Evaluating Uncertainty Resiliency of Type-2 Fuzzy Logic Controllers for Parallel Delta Robot

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Abstract. **As a consequence of recent theoretical advancements in Type-2 (T2) fuzzy logic, applications of T2 Fuzzy Logic Controllers (FLCs) are becoming increasingly popular in various engineering areas. Nevertheless, the qualitative comparison of Type-1 (T1) and T2 FLCs and the assessment of the potential of T2 fuzzy logic can still be considered open questions. Despite this fact, researchers commonly claim superiority of T2 FLC in uncertain conditions based on a very limited exploration of the design parameter space. This manuscript provides a systematic analysis of the uncertainty resiliency of T2 FLC used for position control of parallel delta robot. In order to allow for objective comparison among different T1 and T2 FLCs, the controllers were constructed using a partially-dependent approach. Here, the T2 FLC is created based on an initially optimized T1 FLC. In this, manner the constrained design space allows for its full systematic exploration and analysis. The performance of each controller was evaluated on the real parallel delta robot under various levels of dynamic uncertainty. The experimental results support the theoretical claims about the superiority of T2 FLC. However, it was also demonstrated that there is a clear upper bound on the amount of "type-2 fuzziness" in the controller design, which can result in performance improvement. Exceeding such upper bound leads to performance deterioration.**

***Keywords*: Delta Parallel Robot, Interval Type-2 Fuzzy Logic Control, Resilient Systems.**

I. INTRODUCTION

THE ability to cope with the linguistic uncertainty originating in the imprecise and vague meaning of words, made the Type-1 Fuzzy Logic Systems (T1 FLSs) popular in many engineering areas [1]. However, when various kinds of dynamic uncertainties are encountered, the control performance can significantly deteriorate [2]. In [2] the various sources of uncertainty were identified as follows: i) uncertainty in the linguistic knowledge in FLS (simply different designers might have different opinions about the optimal behavior), ii) uncertainty about the correct output of the system, iii) uncertainty associated with noisy inputs, and iv) uncertainty about the data that were used to tune the parameters of the control system. These sources of uncertainty can lead to performance degradation that is primarily caused by the T1 fuzzy membership functions, which become fixed once the design process is finalized.

As an extension to T1 fuzzy logic, the Type-2 (T2) fuzzy logic was originally proposed by Zadeh [3]. T2 FLSs have recently become the scope of work of many researchers [2], [4]-[8]. T2 FLSs found successful application in many engineering areas, demonstrating ability to perform better than T1 FLSs when facing dynamic uncertainties [9]-[12]. The fundamental difference between T1 and T2 FLSs is in the model of fuzzy sets. T2 fuzzy sets employ membership degrees that are fuzzy sets themselves. This additional uncertainty dimension provides new degrees of freedom for modeling and coping with dynamic input uncertainties.

However, the understanding and correctness of the design process of T2 FLCs can still be considered an open question. In addition, the qualitative comparison of Type-1 (T1) and T2 FLCs and the systematic assessment of the potential of T2 fuzzy logic is currently an active area of research [13]. It is very common that researchers claim superiority of T2 FLC based on a very limited exploration of the space of design parameters. However, as previously demonstrated, T2 FLCs might exhibit slower responsiveness and excessive dumping of the output signal in specific scenarios (e.g. autonomous robot navigation) [6], [12]. It is natural to expect that with the increased amount of "type-2 fuzziness" in the controller design, such negative effects will be further accentuated.

This manuscript provides a systematic analysis of the performance and uncertainty resiliency of T2 FLC used for position control of parallel delta robot. Here, the Interval Type-2 (IT2) FLC was considered [8]. The IT2 FLC assumes only interval membership grades for each fuzzy set. This constitutes a substantial simplification for the design process and implementation. In order to allow for a systematic analysis, the space of design parameters was constrained by considering only partially-dependent design of IT2 FLCs [2]. The partially-dependent approach starts with an optimized T1 FLC and then symmetrically blurs all membership functions. While this design approach is clearly not optimal, it allows for exhaustive exploration of the constrained design space. The authors believe that the presented study of this constrained set of T1 and IT2 fuzzy controllers can provide insights into important conclusions widely applicable to fuzzy logic control.

In this paper, the constructed FLCs were used for position control of the end-effector of a 3DOF parallel delta robot. Such systems are applicable in many engineering and industrial areas, e.g. robotic teleoperation or remote welding [14], [15]. This specific robotic architecture is ideal for the systematic performance evaluation as it allows for automatic testing of the constructed controllers on the real robotic hardware. The controller's performance was subjected to different levels of uncertainty, manifested as different amplitudes of the injected noise combined with the inherent uncertainties of the real robotic hardware.

The rest of the paper is organized as follows. Section II provides background review of T1 and IT2 fuzzy logic control. The robotic platform for the 3DOF parallel delta robot is introduced in Section III. Section IV discusses the design of the fuzzy PD controllers. Finally, the results are presented in Section V and the paper is concluded in Section VI.

II. T1 and IT2 Fuzzy Logic Systems

This section provides fundamental background overview of T1 and IT2 FLSs.

A. Type-1 Fuzzy Logic Systems (T1 FLSs)

Type-1 Fuzzy Logic Systems have been successfully applied to many engineering problems [1]. The primary advantage of T1 FLSs is the ability to encode knowledge via linguistic fuzzy rules so it can be easily understood by humans. Furthermore, T1 FLSs can cope with ambiguity, imprecision and uncertainty [2].

In general, a T1 FLS is composed of four major components – input fuzzification, fuzzy inference engine, fuzzy rule base and output defuzzification [2]. Here, the considered Mamdani FLS maintains a fuzzy rule base populated with fuzzy linguistic rules in an implicative form. As an example consider rule R_k:

Rule R_k: **IF** x_1 is A_1^k **AND** … **AND** x_n is A_n^k **THEN** y_k is B^k (1)

Here, symbol A_j^k and B^k denote the j^{th} input fuzzy set and the output fuzzy set, n is the dimensionality of the input vector $\vec{x}$, and y_k is the associated output variable. Each system input is first fuzzified using the respective fuzzy membership function (e.g. Gaussian, triangular, trapezoidal, etc.). The fuzzification of input x_i into fuzzy set A_i results in a fuzzy membership grade $\mu_{A_i^k}(x_i)$. Using the minimum t-norm, the degree of firing of rule R_k can be computed as follows:

$$\mu_{R_k}(\vec{x}) = \arg\min_i \{\mu_{A_i^k}(x_i)\}, \quad i = 1..n \tag{2}$$

The results of each rule are computed by applying the rule firing strength via the t-norm operator (e.g. minimum or product) to each rule consequent. Then the output fuzzy sets are aggregated using the t-conorm operator (e.g. the maximum operator), resulting in an output fuzzy set B. Detailed description of the fuzzy inference process can be found in [2].

Finally, the defuzzification of the output fuzzy B set yields the crisp output value. Several defuzzification techniques can be found in literature, e.g. centroid defuzzifier, center-of-sums defuzzifier, heights defuzzifier, or the center-of-sets defuzzifier [2]. Assuming that the output domain is discretized into N samples, the centroid defuzzifier can be used to produce the crisp output value y as follows:

$$y = \frac{\sum_{i=1}^{N} y_i \mu_B(y_i)}{\sum_{i=1}^{N} \mu_B(y_i)} \tag{3}$$

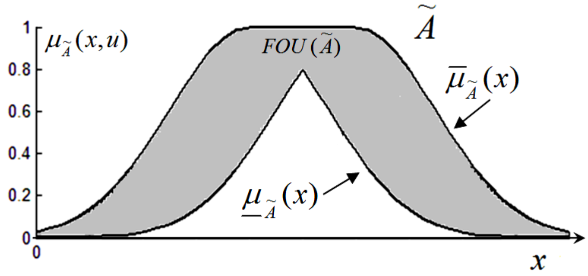


Fig. 1 Interval type-2 fuzzy set $\tilde{A}$.

B. Interval Type-2 Fuzzy Logic Systems

The Interval Type-2 (IT2) FLSs are considered in this paper because of their computational inexpensiveness and ease of implementation [8]. In addition, the space of design parameters of IT2 FLSs is substantially smaller than for full-blown general T2 FLSs. An IT2 fuzzy set $\tilde{A}$ can be expressed as follows:

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} 1/(x,u) \qquad J_x \subseteq [0,1] \tag{4}$$

Here, x and u are the primary and secondary variables, and J_x is the primary membership of x. In case of IT2 fuzzy sets, all secondary grades of fuzzy set $\tilde{A}$ are equal to 1. By instantiating the variable x into a specific value x', the vertical slice of the IT2 fuzzy set can be obtained:

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x') = \int_{u\in J_{x'}} 1/u \qquad J_{x'} \subseteq [0,1] \tag{5}$$

The domain of the primary memberships J_x defines the Footprint-Of-Uncertainty (FOU) of fuzzy set $\tilde{A}$:

$$FOU(\tilde{A}) = \bigcup_{x\in X} J_x \tag{6}$$

The FOU of an IT2 fuzzy set is schematically denoted in Fig. 1. Alternatively, the FOU of an IT2 fuzzy set $\tilde{A}$ can be conveniently and completely described by its upper and lower membership functions:

$$FOU(\tilde{A}) = \bigcup_{\forall x\in X} (\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)) \tag{7}$$

This constitutes a substantial simplification when compared to the general T2 FLS. In this way, only two T1 fuzzy membership functions (the upper and the lower boundary of the FOU) are needed to fully describe the IT2 fuzzy set. This simplification is then transferred through the inference mechanism taking advantage of the modified T2 fuzzy join and meet operations [2]. The interval join and

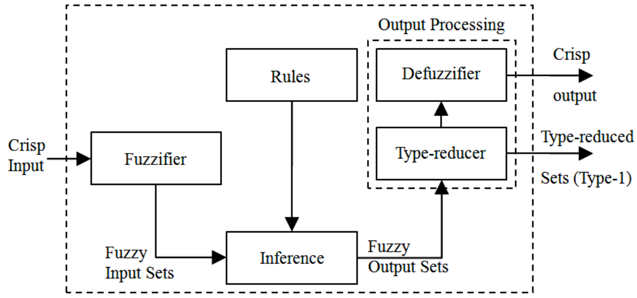


Fig. 2 Type-2 fuzzy logic system [2].

meet operations work exclusively with the FOU of the IT2 fuzzy sets, thus removing much of the computational burden associated with general T2 fuzzy sets.

In order to obtain a crisp output value, the resulting IT2 output fuzzy set $\tilde{B}$ is first type reduced and then defuzzified. The centroid of the IT2 fuzzy set $\tilde{B}$ can be defined as [16]:

$$C_{\tilde{B}} = \int_{\theta_1 \in J_{x_1}} \cdots \int_{\theta_N \in J_{x_N}} 1 \Bigg/ \frac{\sum_{i=1}^{N} x_i \theta_i}{\sum_{i=1}^{N} \theta_i} \tag{8}$$

The centroid $C_{\tilde{B}}$ is an interval T1 fuzzy set. This fuzzy set can be completely described by its left and right end points y_l and y_r. These boundary points can be calculated using the Karnik-Mendel iterative procedure [17]. Using the boundary values of the type-reduced centroid $C_{\tilde{B}}$, the final crisp defuzzified value y can be computed as the mean of the centroid interval:

$$y = \frac{(y_l + y_r)}{2} \tag{9}$$

Fig. 2 shows a block diagram of the T2 FLS.

III. 3DOF Parallel Delta Robot

In this paper, the fuzzy logic control is applied to the problem of precise and robust position control for 3DOF parallel delta robot [18]. In general, parallel robotic architectures feature substantially increased rigidity and allowable workload when compared to their sequential counterparts. However, parallel robots also suffer from constrained workspace and increased complexity of the kinematic and dynamic mechanism [18], [20].

The actual robotic platform consists of Novint Falcon haptic device [19]. This force-controlled parallel delta robot configuration has three force-feedback and tactile sensation

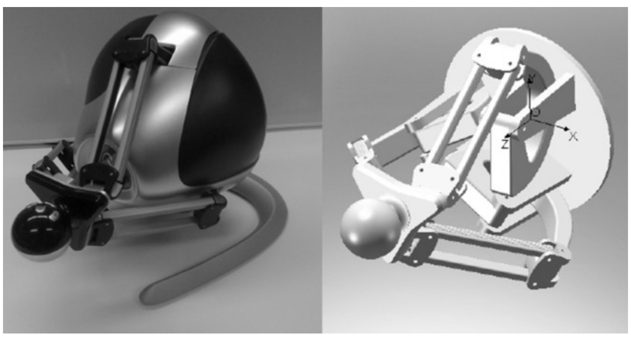


Fig. 3 Novint Falcon haptic device [20].

enabled degrees of freedom. Working frequency of 1kHz and position resolution of 400 dpi within the 4” x 4” x 4” workspace allows for smooth control of the multi-robot system as well as fluent perception of the generated haptic force. The Novint Falcon haptic device is depicted in Fig. 3.

The Falcon typically serves as a 3DOF input device. However, due to the presence of force-feedback, the flow of control can be reversed and a 3DOF robotic manipulator can be created. The Falcon can be then viewed as a 3DOF parallel delta robot. From a robotic control point of view, Novint Falcon constitutes an excellent experimental platform. Sampling of 1kHz and smooth actuation allow for very precise position sensing and high fidelity control. The suitability of the Falcon device for research and application as a robotic manipulator was investigated in [20].

The Novint Falcon is accompanied by a powerful C++ based Software Development Kit (SDK), which allows for programmatic modifications of the robot behavior. The SDK abstracts the user from the inverse kinematics of the robot, which provides convenient control of robot’s motion on the actual three motion axis – x, y, and z. In addition, the SDK also allows for connecting multiple Falcons in one software application. This capability can be utilized for controlling the reversed Falcon robot manipulator (slave) via another Falcon input device (master). In this scenario, the slave Falcon uses the implemented position control to accurately mimic master’s motion (e.g. controlled by the hand of an operator).

IV. Fuzzy PD Controller Design for 3DOF Parallel Robot

This section describes the initial baseline design of the T1 FLC for position control of the delta parallel robot and its subsequent extension to IT2 FLC. The resiliency of the constructed IT2 FLC is then evaluated with respect to the baseline T1 FLC. The design methodology can be summarized in three steps as follows:

Step 1: Manually design an initial T1 FLC.

Step 2: Tune the parameters of the initial T1 FLC using one of the available optimization techniques (here the Particle Swarm Optimization (PSO) algorithm was used).

Step 3: Extend the T1 FLC into an IT2 FLC using the partially dependent design.

Individual steps are described below.

A. Initial Manual Fuzzy Controller Design

The classical way of designing controllers is to use the PID controller (or its PD or PI variants). For the PID controller, the control signal $out(t)$ at time t is proportional to the error $e(t)$ measured by the input sensors. The general control law of the PID controller can be stated as follows [21]:

$$out(t) = K_p\, e(t) + K_i \int e(t)\, dt + K_d \frac{d\, e(t)}{dt} \tag{10}$$

Here, coefficients K_p, K_i and K_d are the gains for the specific actuator (e.g. servo motor). In this paper, the integral term is omitted and only a PD controller is considered. A schematic view of the controller is presented in Fig. 4.

The fuzzy PD controller is commonly adopted in engineering applications for its several advantages: i) it can

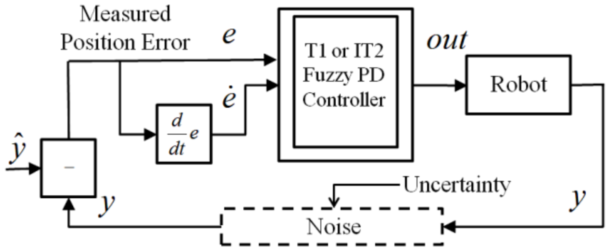


Fig. 4 The control loop of the robot.

be constructed using linguistic knowledge about the controlled system, ii) it features more degrees of freedom, and iii) it was shown to produce smoother control behavior [21], [22].

In the presented paper, the fuzzy controller was designed using two fuzzy sets {*negative*, *positive*} for describing error e and error derivative $\dot{e}$, and three output fuzzy sets {*negative*, *zero*, *positive*} for describing the output signal *out*. The fuzzy rule base presented in Table I provides four linguistic control rules for the control process. This rule table constitutes a commonly adopted set of linguistic rules for fuzzy PD controllers. As derived in the work of Du and Ying [23], the fuzzy PD controller can be understood as a composition of multiple classical PD controllers with variable gains.

In this specific application, three fuzzy PD controllers were implemented for each degree of freedom of the parallel delta robot. The error of the controller was calculated as the difference between the desired position of the robot's end-effector and its actual position sensed by the position sensors of the robot. The fuzzy controller implemented triangular membership function. The initial parameters of the T1 fuzzy sets were determined by a trial

TABLE I
FUZZY RULE TABLE

	$\dot{e}_{neg}$	$\dot{e}_{pos}$
e_{neg}	Out_{neg}	Out_{zero}
e_{pos}	Out_{zero}	Out_{pos}

and error process with the intention to produce preceise and stable initial robot's performance.

B. Optimization of T1 FLC using PSO

Next, the tools of evolutionary computation were used to further optimize the performance of the T1 FLC. Here, the PSO algorithm was used to tune the parameters of the fuzzy membership functions of the respective T1 fuzzy PD controllers. The linguistic rules of the controllers remained fixed during the parameter tuning process.

The PSO algorithm is a biologically and physically inspired paradigm which has been successfully applied to many optimization problems [21]. The algorithm was designed to resemble the patterns observed in social species such as swarms of fish or flocks of birds when searching for food. The PSO algorithm was originally developed by Kennedy and Ebenhart in 1995 [24]. Due to the limited space, the details of the PSO algorithm are not described here but can be found in [21].

Each particle represents a single design of a T1 FLC. The fitness of each particle was evaluated as the total Root Mean Square Error (RMSE) of the respective T1 FLC on the given training dataset. Fig. 5(a)-5(c) depicts the architecture of the optimized T1 FLC for the position control of robot's x-axis. Fig. 6(a) shows the output control surface for the x-axis.

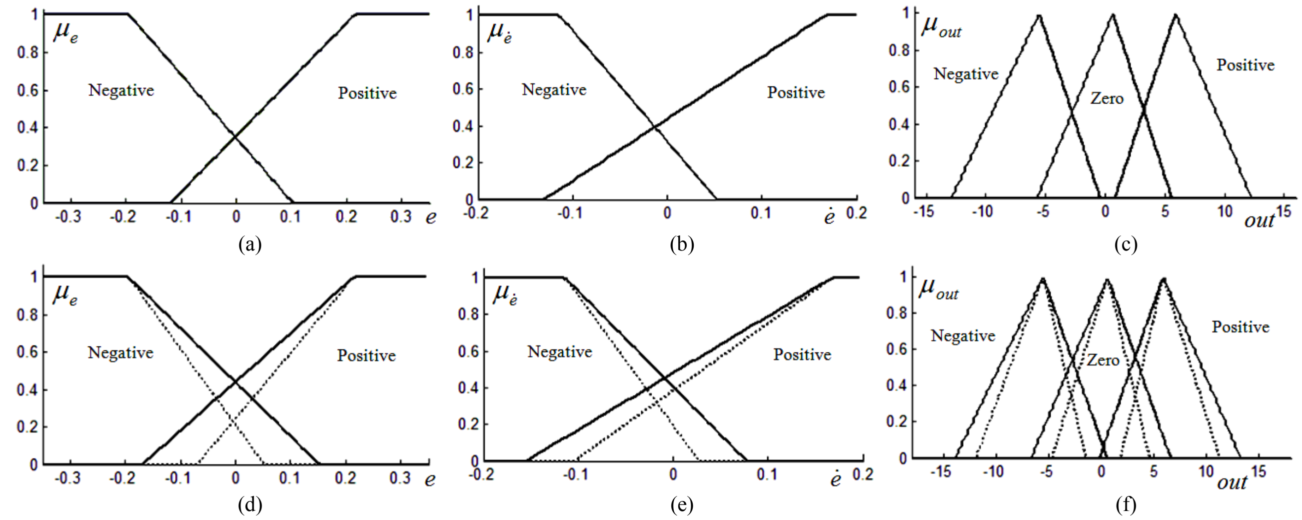


Fig. 5 Optimized design of the T1 FLC (a)-(c) and the IT2 FLC constructed via the partially-dependent approach (d)-(f) for position control of robot's x-axis.

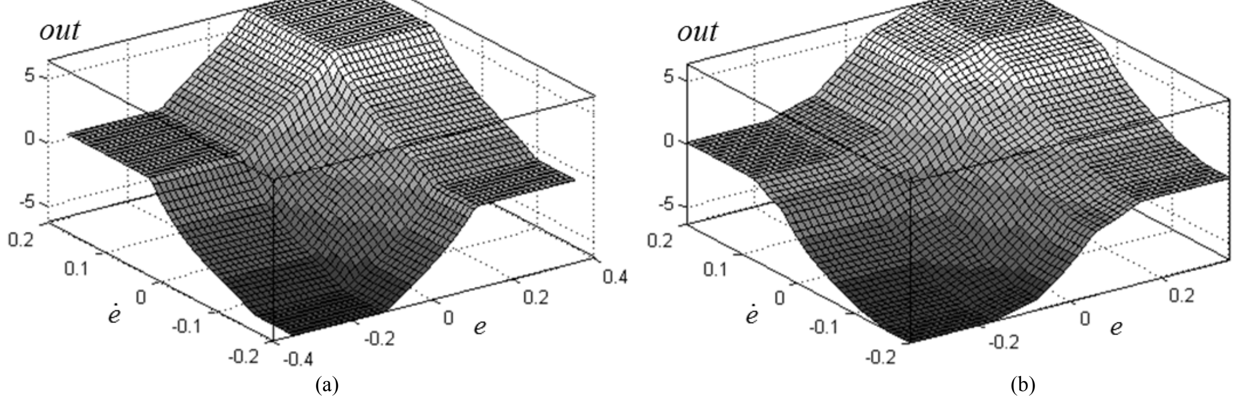


Fig. 6 The output surface of the optimized T1 FLC (a) and the IT2 FLC constructed via the partially-dependent approach (b) for control of robot's x-axis.

C. Partially-Dependent Design of IT2 fuzzy PD controller

In general, there are two available strategies for designing T2 FLCs [2]. Firstly, the controller can be designed via fully-independent approach, when the entire T2 FLC is designed from scratch. Secondly, the partially-dependent approach can be used. In this method, an initial T1 FLC is constructed first and next this controller is extended into the T2 FLC. In this manner, the T2 FLC builds on the architecture of the original T1 FLC and it should provide an additional performance improvement.

The partially-dependent T2 FLC design was favored in this work for two main reasons. First, this design methodology substantially reduces the number of design parameters. Second, the derived T2 FLC can be objectively compared to the original T1 FLC.

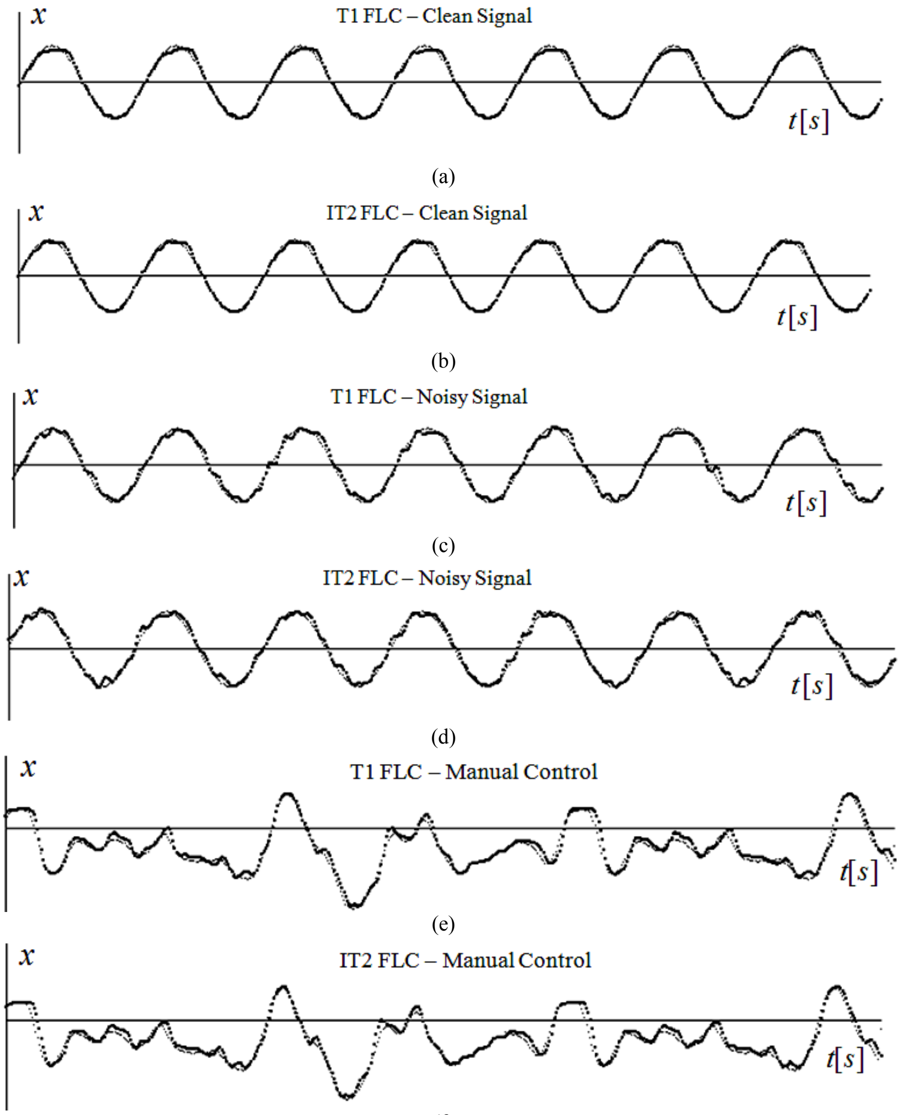
In order to allow for systematic exploration of the design space, the simplest version of the partially-dependent design was used. Here, the trapezoidal and triangular fuzzy sets with uncertain spreads were implemented. The spread of each IT2 fuzzy membership function was calculated by symmetrical blurring the original T1 fuzzy membership function.

First, the maximum allowable spread is determined for each fuzzy set of the optimized T1 FLC, assuring reasonable overlap between neighboring fuzzy sets. Next, a specific ration (0.0-1.0) of the maximum spread called the blurring parameter is used to construct the fuzzy sets of the IT2 FLC. Note that 0.0 spread reduces the IT2 FLS to the original T1 FLC, whereas blurring parameter of 1.0 implements IT2 fuzzy set with the maximum amount of blur. The same blurring parameter was used for all fuzzy membership function of the whole IT2 FLC. In this manner, given the original T1 FLC, the number of design parameters of the partially-dependent approach was reduced to a single parameter – the blurring parameter.

Fig. 5(d)-(f) depict the IT2 FLC designed with the blurring parameter of 0.5 for position control of robot's *x*-axis. Fig. 6(b) plots the respective output control surface. It can be observed that the IT2 FLC offers substantially smoother control performance.

V. EXPERIMENTAL RESULTS

This section presents the results of the experimental performance and uncertainty resilience evaluation of the designed fuzzy controllers. First, the correctness of the proposed fuzzy PD controller design is validated by observing its position tracking performance on the real robot. The robot was controlled with both automated input signal and using operator's hand motion. Next, the uncertainty resiliency of the fuzzy controllers under various



(f)

Fig. 7 Position control for the robot's *x*-axis for the PSO optimized T1 FLC and the IT2 FLC for the clean signal (a), (b), for the noisy signal (c), (d), and for the manual control (e), (f). (Desired position – thin line, robot's position – thick line)

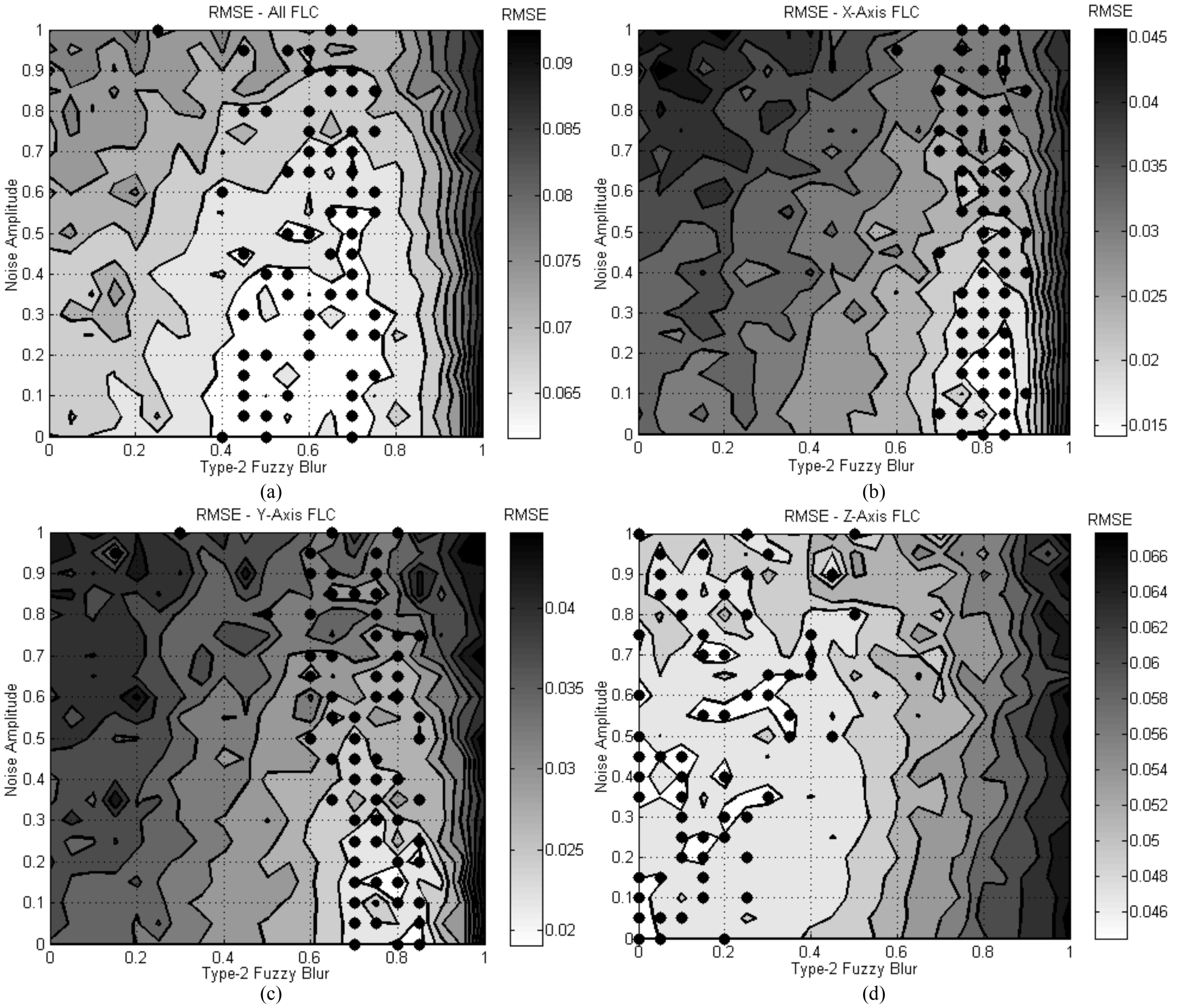


Fig. 8 The noise-blur RMSE surface for the 3D robot position control (a), the *x*-axis control (b), the *y*-axis control (c), and the *z*-axis control (d) (the black points depict 3 levels of blur with the lowest error for the each noise level).

levels of noise is evaluated.

A. Position Control Testing

In order to validate the correctness of the initial design, the performance of the fuzzy PD controllers was first tested on the real robot. In this experiment the PSO optimized T1 FLC and the partially-dependent IT2 FLC constructed with the blurring parameter of 0.5 were used to control the position of the robot's end-effector in the 3D work-space. The testing trajectory was implemented as a sinusoidal signal simultaneously applied to all three control axis resulting in a smooth movement of the end-effector in the 3D work space. The control trajectories for clean signal and for signal with injected noise are depicted in Fig. 7(a)-(d). In addition, Fig. 7(e) and Fig. 7(f) demonstrate the real-world example of the control system performance. In this example, slave Falcon device is following the trajectory produced by a human operator controlling the master Falcon input device.

Fig. 7 demonstrates that both T1 and IT2 FLCs are capable of precise control of the robot's end-effector.

B. Uncertainty Resiliency Evaluation

In this experiment the design space of the IT2 FLC and the space of different uncertainty levels was systematically explored. In this manner, a systematic insight into the performance quality and the uncertainty resiliency (i.g. robustness to injected noise) of IT2 FLC for the parallel delta robot was provided.

For each considered value of the blurring parameter, the IT2 FLC was constructed. This IT2 FLC was then implemented on the robot and used for its position control. The controller's performance was subjected to different levels of uncertainty, manifested as different amplitudes of the injected noise. The amplitude of the injected noise was systematically increased starting from 0% all the way to 5% of the actual signal amplitude. Each constructed IT2 FLC was tested 10 times for each level of uncertainty. The RMSE of the entire run was recorded for all controllers on all three control axis.

Each value of the blurring parameter and the uncertainty level specifies a position on a 3D surface, where the 3rd dimension stores the achieved RMSE of the robot's controllers. Discretization of the interval of possible blurring parameters and possible noise amplitudes allows for systematic reconstruction of this error surface. The reconstructed landscape provides a clear picture of suitability of IT2 FLC architectures for a specific level of noise.

In this experiment, the range of blurring parameter was discretized into 21 values: {0.0, 0.05, 0.1,…,1.0}. The noise amplitude was also discretized into 21 values:

{0.0025, 0.005,…,0.05}. Fig. 8 shows the blur/uncertainty RMSE surface for all three FLCs on individual control axis of the robot together with the accumulated RMSE surface of the 3D position tracking performance. For an ease of understanding, the best three controllers are depicted with black points for each noise amplitude level. Note, that the amplitude of injected noise was normalized into a unit interval. By observing the location of the black points, the most uncertainty resilient designs can be easily determined.

C. Discussion

The initial testing demonstrated that the designed fuzzy PD controller provides robust position control of the end-effector of the 3DOF parallel delta robot. The controllers were capable of following the generated testing trajectory as well as manual control signal provided by an operator.

The observation of the constructed blur/uncertainty RMSE landscapes reveals some interesting conclusions. Firstly, it can be observed that the performance of the IT2 FLC varies for different control axis. This suggests that the performance improvement provided by the IT2 FLC is dependable on the dynamics of the robotic hardware as well as on the initial design of the T1 FLC. Secondly, it can be observed that in all cases the IT2 FLC provided performance benefits over the T1 FLC (equivalent to T2 FLC with 0% blurring parameter). Thirdly, perhaps the most important observation is that excessive blurring of the membership functions results in significant performance deterioration (shown by the high RMSE values for the blurring parameter values close to 1).

The first two observations are in agreement with the recently published study on the robustness of T1 and IT2 fuzzy logic systems in modeling [25]. The last observation clearly demonstrates that the superiority of IT2 FLC over T1 FLC is strongly dependent on careful selection of the parameters of the designed system.

VI. Conclusion

This paper presented a systematic analysis of the performance and uncertainty resiliency of T2 FLC used for position control of parallel delta robot. In order to allow for systematic exploration of the problem domain, the FLC was constrained to IT2 FLC constructed using the partially-dependent approach.

The systematic exploration of the space of blurring parameters and uncertainty levels allowed for global assessment of controllers' performance. The presented results revealed that the IT2 FLC is more resilient than T1 FLC when dealing with dynamic uncertainties. However, the experimental testing also showed that excessive blurring of the IT2 fuzzy membership functions might likely lead to significant performance deterioration.

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