Comparative Analysis of Type-1 and Type-2 Fuzzy Control in Context of Learning Behaviors for Mobile Robotics

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***Abstract*—Dynamic uncertainties, manifested as input noise or variable environment conditions, are an inherent part of most real world control applications. Recently, several researchers demonstrated that Type-2 Fuzzy Logic Controllers (T2 FLC) are able to cope with such uncertainty and reduce its negative effects. However, the design and optimization of T2 FLC and its subsequent unbiased comparison to T1 FLC are still an open question. This paper presents a comparative analysis of interval T2 (IT2) and T1 FLCs in the context of learning behaviors for mobile robotics. First, a T1 FLC is optimized using the Particle Swarm Optimization algorithm to mimic a wall-following behavior performed by an operator. Next, an IT2 FLC is constructed by symmetrically blurring the fuzzy sets of the original T1 FLC. The performance of the fuzzy controllers is compared using a wall-following sonar-equipped mobile robot in both noise-free and noisy environments. It is experimentally demonstrated that the IT2 FLC can cope better with dynamic uncertainties in the sensory inputs due to the softening and smoothing of the output control surface by the IT2 fuzzy sets. However, the IT2 FLC is outperformed by the T1 FLC when sudden and fast response of the controller is required, such as in the case of turning around corners. Those results suggest the difficulties of symmetrical blurring of T1 FLC (although commonly used) as a design methodology for obtaining the architecture of an IT2 FLC.**

***Index Terms*—Fuzzy Control, Interval Type-2 Fuzzy Systems, Mobile Robots, Behavior Learning, PSO.**

I. Introduction

TYPE-1 Fuzzy Logic Control (T1 FLC) have been used in many engineering applications [1]. Its main advantages are the abilities to incorporate human-understandable knowledge in the form of linguistic fuzzy rules and to cope with ambiguity, imprecision and uncertainty. However, dynamic uncertainties, inherent to many real world applications, can negatively affect the resulting performance of the control system [1]. The T1 FLC using crisp fuzzy memberships cannot directly address such variable conditions. Neglecting this uncertainty can lead to a subsequent deterioration of the system's quality.

Type-2 fuzzy logic, introduced by Zadeh, has the potential to handle dynamic uncertainties of unstructured environments [2], [3]. T2 FLC uses additional dimension of uncertainty, where the fuzzy membership degrees are themselves fuzzy. To deal with the increased computational burden, Mendel et al. introduced the Interval T2 (IT2) fuzzy systems [1], [4]. Several researchers worked in recent years towards demonstrating advantageous properties of the IT2 FLC over the T1 counterparts. However, the design and optimization of IT2 FLC and its subsequent unbiased comparison to the original T1 FLC are still an open question [5].

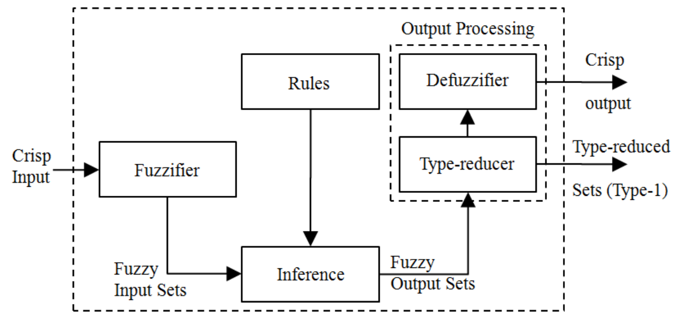


Fig. 1 Type-2 fuzzy logic system [1].

Several authors implemented the IT2 fuzzy systems in the context of mobile robotics [6] - [11]. In addition, Hagras et al. showed their superior performance in control of marine diesel engines [12]. However, in the work of Birkin and Garibaldi [13], [14], no statistical difference between the T1 and T2 controllers is reported in a micro-robot context.

The variability of the reported results can be attributed to a missing established methodology of designing the T1 and T2 FLC and their subsequent comparison [5]. One commonly adopted approach for constructing the IT2 FLC is the partially independent approach of symmetrically blurring the membership function of the original T1 FLC. This paper contributes by providing an experimental analysis of the performance of T1 and the respective "blurred" IT2 FLC in the context of learning behaviors for mobile robotics.

Here, fuzzy T1 and IT2 controllers are constructed using recorded experimental data. The data are sampled from control maneuvers of an operator performing a wall-following behavior with a tele-operated mobile robot. Such learning task inherently requires mechanisms for coping with noisy, imprecise and uncertain data. First, a T1 FLC is optimized using the PSO algorithm [15], [16]. Next, two IT2 FLCs are constructed by symmetrically blurring the T1 fuzzy membership functions. The two IT2 FLCs use T1 and IT2 fuzzy consequents, respectively. The control performance of all three fuzzy controllers was compared in both noise-free and in noisy environment. The experimental results revealed the advantageous properties of the IT2 fuzzy controller when dealing with dynamic noise around the set-point of the controller. However, the IT2 FLC was outperformed by its T1 counterpart, when fast response was required while turning around corners. Such results experimentally demonstrate some of the difficulties of the symmetrical blurring method as a design tool for deriving an IT2 FLC from the original T1 FLC.

The rest of the paper is organized as follows. Section II provides a primer on T1 and IT2 FLC. Section III describes the optimization of T1 FLC and the subsequent blurring construction of the IT2 FLC. Experimental results are presented in Section IV. The paper is concluded in Section V.

II. Review of Type-1 and Type-2 Fuzzy Control

This section reviews fundamentals of T1 and IT2 fuzzy control.

A. T1 Fuzzy Control

In general, T1 fuzzy controller is composed of four major parts – input fuzzification, fuzzy inference engine, fuzzy rule base and output deffuzification. The considered Mamdani FLC maintains a fuzzy rule base populated with fuzzy linguistic rules R_k in the implicative form as follows:

$$\text{Rule } R_k\text{: } \textbf{IF } x_1 \text{ is } A_1^k \textbf{ AND } \ldots \textbf{ AND } x_n \text{ is } A_n^k \textbf{ THEN } y_k \text{ is } B^k \quad (1)$$

Here, symbol A_j^k and B^k denote the j^{th} input fuzzy set and the output fuzzy set of the k^{th} rule, respectively and n is the dimensionality of the input vector $\vec{x}$. Using the minimum t-norm the degree of firing of rule R_k can be calculated as:

$$\mu_{R_k}(\vec{x}) = \min_{i=1\ldots n}\{\mu_{A_i^k}(x_i)\} \quad (2)$$

The output fuzzy sets are aggregated using the maximum operator, resulting in a fuzzy output set B. Upon discretizing the output domain into N samples, the centroid defuzzifier can be used to produce a crisp output value y as follows:

$$y = \frac{\sum_{i=1}^{N} y_i \mu_B(y_i)}{\sum_{i=1}^{N} \mu_B(y_i)} \quad (3)$$

B. IT2 Fuzzy Control

T2 FLC uses T2 fuzzy sets, which introduce additional dimension of uncertainty – the secondary membership grade. The structure of T2 FLC is depicted in Fig. 1 [1]. The interval T2 FLC is considered here due to its computational in-expensiveness. This computational efficiency is achieved by using the Footprint of Uncertainty (FOU) for describing each IT2 fuzzy set. The FOU of an IT2 fuzzy set $\tilde{A}$ can be conveniently described by its upper and lower membership functions:

$$FOU(\tilde{A}) = \bigcup_{\forall x \in X} (\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)) \quad (4)$$

The system then uses similar inference mechanism utilizing the modified IT2 fuzzy join and meet operations [1]. The resulting IT2 output fuzzy set must be first type reduced and then defuzzified in order to obtain the crisp output value. Despite the widespread use of the Karnik-Mendel iterative procedure, the Nie-Tan (N-T) type reduction method is considered in this work [17]. This method was selected due to its lower computational complexity and the lack of need to calculate the actual centroid of the output IT2 fuzzy set in the presented application. Briefly, the N-T method first computes the centroid of each vertical slice of the output fuzzy set and then defuzzifies the resulting type-reduced set. For the IT2 fuzzy sets, the centroid u_j of the j^{th} vertical slice is expressed as:

$$u_j = \frac{1}{2}(\overline{u}_j + \underline{u}_j) \quad (5)$$

Here, $\overline{u}_j$ and $\underline{u}_j$ are the membership grades of the upper and the lower membership functions at the j^{th} vertical slice. Assuming that the output domain was discretized into N samples, the defuzzified output is obtained as [17]:

$$y = \frac{\sum_{j=1}^{N} y_j u_j}{\sum_{j=1}^{N} u_j} \quad (6)$$

III. Learning Robotic Behaviors using PSO Algorithm

The PSO algorithm is used to obtain the design of the T1 FLC [15], [16]. The partial independent approach is used for the subsequent design of the IT2 FLC via symmetrical blurring the membership functions. Using a fully independent design using the PSO algorithm for the IT2 FLC optimization would introduce variance associated with the optimization method. By using a partially independent approach, the difference in the performance of both controllers can be fairly attributed to the controller's architecture rather than to the optimization method. However, as demonstrated by the experimental results, the widely adopted IT2 FLC design via

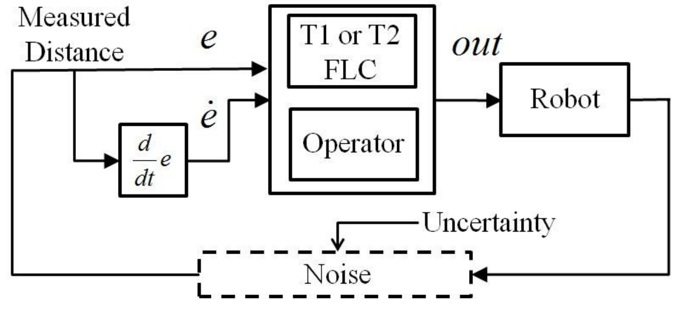


Fig. 2 The wall-following control loop.

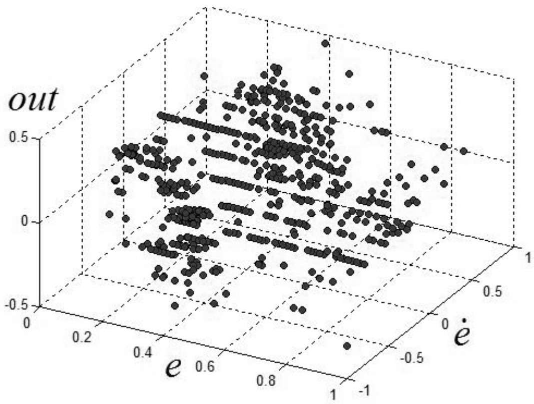


Fig. 3 Recorded operator's wall-following behavior.

symmetrical blurring of membership functions can impose some limitations to the controller's performance.

A. Experimental Setup

The experimental problem considered in this paper is learning a wall-following behavior performed by an operator. A remotely controlled Lego NXT mobile robot equipped with a sonar sensor was used in this experiment. The operator drove the robot starting from various distances along the wall. The goal of the operator was to stabilize the robot at a predefined distance from the wall by controlling its differential motor drives. The system control loop is depicted in Fig. 2. During the data collection stage, the operator was determining the control output based on a visual inspection of the robot's performance. In the following experimental stage, the T1 and IT2 fuzzy controllers were used to reproduce the same wall-following behavior autonomously. Additional uncertainty can be introduced into the system by adding a specified amount of noise into the sensory inputs.

The training data were recorded from multiple runs. The sonar distance input and its derivative together with the operator's control actions were recorded. Altogether 1300 data samples were obtained. The normalized recorded data are plotted in Fig. 3.

B. PSO Optimization of Type-1 FLC

A simple T1 FLC was constructed for reproducing the observed wall-following behavior. The controller fuzzifies both inputs e and $\dot{e}$ using three input Gaussian fuzzy sets. Each fuzzy set is defined by its mean and standard deviation. Each combination of input fuzzy sets produces a unique fuzzy rule. The output of each fuzzy rule is modeled as an output Gaussian fuzzy set. Altogether, the structure of the system is defined by 6 input fuzzy sets and 9 output fuzzy sets. This structure yields 30 control parameters (mean and standard deviation parameters for each fuzzy set).

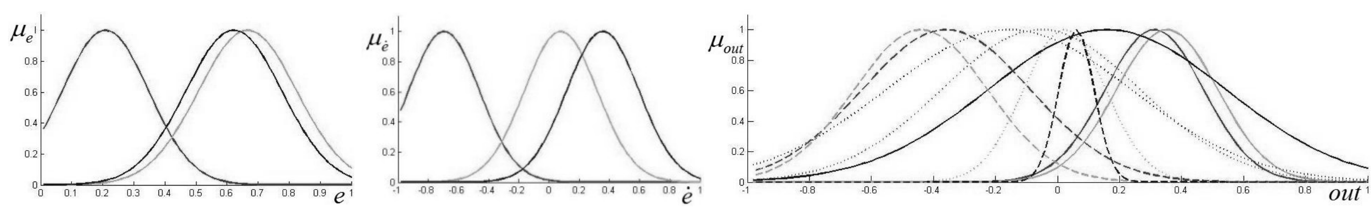
The PSO algorithm was used as a suitable optimization technique for learning the optimal structure of the T1 FLC [15], [16]. The PSO algorithm was chosen due to its stable convergence and its known ability to generate solutions for many non-linear highly complex problems. Due to the limited space, the details of the PSO algorithm are not described here.

Each particle represents a single design of a T1 FLC, determined by its 30 parameters. The fitness of each particle is evaluated as the total RMSE of the respective T1 FLC on the given training dataset. The optimized input and output fuzzy sets of the T1 FLC are depicted in Fig. 4.

C. Partially Independent Design of IT2 FLC

In order to prevent biasing the performance of the IT2 FLC by the optimization method, the IT2 FLC architecture is directly derived from the optimized T1 FLC. Each T1 fuzzy set was transformed into its IT2 counterpart by symmetrically blurring the respective fuzzy membership function. The IT2 fuzzy sets with uncertain mean were implemented.

The mean m_k of the T1 fuzzy set μ_k was shifted symmetrically to both sides by a specified offset. The blurring offset was selected arbitrary to provide reasonable overlap between the neighboring fuzzy sets. This resulted in an interval of means [m_{k1}, m_{k2}]. The upper and the lower membership function of the resulting IT2 Gaussian fuzzy set can then be analytically expressed as [1]:



(a) (b) (c)

Fig. 4 Optimized input (a), (b) and output (c) fuzzy sets of the wall-following T1 FLC.

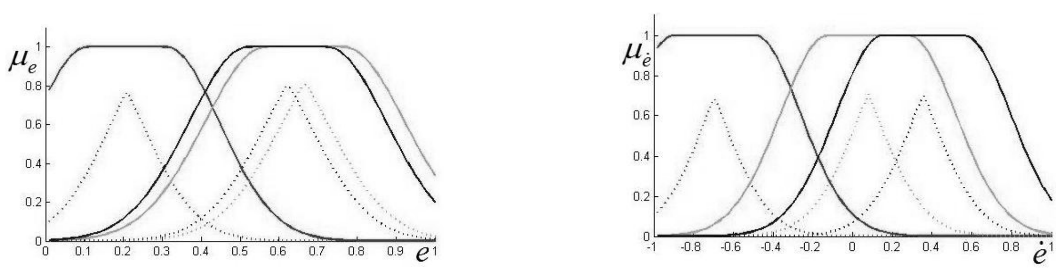
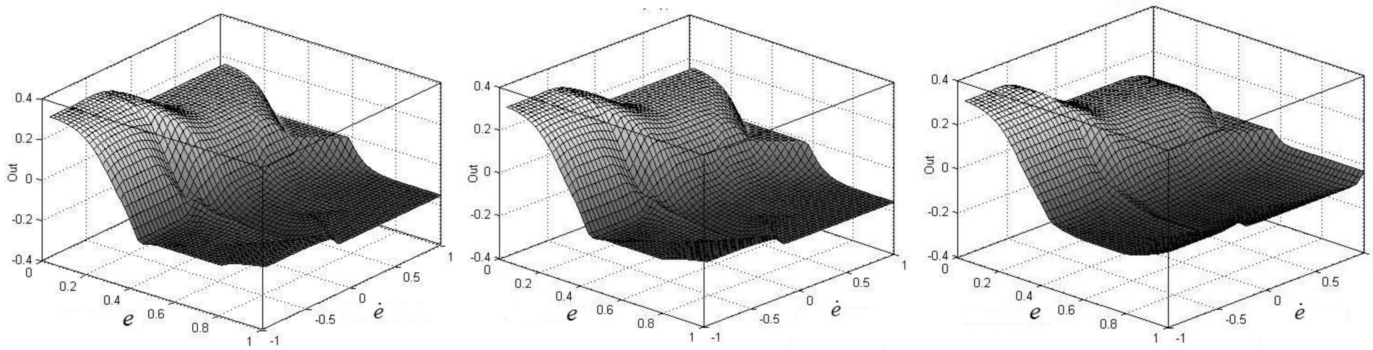
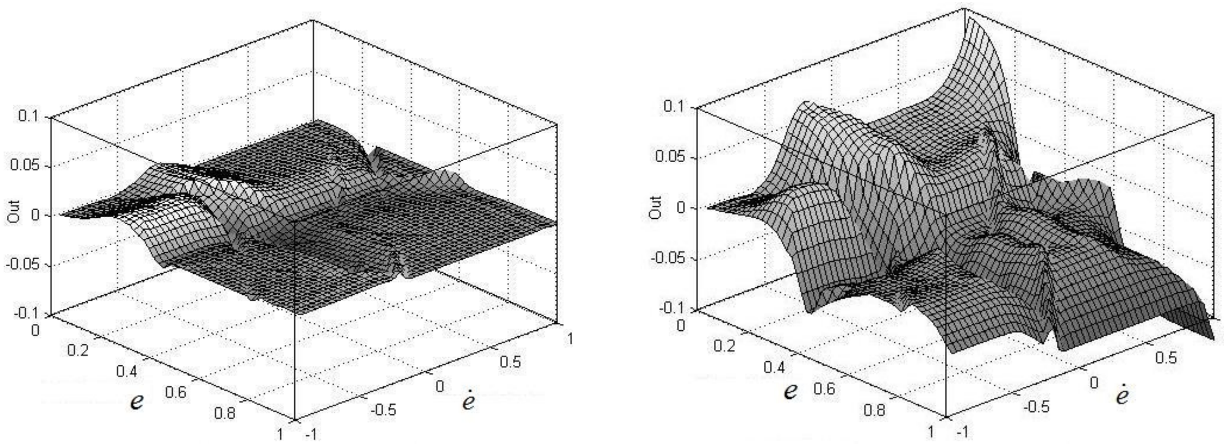


Fig. 5 Input T2 fuzzy sets obtained by blurring the T1 fuzzy sets.



(a) (b) (c)

Fig. 6 Comparison of control surfaces of FLC1 (a), FLC2 (b) and FLC3 (c).



(a) (b)

Fig. 7 Differences between FLC1 and FLC2 (a) and differences between FLC1 and FLC3 (b).

$$\overline{\mu}_k(x) = \begin{cases} N(m_{k1}, \sigma_k, x) & x < m_{k1} \\ 1 & m_{k1} \le x \le m_{k2} \\ N(m_{k2}, \sigma_k, x) & x > m_{k2} \end{cases} \tag{7}$$

$$\underline{\mu}_k(x) = \begin{cases} N(m_{k2}, \sigma_k, x) & x \le \dfrac{m_{k1} + m_{k2}}{2} \\ N(m_{k1}, \sigma_k, x) & x > \dfrac{m_{k1} + m_{k2}}{2} \end{cases} \tag{8}$$

As a demonstration, the IT2 counterparts of the optimized input T1 fuzzy sets are depicted in Fig. 5. Three wall-following controllers were constructed in this work. FLC1 is a purely T1 FLC that has all membership functions of T1 and uses the centroid defuzzifier (3). FLC2 uses blurred IT2 rule antecedents, but it maintains the T1 consequents. In this manner the controller handles uncertainty by its IT2 input fuzzy sets, but the output of the inference process is a T1 fuzzy set and the centroid defuzzification is used. Finally, FLC3 is a purely IT2 FLC, which uses IT2 membership functions for both rule antecedents and consequents. The result of the inference process is an output IT2 fuzzy set, which is defuzzified by the N-T method (6).

IV. Experimental Results

The three types of FLCs were used to implement the autonomous wall-following behavior of the mobile robot. First, the control surfaces produced by the respective FLCs are compared. Next, the performance of each fuzzy controller autonomously driving a sonar-equipped mobile robot using the input sonar measurements is evaluated.

A. FLCs Structure Comparison

The control surfaces of the all FLCs are plotted in Fig. 6. Recall, that FLC1 was fully T1 FLC, FLC2 used IT2 fuzzy antecedents, and FLC3 was fully IT2 FLC. It can be observed that there is only a negligible difference between the FLC1's and FLC2's control surfaces. Both controller surface feature several abrupt changes. On the other hand, the FLC3's control surface is rather smooth. Thus, less performance variation can be expected from the purely IT2 FLC, should the environment conditions change.

Such conclusions are further supported by inspecting the differences between pairs of control surfaces, as shown in Fig. 7. In Fig. 7(a), the FLC2 control surface was subtracted from FLC1 control surface. Here, it is visible that only subtle changes were introduced into the control performance by using the IT2 antecedent membership functions. However, when comparing the control surfaces of FLC1 and FLC3 in

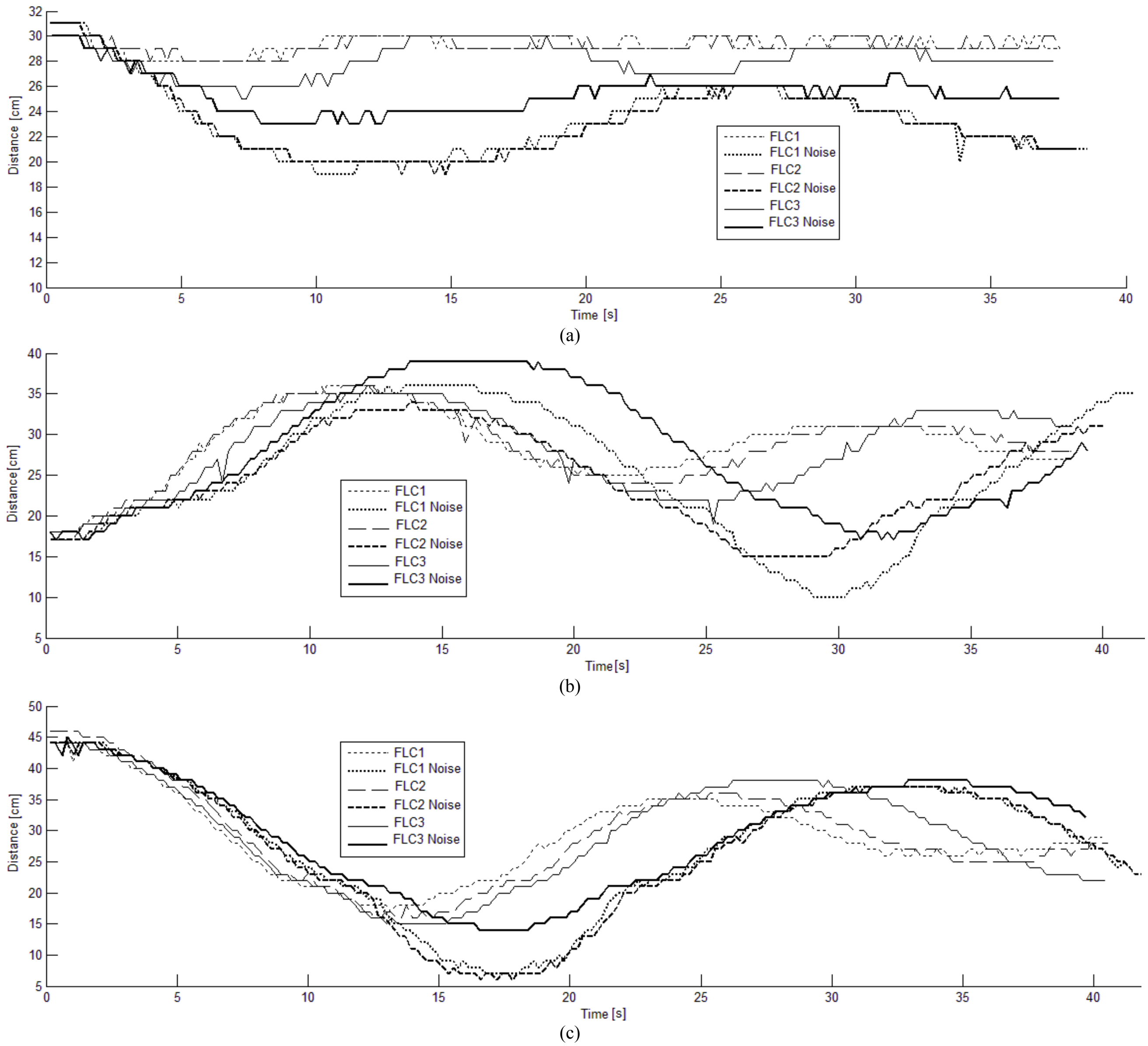


Fig. 8 Exemplary wall-following control performance of all three FLCs in three testing cases.

TABLE I
RMSE OF THE WALL-FOLLOWING BEHAVIOR

Test Case	FLC1		FLC2		FLC3	
	Mean	Std	Mean	Std	Mean	Std
Test A	10.1115	2.1655	10.3832	2.0573	8.0934	3.0451
Test B	7.9698	1.5157	7.9178	0.8733	4.5375	1.2324
Test C	7.1617	2.6356	7.5100	1.6693	5.4681	1.5455
Total	8.4143	2.1056	8.6037	1.5333	6.0330	1.9410

Fig. 7(b), substantially greater changes can be observed. The amount of introduced smoothing is apparent from the plotted difference surface.

B. Performance in Noisy Environment

This experiment investigated the capabilities of each controller to cope with unexpected dynamic uncertainties in the real-world applications. Here, the constructed FLCs were autonomously controlling the mobile robot. The control loop is displayed in Fig. 2. The controllers took the measurements from the mounted sonar sensor together with the derivative of the signal as the inputs. The output of the controllers was a control signal for the differential drives of the robot. Based on the provided training data, it was expected that the controllers should perform a wall-following behavior, resembling the behavior performed by the operator.

First, noise-less sonar measurements were supplied to the controllers and the distance to the wall of the autonomously

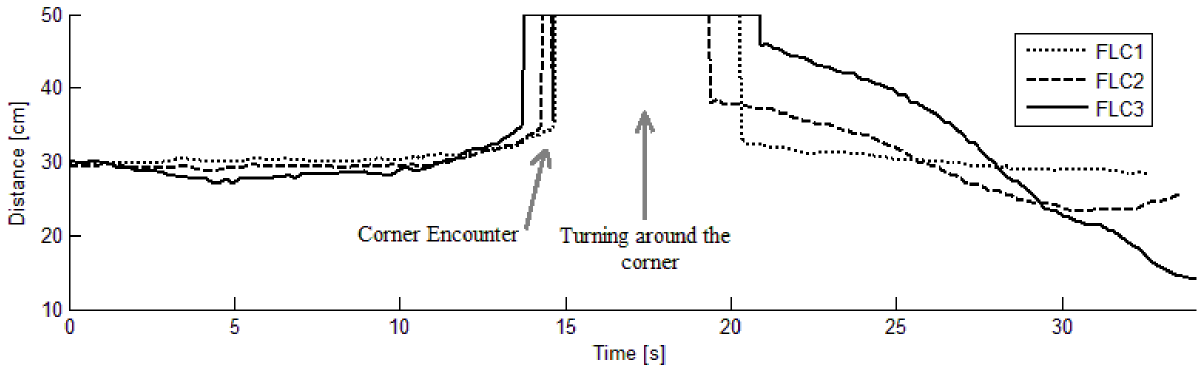


Fig. 9 Averaged sonar distance measurement for the wall-following robot when turning around a corner.

controlled robot was recorded. In the second set of experiments, the environmental conditions were artificially altered by introducing a uniform noise distribution into the incoming sonar measurements. The amplitude of the introduced noise reached 20% of the desired wall following distance. The experiment was started from three different initial positions: near the wall, far from the wall and at the desired wall-following distance. The performance deterioration in the noisy environment of each FLC was then studied. Each set of runs was repeated 5 times. Fig. 8 shows an example of the recorded driving performance for all three controllers during one set of runs. The performance of each controller in noise-less conditions is depicted by thin lines, while the performance when using the noisy sonar measurements is visualized using bold lines.

First thing to notice is that the wall-following performance of all three controllers is far from perfect. Nevertheless, this was expected since the training data provided by the operator were far from perfect as well. However, the attempts to stabilize the robot at a certain optimal wall-following distance are apparent. It is important to point out, that the purpose of this paper is not to design an optimal wall-following controller. This paper investigates the amount of performance deterioration under dynamic uncertainties in the environment conditions. This was achieved by matching the performance of each FLC under the noisy conditions with its noise-less counterpart. It can be observed that the FLC3 features substantially lower performance deterioration and overshooting when the noisy sonar measurements are used (e.g. substantial smaller overshoot at t=15s in Fig. 8(c)).

In order to quantify the performance deterioration, the RMSE measure was calculated for all pairs of noise-less/noisy signals. Here, the error is defined as the change in the robot's position under the altered environmental conditions. This measure reports the expected performance variation under dynamic uncertainty. The RMSE for all controllers in the three testing cases is summarized in Table I.

It can be seen that the fully IT2 FLC outperformed both of the other two controllers by nearly 30% lower RMSE, in terms of the performance deterioration. The FLC3 controller featured steadily the smallest RMSE during all three test cases. This suggests that the designed IT2 was capable of coping with the dynamic uncertainties by the means of its smoother and softer control surface.

C. Controller Responsiveness Analysis

Next, the FLC-controlled robot was forced to navigate into a situation where sudden and responsive action is needed. Such situation was encountering a corner during the wall-following behavior. The desired response of the controller was to turn around the corner and continue in the wall-following behavior. The experiment was run 5 times for each of the 3 considered FLCs. The distance to the wall was recorded. The average distances over all test runs are plotted in Fig. 9. The corner encounter is apparent as the sudden increase in the measured distance.

Analysis of the measured signals reveals an interesting result. The IT2 FLC was vastly outperformed by the T1 FLC, due to its slow responsiveness. Since the smoothed IT2 control surface was not able to react fast enough to the abrupt input signal change. The robot was not able to properly turn and the controller overshot the turning maneuver and consequently must have compensated for this. On the other hand, the relatively rough control surface of the T1 FLC enabled fast response and the robot turned around the corner with minor difficulties. Similar observation is reported by Wagner in [11].

V. CONCLUSION

This paper presented a comparative analysis of Type-1 and Interval Type-2 FLCs in context of learning behaviors for mobile robotics. The controllers were trained to autonomously perform a wall-following behavior for a sonar-equipped mobile robot. First, the PSO algorithm was used to optimize a T1 FLC using recorded data of a wall-following behavior. Next, the IT2 FLC was constructed by symmetrically blurring the membership functions of the original T1 FLC.

First the smoothing of the control surface introduced by blurring the T1 membership functions was demonstrated. Next, it was experimentally verified that the IT2 FLC outperforms the T1 FLC near the set point of the controller when coping with dynamic uncertainties such as noisy inputs. However, it was also experimentally demonstrated that the smoothing of the IT2 control output (obtained by the symmetrical blurring design) reduces the ability of the controller to quickly react to sudden and abrupt changes in the input signal (e.g. when turning around corners). In this case the T1 FLC outperformed the IT2 FLC.

Besides the comparative analysis of T1 and IT2 FLC, this paper demonstrated the tradeoffs of the symmetrical blurring process as a methodology for obtaining an IT2 FLC from a T1 FLC. Such blurring method results in a global smoothing of the output surface, which is advantageous when dealing with dynamic uncertainties near the controller set point, but it can result in difficulties when fast controller response is required.

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