the previous work on uncertainty modeling with IT2 FSs and IT2 FLSs. Two novel uncertainty quantifiers were proposed to track the uncertainty propagation throughout the inference process: antecedent uncertainty and

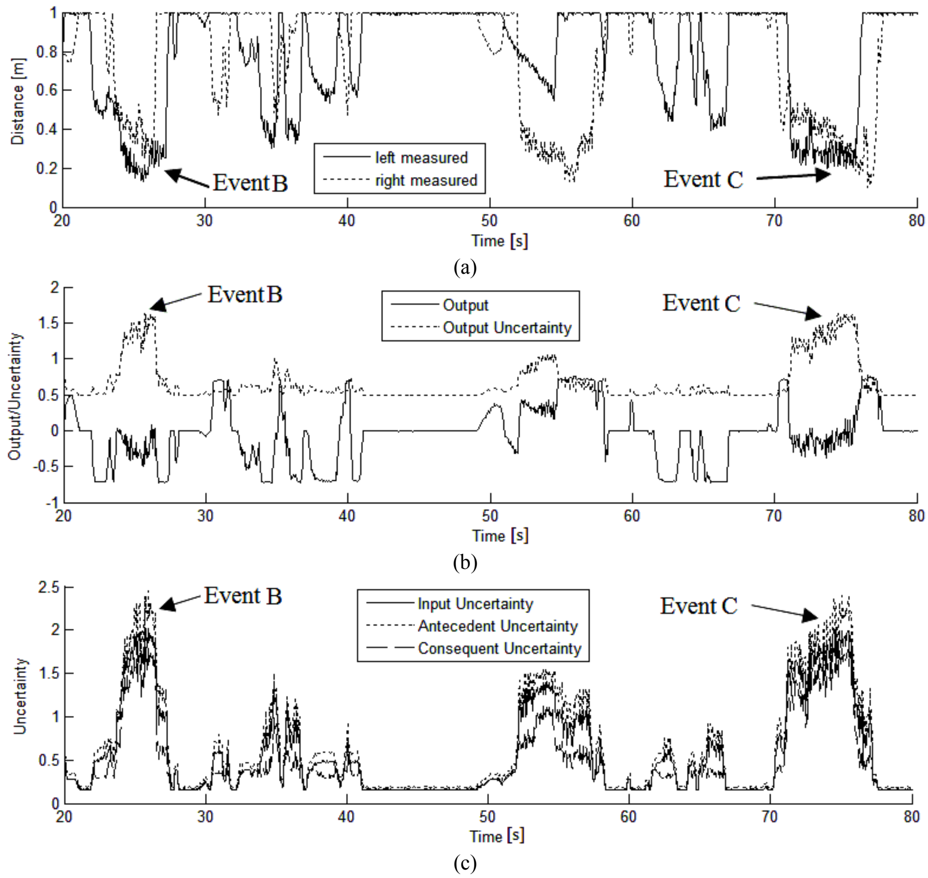


Fig. 8 The input signals (a), the output signal and its uncertainty (b), and the uncertainty quantifiers (c) during an autonomous navigation of a mobile robot using the proposed uncertainty-based design of IT2 FLS.

consequent uncertainty. The novel design methodology for constructing IT2 FLSs based on measured distribution of input uncertainty was used to design a wall-following behavior for an autonomous mobile robot. The experimental results verified that the proposed design methodology for IT2 FLS offers more accurate interpretation of the uncertainty associated with the output of the IT2 FLS. Such additional information can be further utilized for improved control of the mobile robot (e.g. reduce robot's speed in more uncertain conditions).

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