Performance analysis of 3 DOF Delta parallel robot

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Abstract. **Parallel robots have inherent advantages for many applications in the fields of robotics. They offer high dynamic capabilities combined with high accuracy and stiffness. There are a lot of performance criteria which have to be taken into account and which are pose dependent. The main idea of this paper is to present the fundamentals for a performance evaluation of the 3 DOF Delta parallel robot. Therefore we discuss a large number of performance criteria dealing with workspace, quality transmission, manipulability, dexterity and stiffness.**

***Keywords*: performance analysis, Delta parallel robot, 3DOF, workspace, kinematics.**

I. Introduction

VARIOUS performance analyses for parallel robots have been proposed in the literature. Early investigations of robot workspace were reported by Merlet [2], Kumar and Waldron [3], Tsai and Soni [4], Gupta [1] and Roth [5], Sugimoto and Duffy [6], Gupta [7], and Davidson and Hunt [8]. The consideration of joint limits in the study of the robot workspaces was presented by Delmas and Bidard (1995). Other works that have dealt with robot workspace are reported by Agrawal [9], Gosselin and Angeles [10], Cecarelli [11]. Various numerical methods for determination of the workspace of parallel robots have been developed in the recent years. For example, Stan [14] presented a genetic algorithm approach for multi-criteria optimization of PKM (Parallel Kinematics Machines).

The majority of numerical methods used for parallel manipulator workspace boundary determination typically rely on manipulator's pose parameter discretization. [15, 16]. With the discretization approach, the workspace is envisioned as the uniform grid of nodes in Cartesian or polar coordinate system. Each node is then examined in order to determine whether it belongs to the workspace or not.

The paper presents several contributions. First, the paper introduces some evaluation index metrics as the performance measure of the 3DOF Delta parallel robot. Secondly, the dimensioning of the 3 DOF parallel robot of type DELTA with revolute actuators for the largest workspace, best stiffness and transmission quality is presented. The results shown in this paper demonstrate a novel approach that improves the workspace performances.

Section II describes the 3DOF parallel robot. The third section introduces the performance evaluation indexes under mathematical form. The fourth section presents the performance evaluation results, while the final, fifth section concludes this paper.

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II. 3 DOF Delta parallel robot

Parallel robots with 3 degrees-of-freedom are parallel manipulators comprising a fixed base platform and a payload platform linked together by three independent, identical, and open kinematic chains (Fig. 1).

The DELTA parallel robot consists of a spatial parallel structure with three degrees of freedom, and is driven by three revolute actuators. The platform is connected with each drive by two links forming a parallelogram, allowing only translational movements of the platform and keeping the platform parallel to the base plane.

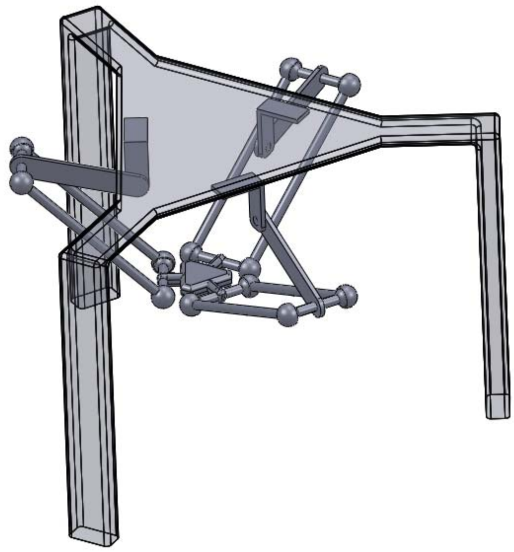

Fig. 1. Delta parallel robot with 3 degrees-of-freedom (CAD model)

A. Mathematical model

To analyze the kinematic model of the parallel robot, two relative coordinate frames are assigned, as shown in Fig. 2c).

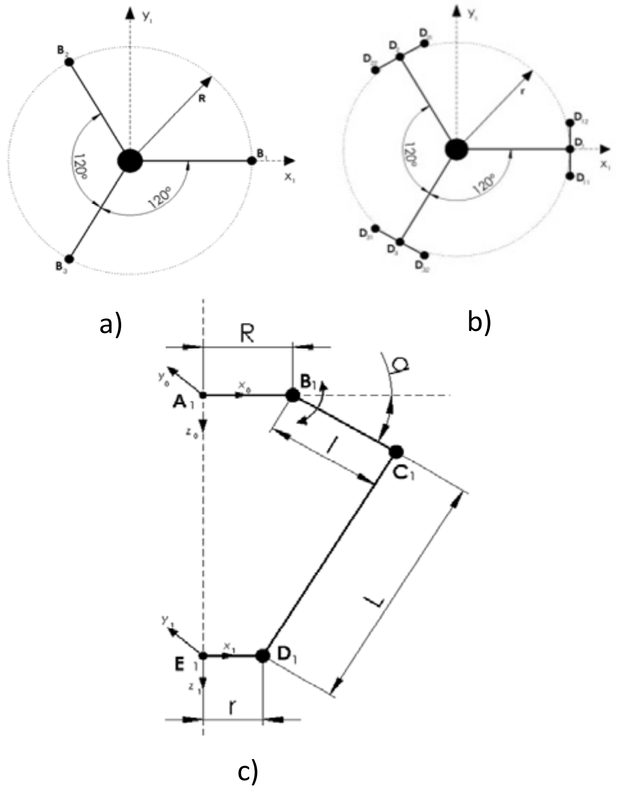

Fig. 2. Schematic diagram of DELTA parallel robot; a) and b) mobile and fixed platform for DELTA parallel robot; c) kinematic scheme of DELTA parallel robot

TABLE 1: CONSTRUCTION PARAMETERS.

Construction parameters	Li, Ri, li, ri, qi, i = 1…3 in [mm]
Parameterization	AiBi=R1; BiCi=L1; CiDi=l1; DiEi=r1;

A static Cartesian coordinate frame XYZ is fixed at the center of the base, while a mobile Cartesian coordinate frame $X_PY_PZ_P$ is assigned to the center of the mobile platform. B*i*, *i* = 1, 2, 3, and D*i*, *i* = 1, 2, 3, are: the joints located at the center of the base, as presented in Fig. 2 a) and b), and the platform passive joints, respectively.

B. Inverse Kinematics Problem of 3DOF Delta parallel robot

Inverse kinematics problem results from determination of angle values q_i $(i = 1,2,3)$ when the position of the characteristic point or the final effector (TCP – Tool Centre Point), respective the general coordinates: x_p, y_p, z_p.

For the solution of q_1 angle the next equations will be used:

$$a_1 = x^2 + y^2 + z^2 + \frac{2yl}{\sqrt{3}} - \frac{2yL}{\sqrt{3}} + \frac{l^2}{3} - \frac{2lL}{3} + \frac{L^2}{3} + l_1^2 - l_2^2 \quad (1)$$

$$b_1 = 2yl_1 + \frac{2ll_1}{\sqrt{3}} - \frac{2Ll_1}{\sqrt{3}} \quad (2)$$

$$c_1 = 2zl_1 \quad (3)$$

Where a_1, b_1 and c_1 variables will be used in final equation that will solve q_1 angle, the angle from the first link motor. For q_1 angle appears to be two solutions:

$$q_1 = -a\tan(c_1, b_1) + a\cos\left(\frac{a_1}{\sqrt{b_1^2 + c_1^2}}\right) \quad (4)$$

$$q_1 = a\tan(c_1, b_1) - a\cos\left(\frac{a_1}{\sqrt{b_1^2 + c_1^2}}\right) \quad (5)$$

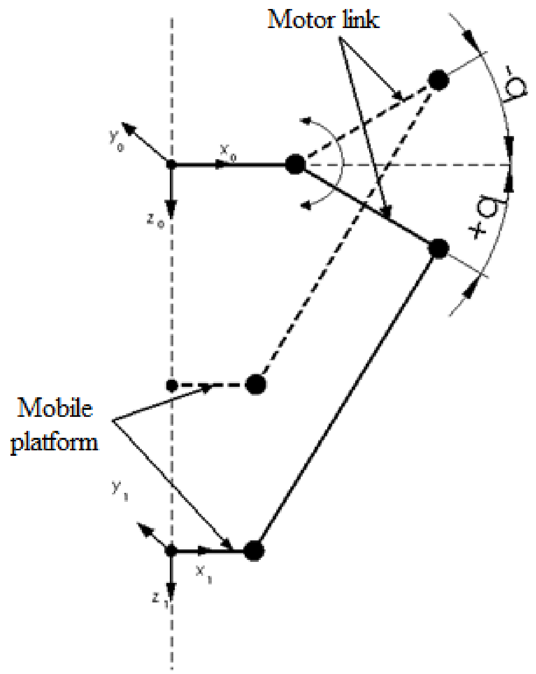

Fig. 3. Representation of the two solutions for the first motor link of Delta parallel robot (4) and (5)

As it can be observed are admitted two solutions for two possible position solutions for the first motor link. From these two solutions it will be chosen the first solution because with this one the result of the equation is the correct value for our angle. The same procedure will be applied to compute next two angles of the other two motor links.

In conclusion, equations (q_1, q_2, q_3) represent the analytic solutions for inverse kinematic models of Delta parallel robot.

For the implementation and resolution of forward and inverse kinematic problems of a parallel robot, a MATLAB environment was chosen. This is where a user friendly graphical user interface was developed, as well (Fig.4).

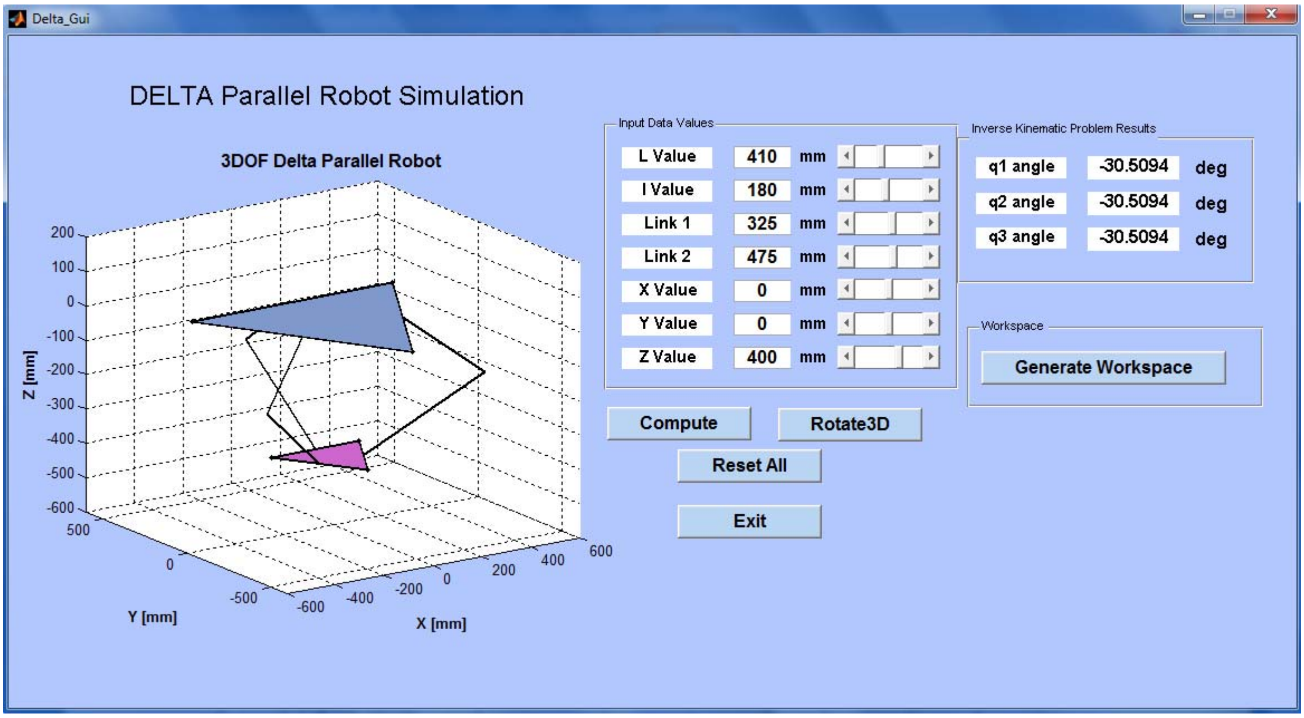

Fig. 4. Graphical User Interface for IKP and workspace determination of 3 DOF Delta parallel robot

III. PERFORMANCE EVALUATION

In this paper, the local value of manipulability, dexterity, stiffness and transmission quality index are defined as measure to evaluate the performance of the 3DOF Delta parallel robot. Another contribution is the determination of the workspace by means of discretization method in Matlab.

In addition to important design criterion such as the workspace, another important criterion, transmission quality index, has been considered. The transmission quality index, *T,* couples velocity and force transmission properties of a parallel robot, i.e. power features [14]. Its definition is:

$$T = \frac{\|I\|^2}{\|J\| \cdot \|J^{-1}\|} \tag{6}$$

where *I* is the unity matrix, and *J* is Jacobian matrix.

The values transmission quality index, *T,* are within a range 0<*T*<1, where *T*=0 characterizes a singular pose and *T*=1 characterizes an optimal value, therefore reflecting the isotropy of the system [14]. Here $\|J\|$ is calculated as:

$$\|J\| = \sqrt{tr\left(J^T wJ\right)};\ w = \frac{1}{n} I \tag{7}$$

where *n* is the dimension of the Jacobian matrix, and *I* the *n* x *n* identity matrix.

The manipulability condition number is a quality number in the sense of Yoshikawa, can be defined in terms of the ratio of a measure of performance in the task space and a measure of effort in the joint space.

$$M = \sqrt{\det(J \cdot J^T} \tag{8}$$

If J is quadratic. Eq. (8) reduces to *M*=det(*J*). The goal is to have a value of *M* as large as possible.

The stiffness condition number runs using the matrix *K:*

$$S = \left\|K^{-1}\right\| \cdot \|K\| = \left\|J \cdot J^T\right\| \cdot \left\|(J \cdot J^T)^{-1}\right\| \tag{9}$$

If the guiding chains of the machine between frame and working platform have different stiffness, the matrix *K* must be replaced by the matrix:

$$K_C = (J^T)^{-1} \cdot C \cdot J^{-1} \tag{10}$$

where the diagonal matrix *C* contains the stiffness of the single guiding chain. The reciprocal value of *S* is between $0 < 1/S \leq 1$; a singular pose is again characterized by $1/S = 0$, whereas $1/S = 1$ is the optimal (isotropic) index.

C. Workspace evaluation

In this section, the workspace of the proposed robot will be discussed in details. For a robot in the context of industrial application and given parameters, it is very important to analyze the area and the shape of its workspace. Calculation of the workspace and its boundaries with perfect precision is crucial, because they influence the dimensional design, the manipulator's positioning in the work environment, and its dexterity to execute tasks.

The workspace is limited by several conditions. The prime limitation is the boundary obtained through solving inverse kinematics. Further, the workspace is limited by the reachable extent of drives and joints, then by the occurrence of singularities, and finally by the link and platform collisions. The parallel robot DELTA linear realizes a wide workspace, as presented in Fig. 6. Analysis, i.e. visualization of the workspace is an important aspect of performance analysis. In order to generate a reachable workspace of parallel manipulators, a numerical algorithm was introduced. For the sake of simplicity, other design specific factors such as the end-effector size, drive volumes have been ignored. The following figures visualize the 3D robot workspace (Fig. 5).

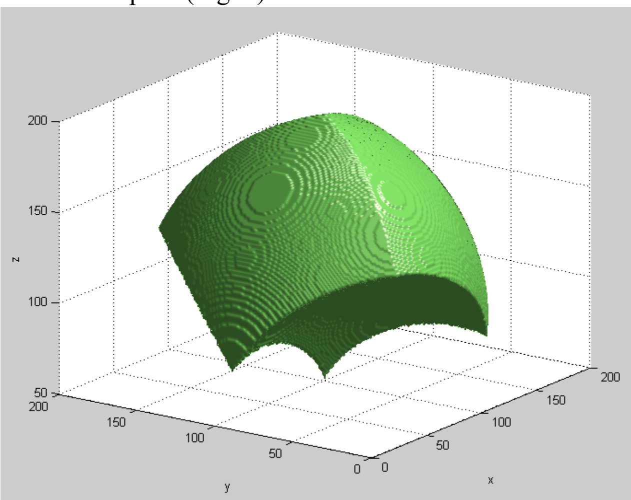

Fig. 5. Workspace for the DELTA 3 DOF parallel robot.

Ideally it's preferred those criteria to be like:

1. *transmission quality index* → *T*=1 the best value and the maximum one;
2. *workspace* → a higher value is desirable;
3. *stiffness index* → a higher value is desirable;
4. *manipulability* index → a higher value is desirable.

IV. Performance Evaluation Results

In the following figures, the performances evaluation throughout the workspace of the 3 DOF DELTA parallel robot is presented.

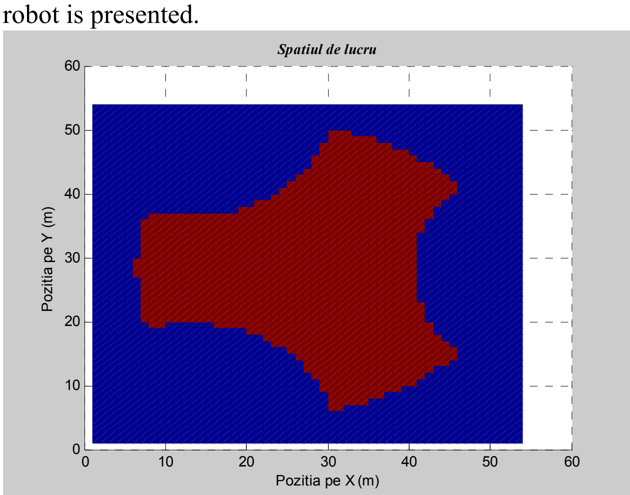

Fig. 6. Workspace for the DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

Figures 7 & 8 demonstrate better performances in terms of transmission quality of the DELTA parallel robot in the central part of the workspace approaching to the isotropic configuration.

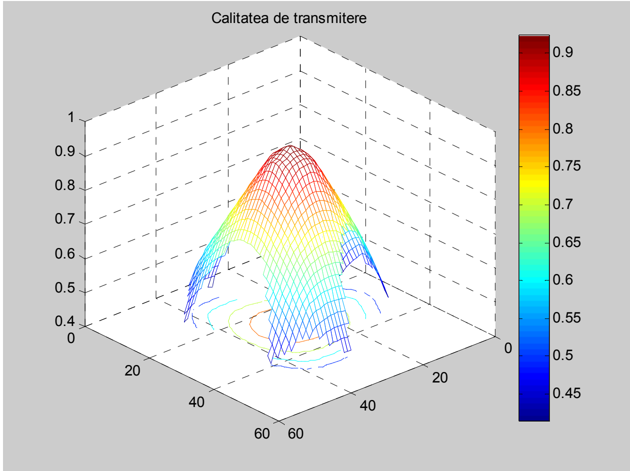

Fig. 7. Transmission quality index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

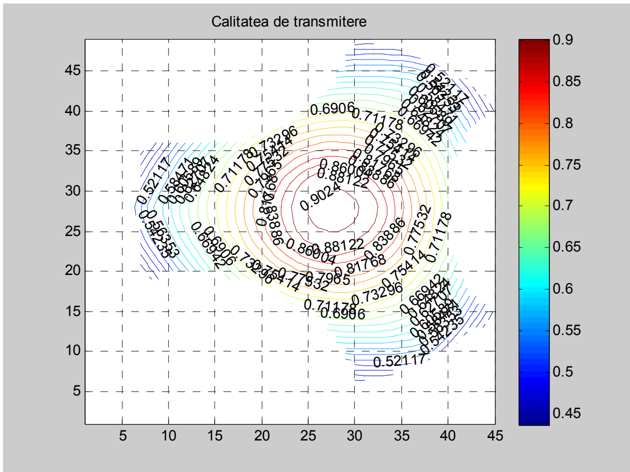

Fig. 8. Transmission quality index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

When a robot architecture admits a condition number equal to one, it is called an isotropic architecture, based on the isotropy of its relative Jacobian matrix.

Stiffness is one of the most important performances of parallel mechanisms, particularly for those which are used as machine tools, because higher stiffness allows higher machining speeds with higher accuracy of the end-effector. Therefore, it is necessary to perform the stiffness modeling, as well as the evaluation of the parallel robot in the early design stage.

With regards to stiffness evaluation, several different performance indices have been proposed and utilized in the literatures.

A simple way to predict the stiffness is to use the interested stiffness factors, i.e., the terms of the stiffness matrix [14]. Figures 9 & 10 demonstrate better performances in terms of stiffness of the DELTA parallel robot in the central part of the workspace.

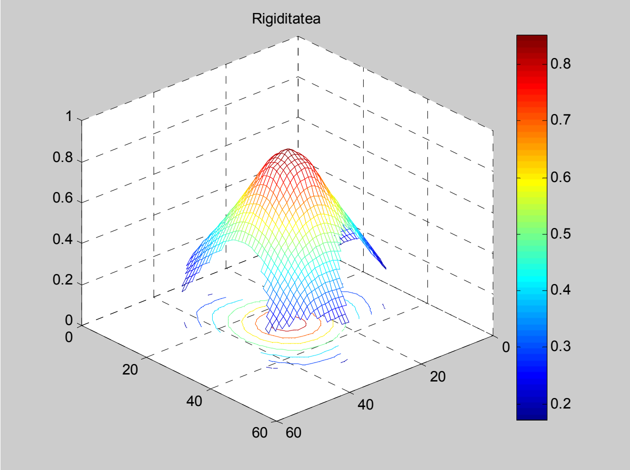

Fig. 9. Stiffness index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

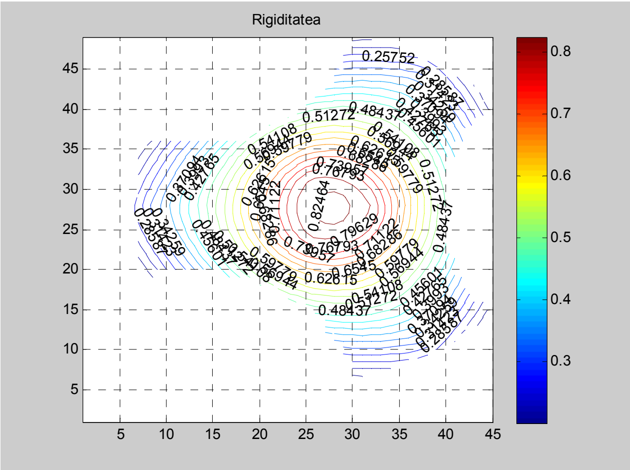

Fig. 10. Stiffness index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

Furthermore, similar to the condition number of Jacobian matrix, the condition number of the stiffness matrix was introduced (Fig.9 & 10). Accuracy of the control of the robot is dependent on the condition number of the Jacobian matrix.

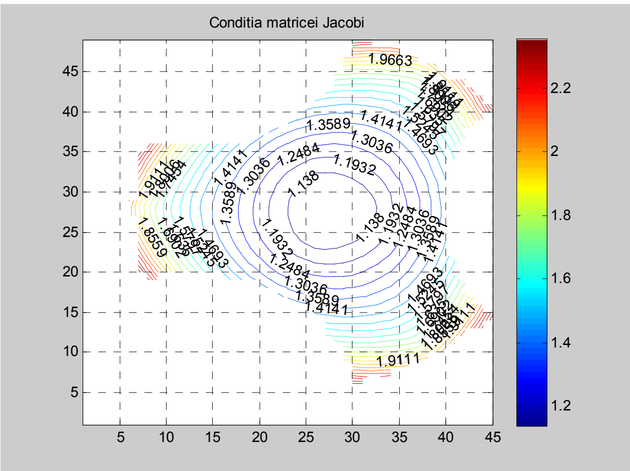

Fig. 11. Condition number of Jacobian matrix index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

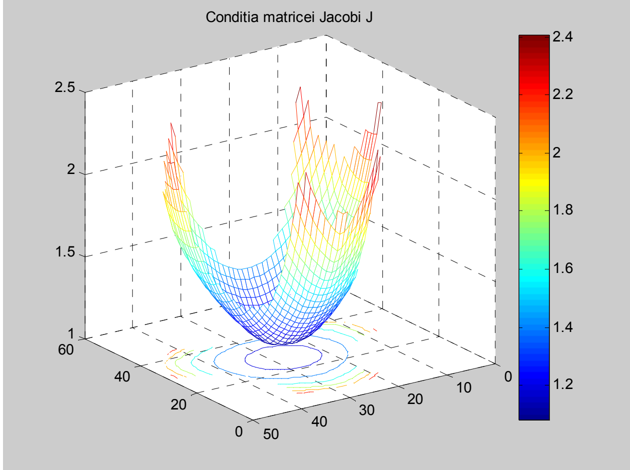

Fig. 12. Condition number of Jacobian matrix index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

It is well known that the numerical stability of the mapping from end-effector velocity to joint velocity is closely related to the kinematic performance of a robot. This mapping is described by the so-called Jacobian matrix.

Thus, closely relating the kinematic performance of a robot to the numerical condition of its Jacobian matrix. The performance of the robot is better, when the condition number is smaller.

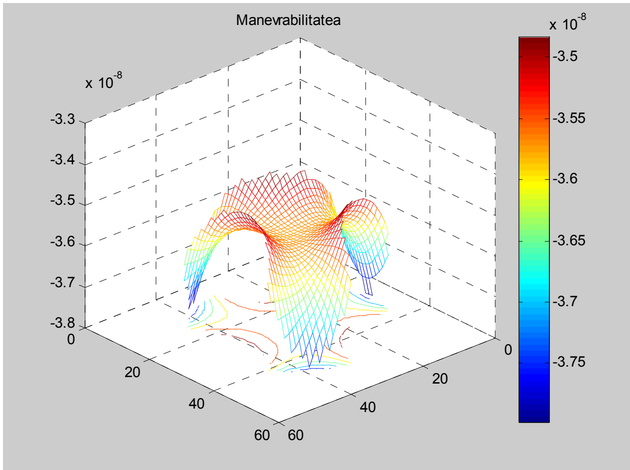

Fig. 13. Manipulability index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

Quite all the performance criteria that are in use are referred to the Jacobian matrix and to the condition number.

The evaluation of the parallel robots is the key issue for an efficient use of parallel robots. This is a complex and hard task. In the paper was presented a framework for the performance evaluation considering basic characteristics of:

- workspace;
- stiffness;
- isotropy;
- manipulability.

This can be useful for future optimal synthesis of 3DOF Delta parallel robot.

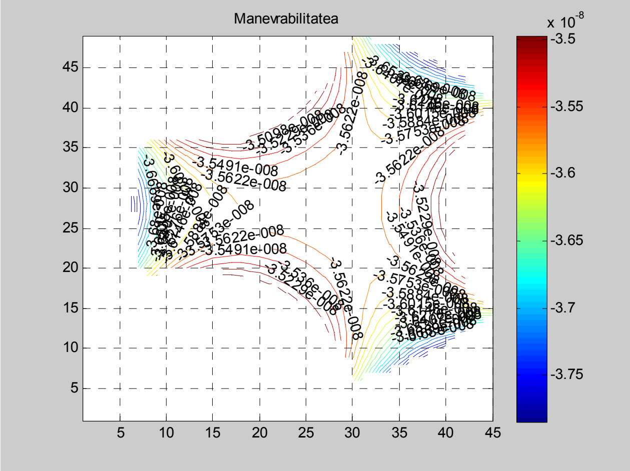

Fig. 14. Manipulability index for DELTA 3 DOF parallel robot, section through Z-plane, height 333mm.

V. CONCLUSION

The assessment based on manipulability, transmission quality, stiffness, dexterity and workspace are defined to evaluate the performance of the 3DOF Delta parallel robot. The evaluation measures consist of the local manipulability, local transmission quality, local stiffness and local dexterity. The evaluation measures can be used as the constraint criteria for future optimal synthesis of 3DOF Delta parallel robot. The goal of the future optimization will be to determine the dimensions and initial posture of the 3 DOF Delta parallel robot that has the largest workspace and isotropy status of performance. In the future, it can be solved optimization for enlarging the height of the workspace or for the optimal global manipulability, global dexterity, global transmission quality, global stiffness with expected workspace.

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REFERENCES

[1] Gupta, A., et al., “Design, Control and Performance of RiceWrist: A Force Feedback Wrist Exoskeleton for Rehabilitation and Training”, *The Int. J. of Robotics Research*, Vol. 27, No. 2, 233-251 (2008).

[2] J. P. Merlet. “Determination of the orientation workspace of parallel manipulators”. *Journal of intelligent and robotic systems*, 13:143–160, 1995.

[3] A. Kumar, KJ. Waldron. “The workspace of mechanical manipulators”. *ASME J. Mech. Des*. 1981; 103:665-672.

[4] YC. Tsai, AH. Soni. “Accessible region and synthesis of robot arm”. *ASME J. Mech Des*. 1981, 103: 803-811.

[5] KG. Gupta, Roth B., “Design considerations for manipulator workspace”. *ASME J. Mech. Des.* 1982, 104(4), 704-711.

[6] K. Sugimoto, Duffy J, Hunt KH, “Special configurations of spatial mechanisms and robot arms”. *Mech Mach Theory* 1982, 117(2); 119-132.

[7] KC. Gupta. “On the nature of robot workspaces”, *Int. J. Rob. Res.* 1986; 5(2): 112-121

[8] JK. Davidson, KH. Hunt, “Rigid body location and robot workspace: some alternative manipulator forms”. *ASME J. Mech. Transmissions Automat Des* 1987, 109(2); 224-232.

[9] SK. Agrawal, “Workspace boundaries of in-parallel manipulator systems”. *Int. J. Robotics Automat* 1990, 6(3) 281-290.

[10] C. Gosselin, Angeles J. “Singularities analysis of closed loop kinematic chains”.*IEEE-T.Robotics Automat* 1990; 6(3) 281-290.

[11] M. Cecarelli, “A synthesis algorithm for three-revolute manipulators by using an algebraic formulation of workspace boundary”. *ASME J. Mech. Des.* 1995; 117(2(A)): 298-302.

[12] S. K. Agrawal. “Workspace boundaries of in-parallel manipulator systems”. *IEEE Transactions on Robotics and Automation*, 7(2):94–99, 1991.

[13] F. Pernkopf and M. Husty, ”Reachable Workspace and Manufacturing Errors of Stewart-Gough Manipulators”, *Proc. of MUSME 2005, the Int. Sym. on Multibody Systems and Mechatronics* Brazil, 2005, p. 293-304.

[14] S. Stan, Diplomarbeit, *Analyse und Optimierung der strukturellen Abmessungen von Werkzeugmaschinen mit Parallelstruktur*, IWF-TU Braunschweig, 2003, Germany.

[15] K. Cleary and T. Arai. “A prototype parallel manipulator: Kinematics, construction, software, workspace results, and singularity analysis”. In *Proceedings of International Conference on Robotics and Automation*, pages 566–571, Sacramento, California, April 1991.

[16] M. Ceccarelli, G. Carbone, E. Ottaviano, “An Optimization Problem Approach For Designing Both Serial And Parallel Manipulators”, *Proc. of MUSME 2005, the Int. Sym. on Multibody Systems and Mechatronics* Uberlandia, Brazil, 6-9 March 2005

[17] M. Ceccarelli, *Fundamentals of Mechanics of Robotic Manipulation*, Dordrecht, Kluwer/Springer, 2004.

[18] G. Gogu, “Evolutionary morphology: a structured approach to. inventive engineering design”, 5th. International Conference. IDMME 2004, France.

[19] Verdes Dan, Sergiu-Dan Stan, Milos Manic, Radu Balan, Mechatronic design, kinematics analysis of a 3 DOF medical parallel robot, IEEE-ISRCS’10, the 3rd IEEE Symposium on Resilience Control Systems, Idaho Falls, Idaho, USA, Aug. 10-12, 2010.

[20] Rad, C.-R.; Manic, M.; Balan, R.; Stan, S.-D.; Real time evaluation of inverse kinematics for a 3-RPS medical parallel robot usind dSpace platform, 2010 3rd Conference on Human System Interactions (IEEE-HSI 2010), 13-15 May 2010, page(s): 48 - 53, Rzeszow, Poland, ISBN: 978-1-4244-7560-5, Digital Object Identifier: 10.1109/HSI.2010.5514590

[21] Nadia Rat, Mircea Neagoe, Dorin Diaconesc, Sergiu-Dan Stan, Radu Balan, Dynamic Modeling and VR Simulation of 3DOF Medical Parallel robots, M. Iskander et al. (eds.), Technological Developments in Education and Automation, DOI 10.1007/978-90-481-3656-8_55, pag. 297-302.© Springer-Verlag Berlin Heidelberg 2010.

[22] Stan, S.-D., Gogu, G., Manic, M., Balan, R., Rad, C., Fuzzy Control of a 3 Degree of Freedom Parallel Robot, M. Iskander et al. (eds.), Technological Developments in Education and Automation, DOI 10.1007/978-90-481-3656-8_55, pag. 437-442 © Springer-Verlag Berlin Heidelberg, 2010, ISBN 978-90-481-3655-1 (Print) 978-90-481-3656-8 (Online), 2010.

[23] Nadia Ramona Raţ, Mircea Neagoe, Sergiu-Dan Stan, Comparative dynamic analysis of two parallel robots, Solid State Phenomena Vols. 166-167 (2010) pp 345-356 (2010) Trans Tech Publications, Switzerland doi:10.4028/www.scientific.net/SSP.166-167.345.

[24] Mircea Coman, Sergiu-Dan Stan, Milos Manic, Radu Balan, Application of Distance Measuring with Matlab/Simulink, IEEE-HSI 2010, the 3rd International Conference on Human System Interaction, Rzeszow, Poland, May 13-15, 2010, ISBN: 978-1-4244-7560-5.

[25] Khaled ASSAD ARROUK, Belhassen CHEDLI BOUZGARROU, Sergiu-Dan STAN, Grigore GOGU, CAD Based Design Optimization of Planar Parallel Manipulators, Solid State Phenomena Vols. 166-167 (2010) pp 33-38 © (2010) Trans Tech Publications, Switzerland, doi:10.4028/www.scientific.net/SSP.166-167.33.

[26] Szep, C. Stan, S.-D. Csibi, V. Manic, M. Balan, R. , Kinematics, workspace, design and accuracy analysis of RPRPR medical parallel robot, IEEE- HSI '09. 2nd Conference on Human System Interactions, 2009, 21-23 May 2009 Catania, page(s): 75-80, ISBN: 978-1-4244-3959-1, Digital Object Identifier: 10.1109/HSI.2009.5090957.