Evolutionary Approach to Optimal Design of 3 DOF Translation Exoskeleton and Medical Parallel Robots

Sergiu-Dan Stan[†], Member, IEEE, Milos Manic[‡], Senior Member, IEEE, Vistrian Mătieş, and Radu Bălan[†] † Technical University of Cluj-Napoca, Cluj-Napoca, Romania sergiustan@ieee.org, matiesvistrian@yahoo.com, radubalan@yahoo.com [‡] University of Idaho, Idaho, USA, misko@uidaho.edu

Abstract — Parallel robots find many applications in human-systems interaction, medical robots, rehabilitation, exoskeletons, to name a few. These applications are characterized by many imperatives, with robust precision and dynamic workspace computation as the two ultimate ones. This paper presents a multi-objective optimum design procedure to 3 degrees of freedom (DOF) parallel robots with regards to four optimality criteria: workspace boundary, transmission quality index, stiffness. A kinematic optimization was performed to maximize the workspace of the parallel robot. In order to perform an optimal design of 3 DOF parallel robots, an objective function was developed first, and then Genetic Algorithms applied in order to optimize the objective function. The experimental results demonstrate the advantages of the presented optimization procedure in design of 3 DOF parallel robots, specifically TRIGLIDE and DELTA robots. These advantages are reflected in a presented framework for robust, precise, and dynamically calculated workspace boundaries. Therefore, the performances of the 3 DOF translation parallel robots provide high potential and good prospects for their practical implementation in human-systems interaction.

Keywords — optimization, Triglide parallel robot, Delta parallel robot, 3 degrees of freedom, Genetic Algorithms.

I. INTRODUCTION

FOR medical robots or exoskeleton applications, the environment constantly changes [1]. Therefore, there are two crucial needs for such an application: precision (since used in conjunction with human actions), and dynamic workspace determination, since the environment constantly changes. This paper addresses both needs. Parallel robotic structures are characterized by many inherent advantages, such as rigidity, accurate positioning, or high velocities. In order to achieve superior robotic performances, two steps in the design process are of utmost importance:

- 1. The choice of appropriate mechanical structure;
- 2. The choice of right dimensions.

The latter one is very important, since with parallel structures, there is a much higher performance variability along the different dimensions, than it is the case with classical, serial structures. Therefore, the first stage in the design of a specified kinematic structure is establishing its architecture, i.e. the joint and link layout and the dimensions of the robot. Also, choosing the best kinematic-dimensions for a specified machining application is a difficult problem for many reasons. In most cases we are able to compute the performance criteria only for a given pose of the robot, which means local performances. To evaluate the robot, global performances are needed, and therefore more efficient algorithms.

Various methods based on geometric or numerical approaches to determine workspace of a parallel robot have been proposed in the literature. Early investigations of robot workspace were reported by Merlet [2], Kumar and Waldron [3], Tsai and Soni [4], Gupta and Roth [5], Sugimoto and Duffy [6], Gupta [7], and Davidson and Hunt [8]. The consideration of joint limits in the study of the robot workspaces was presented by Delmas and Bidard (1995). Other works that have dealt with robot workspace are reported by Agrawal [9], Gosselin and Angeles [10], Cecarelli [11], Agrawal [12] determined the workspace of an in-parallel manipulator system using a different concept. Specifically, when a point is at its workspace boundary, it does not have a velocity component along the outward normal to the boundary. Therefore, configurations are determined in such a way that the velocity of the end-effector satisfies this property. Pernkopf and Husty [13] presented an algorithm to compute the reachable workspace of a spatial Stewart Gough-Platform with planar base and platform (SGPP) taking into account active and passive joint limits. Other approaches where optimization methods were used for the workspace boundary determination can be also found in the literature. Various numerical methods for determination of the workspace of parallel robots have been developed in the recent years. For example, Stan [14] presented a genetic algorithm approach for multicriteria optimization of PKM (Parallel Kinematics Machines).

The majority of numerical methods used for parallel manipulator workspace boundary determination typically rely on manipulator's pose parameter discretization. [15, 16]. With the discretization approach, the workspace is envisioned as the uniform grid of nodes in Cartesian or polar coordinate system. Each node is then examined in order to determine whether it belongs to the workspace or not.

The paper presents several contributions. First, the paper introduces the optimization workspace index metrics as the performance measure of the 3 DOF parallel robots. Secondly, the optimal dimensioning of the 3 DOF parallel robots of type TRIGLIDE and DELTA with translation actuators for the largest workspace, best stiffness and transmission quality is presented. Thirdly, the relationship between link's lengths and robot's performances is also introduced. This relationship enables optimum results with respect to a desired workspace. The results shown in this paper demonstrate a novel approach that resolves the singularitv workspace problems. improves the performances, and finally results in the optimum design of the robots.

Section II describes the 3 DOF parallel robots. The third section introduces the performance evaluation. The fourth section presents the optimization results, while the final, fifth section concludes this paper.

II. 3 DOF DEGREE OF FREEDOM PARALLEL ROBOTS

The most important requirements of parallel robots are workspace, accuracy, stiffness, and velocity. In order to be used in parallel robot control system, these requirements need to be mathematically expressed and precisely described.

Choosing the optimal robot dimensions for the best performance is still a challenging task. There are a lot of performance criteria which have to be taken into account and which are pose (position and orientation) dependent. These characteristic functions or performance criteria are crucial in establishing the degree of fulfillment of a parallel robot requirement.

The requirements and developed characteristic functions are in general not constant (isotropic) and depend on the location or pose of the mobile platform in plane or space. Isotropic behavior is strongly desired. In isotropic configuration, the Jacobian matrix has the condition number as well as the determinant equal to one, and the robot performs very well with regards to its force and motion transmission capabilities.

A. Three DOF parallel robots

Parallel robots with 3 degrees-of-freedom are parallel manipulators comprising a fixed base platform and a payload platform, linked together by three independent, identical, and open kinematic chains (Fig. 1 & Fig. 2).

The TRIGLIDE parallel robot consists of a spatial parallel structure with three translational degrees of freedom, and is driven by three linear actuators. The platform is connected with each drive by two links forming a parallelogram, allowing only translational movements of the platform and keeping the platform parallel to the base plane. An additional rotational axis can be mounted on the working platform to adjust the orientation of the end-effector. The three drives of the structure are star-shaped and arranged in the base plane at 120 degree intervals. Thus, the structure has a workspace which is nearly round or triangle-shaped (Fig. 1).

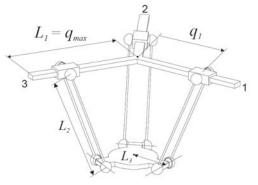


Fig. 1. TRIGLIDE parallel robot with 3 DOF.

The DELTA linear parallel robot with 3 DOF is shown in Fig. 2. Geometric parameters are illustrated by Fig. 3, where the moving platform is connected to the base platform via three identical serial chains.

Each of the three chains contains one spatial parallelogram. The parallelogram is actually composed of the vertices of which are actually four spherical joints.

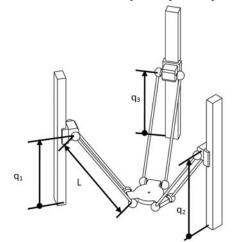


Fig. 2. DELTA parallel robot with linear actuators.

B. Mathematical model

To analyze the kinematic model of the parallel robots, two relative coordinate frames are assigned, as shown in Fig. 3.

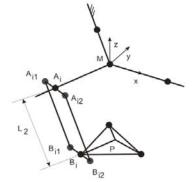


Fig. 3. Schematic diagram of TRIGLIDE parallel robot.

A static Cartesian coordinate frame XYZ is fixed at the center of the base, while a mobile Cartesian coordinate frame $X_PY_PZ_P$ is assigned to the center of the mobile platform. A*i*, *i* = 1, 2, 3, and B*i*, *i* = 1, 2, 3, are: the joints located at the center of the base, as presented in Fig. 4 & 5, and the platform passive joints, respectively. A middle link L₂ is installed between the mobile and fixed platform.

Let L_1 , L_2 , L_3 be the link's lengths as expressed in (1):

$$L_{1} = A_{i1}A_{i2} = B_{i1}B_{i2}$$

$$L_{2} = A_{i1}B_{i1} = A_{i}B_{i}$$

$$L_{3} = B_{i}P$$

$$M \xrightarrow{120^{\circ}} A_{1} \xrightarrow{\alpha} B_{3}$$
Fixed platform
$$M \xrightarrow{120^{\circ}} B_{12}$$

$$B_{31} \xrightarrow{B_{32}} B_{31}$$

$$M \xrightarrow{B_{32}} B_{32}$$

$$M \xrightarrow{B_{32}} B_{32}$$

Fig. 4. Schematic diagram of mobile and fixed platform for TRIGLIDE parallel robot.

In order to compute $r_{A_iB_i} = r_{B_i} - r_{A_i}$ for i=1, 2, 3, r_{B_i} and r_{A_i} need to be computed first. First, r_{B_i} is defined as:

$$r_{B_i} = r_P + r_{PB_i} = \begin{pmatrix} x_P + L_3 \cdot \cos \beta_i \\ y_P + L_3 \cdot \sin \beta_i \\ z_P \end{pmatrix}$$
(2)

where β_i is computed as $\beta_i = (i-1) \cdot 120^\circ$.

Then, r_{A_i} is calculated as:

$$r_{A_i} = \begin{pmatrix} q_i \cdot \cos \alpha_i \\ q_i \cdot \sin \alpha_i \\ 0 \end{pmatrix}$$
(3)

where α_i is computes as $\alpha_i = (i-1) \cdot 120^\circ$. From (3) yields f_i :

$$\begin{split} f_{i} &= \begin{pmatrix} x_{P} + L_{2} \cdot \cos\beta_{i} - q_{i} \cdot \cos\alpha_{i} \\ y_{P} + L_{2} \cdot \sin\beta_{i} - q_{i} \cdot \sin\alpha_{i} \end{pmatrix}^{2} - L_{2}^{2} = 0; \ L_{2} = |r_{A_{i}B_{i}}| \\ (4) \\ (x_{p} + L_{3} \cdot \cos\beta_{i})^{2} + q_{i}^{2} \cdot \cos^{2}\alpha_{i} + 2 \cdot (x_{p} + L_{3}\cos\beta_{i}) \cdot q_{i} \cdot \cos\alpha_{i} \\ + (y_{p} + L_{3} \cdot \sin\beta_{i})^{2} + q_{i}^{2} \cdot \sin^{2}\alpha_{i} + 2 \cdot (y_{p} + L_{3}\sin\beta_{i}) \cdot q_{i} \cdot \sin\alpha_{i} \\ + z_{P}^{2} - L_{2}^{2} = 0 \\ (5) \end{split}$$

From (4) we obtain (5) and by reformulating (5), (6) is obtained:

$$q_{i}^{2} + q_{i} \cdot 2 \cdot \left[(x_{p} + L_{3} \cdot \cos \beta_{i}) \cdot \cos \alpha_{i} + (y_{p} + L_{3} \cdot \sin \beta_{i}) \cdot \sin \alpha_{i} \right] + (x_{p} + L_{3} \cdot \cos \beta_{i})^{2} + (y_{p} + L_{3} \cdot \sin \beta_{i})^{2} + z_{p}^{2} - L_{2}^{2} = 0$$
(6)

By substituting (7): $u_{i} = (x_{p} + L_{3} \cdot \cos\beta_{i}) \cdot \cos\alpha_{i} + (y_{p} + L_{3} \cdot \sin\beta_{i}) \cdot \sin\alpha_{i}$ $v_{i} = (x_{p} + L_{3} \cdot \cos\beta_{i})^{2} + (y_{p} + L_{3} \cdot \sin\beta_{i})^{2} + z_{p}^{2} - L_{2}^{2}$ (7)

in (6), we obtain the inverse kinematics problem of the TRIGLIDE parallel robot from Fig. 1:

$$q_i = u_i \pm \sqrt{u_i^2 - v_i} \tag{8}$$

For the implementation and resolution of forward and inverse kinematic problems of a parallel robot, a MATLAB environment was chosen. This is where a user friendly graphical user interface was developed, as well.

For the DELTA linear robot, closed-form solutions for both the inverse and forward kinematics have been developed in [14]. Here, for convenience, we recall the inverse kinematics briefly.

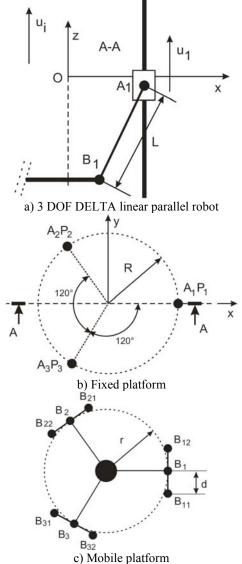


Fig. 5. Schematic diagram of mobile and fixed platform for DELTA linear parallel robot.

Let *R* and *r* be the radii of the base and the platform passing through joints P_i and B_i , for i = 1, 2, 3:

$$\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} = \begin{bmatrix} R & -\frac{R}{2} & -\frac{R}{2} \\ 0 & R\frac{\sqrt{3}}{2} & -R\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \end{bmatrix}$$
(9)

$$\begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix} = \begin{bmatrix} r & -\frac{r}{2} & -\frac{r}{2} \\ 0 & r\frac{\sqrt{3}}{2} & -r\frac{\sqrt{3}}{2} \\ 0 & 0 & 0 \end{bmatrix}$$
(10)
$$\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$
(11)

After computing positions of joints P_i and B_i , the inverse kinematics of the DELTA parallel robot with linear actuators can be solved via equations (12):

$$q_1 = z_p + \sqrt{L^2 - (r - R + x_p)^2 - y_p^2}$$
(12)

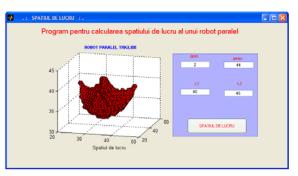
$$q_{2} = z_{p} + \sqrt{L^{2} - \left((R - r) \cdot \frac{1}{2} + x_{p}\right)^{2} - \left((r - R) \cdot \frac{\sqrt{3}}{2} + y_{p}\right)^{2}}$$
$$q_{3} = z_{p} + \sqrt{L^{2} - \left((R - r) \cdot \frac{1}{2} + x_{p}\right)^{2} - \left((R - r) \cdot \frac{\sqrt{3}}{2} + y_{p}\right)^{2}}$$

These equations represent the inverse kinematics problem of the DELTA linear parallel robot.

C. Workspace evaluation

In this section, the workspace of the proposed robots will be discussed in details. For a robot in the context of industrial application and given parameters, it is very important to analyze the area and the shape of its workspace. Calculation of the workspace and its boundaries with perfect precision is crucial, because they influence the dimensional design, the manipulator's positioning in the work environment, and its dexterity to execute tasks.

The workspace is limited by several conditions. The prime limitation is the boundary obtained through solving inverse kinematics. Further, the workspace is limited by the reachable extent of drives and joints, then by the occurrence of singularities, and finally by the link and platform collisions. The parallel robots TRIGLIDE and DELTA linear realize a wide workspace, as presented in Fig. 6 & 7. Analysis, i.e. visualization of the workspace is an important aspect of performance analysis. In order to generate a reachable workspace of parallel manipulators, a numerical algorithm was introduced. For the sake of simplicity, other design specific factors such as the end-effector size, drive volumes have been ignored.





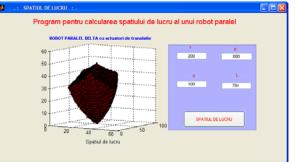


Fig. 7. The GUI for calculus of workspace for the DELTA linear 3 DOF parallel robot

III. PERFORMANCE EVALUATION

In addition to important design criterion such as the workspace, another important criterion, transmission quality index, has been considered. The transmission quality index, *T*, couples velocity and force transmission properties of a parallel robot, i.e. power features [14]. Its definition is:

$$T = \frac{\|I\|^2}{\|J\| \cdot \|J^{-1}\|}$$
(13)

where *I* is the unity matrix, and *J* is Jacobian matrix.

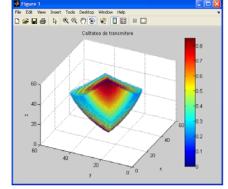


Fig. 8. Transmission quality index for DELTA linear parallel robot

The values transmission quality index, *T*, are within a range 0 < T < 1, where *T*=0 characterizes a singular pose and *T*=1 characterizes an optimal value, therefore reflecting the isotropy of the system [14]. Here ||J|| is calculated as:

$$\|J\| = \sqrt{tr(J^T w J)}; \ w = \frac{1}{n}I \tag{14}$$

where n is the dimension of the Jacobian matrix, and I the $n \ge n$ identity matrix.

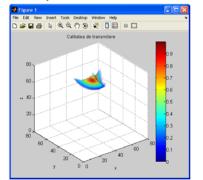


Fig. 9. Transmission quality index for TRIGLIDE parallel robot

Figures 8 & 9 demonstrate better performances in terms of transmission quality of the DELTA linear and TRIGLIDE parallel robots in the central part of the workspace approaching to the isotropic configuration.

Stiffness is one of the most important performances of parallel mechanisms, particularly for those which are used as machine tools, because higher stiffness allows higher machining speeds with higher accuracy of the endeffector. Therefore, it is necessary to perform the stiffness modeling, as well as the evaluation of the parallel robot in the early design stage. With regards to stiffness evaluation, several different performance indices have been proposed and utilized in the literatures. A simple way to predict the stiffness is to use the interested stiffness factors, i.e., the terms of the stiffness matrix [14]. Figures 10 & 11 demonstrate better performances in terms of stiffness of the DELTA linear and TRIGLIDE parallel robots in the central part of the workspace.

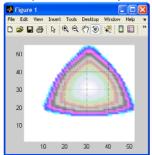


Fig. 10. Stiffness index for TRIGLIDE parallel robot

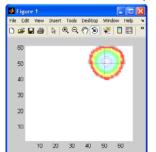


Fig. 11. Stiffness index for DELTA linear parallel robot

Furthermore, similar to the condition number of Jacobian matrix, the condition number of the stiffness matrix was introduced.

IV. OPTIMIZATION RESULTS FOR 3 DOF TRANSLATION PARALLEL ROBOTS

The design of the robots can be made based on any particular criterion. For simplicity of the optimization calculus, a symmetric design of the structure was chosen for both types of robots. The chosen performance indexes for both parallel robots were W (Workspace), T (Transmission quality index), and S (Stiffness).

For the optimization purposes, an objective function was defined. This objective function corresponds to optimal stiffness in workspace and transmission quality index. Hence, the design optimization problem can be defined as following:

$$ObjFun=T+S$$
 (15)

Here, the objective is to evaluate optimal link lengths which maximize (15). The design variables (the optimization factor), is the link length L for DELTA linear parallel robot, and L_2 for TRIGLIDE parallel robot.

Constraints to the design variables are:

$$0.8 \le L \le 2. \tag{16}$$

for the DELTA linear parallel robot, and:

$$100 \le L_2 \le 450.$$
 (17)

for the TRIGLIDE parallel robot.

During the optimization process, a genetic algorithm (GA) was used with the GA parameters from Table 1. A genetic algorithm was used for its robust convergence properties. The obvious advantage of GA approach over conventional optimization approaches lays in the fact that GA examines a number of solutions in a single design cycle, therefore ensuring a near optimal solution as the result of the optimization process.

One of the pitfalls of traditional methods is that they search optimal solutions from point to point, and often times get stuck in local optimal points. Using a population size of 50, the GA was run for 100 generations. A list of the best 50 individuals was continually maintained during the execution of the GA, allowing the final selection of solution to be made from the best structures found by the GA over all generations.

TABLE 1: GA PARAMETERS.		
Generations	100	
Crossover rate	0.08	
Mutation rate	0.005	
Population	50	

A kinematic optimization was performed in such a way that the objective function was maximized. Different values of the parameter optimization L and L_2 were obtained for different objective functions. The following table presents the results of optimization for different goal functions. Here, W₁ and W₂ are the weight factors.

TABLE 2: RESULTS OF OPTIMIZATION FOR DIFFERENT GOAL	
FUNCTIONS FOR DELTA LINEAR PARALLEL ROBOT	

	Method	GAOT Toolbox MATLAB
Goal	Z= $W_1 \cdot \ddot{U} + W_2 \cdot S$, W ₁ =0,7 și	<i>L</i> = 1,2 (m)
functions	W ₂ =0,3 Z=W ₁ ·Ü+W ₂ ·S, W ₁ =0,3 și	L = 1,8 (m)
	$W_2=0,7$ Z= W ₁ ·Ü, W ₁ =1 şi W ₂ =0	L = 0.9 (m)
	Z=W ₂ ·S, W ₁ =0 și W ₂ =1	L = 2 (m)

TABLE 3: RESULTS OF OPTIMIZATION FOR DIFFERENT GOAL
FUNCTIONS FOR TRIGLIDE PARALLEL ROBOT

Method	GAOT Toolbox MATLAB
$Z=W_1 \cdot \ddot{U}+W_2 \cdot S,$ W_1=0.7 si	L ₂ = 309,7359
$ W_2=0,3 Z=W_1 \cdot \ddot{U}+W_2 \cdot S, $	$L_2 = 450$
$W_1=0,3$ §1 $W_2=0,7$ $Z=W_1 \cdot \dot{U},$	L_2 = 169,2995
W ₁ =1 și W ₂ =0 Z=W ₂ ·S, W ₁ =0 și W ₂ =1	L ₂ = 450
	$Z=W_{1}\cdot\ddot{U}+W_{2}\cdot S,$ $W_{1}=0,7 \text{ si}$ $W_{2}=0,3$ $Z=W_{1}\cdot\ddot{U}+W_{2}\cdot S,$ $W_{1}=0,3 \text{ si}$ $W_{2}=0,7$ $Z=W_{1}\cdot\ddot{U},$ $W_{1}=1 \text{ si } W_{2}=0$

If an elitist GA is used, the best individual of the previous generation is kept and compared to the best individual of the new one. If the performance of the previous generation's best individual is found to be superior, it is passed on to the next generation instead of the current best individual. The experimental results demonstrate that GA was able to successfully determine the architectural parameters of the robot that would provide an optimized workspace. Since the workspace of a parallel robot is far from being intuitive, the developed method represents a useful, long needed design tool for an optimized parallel workspace calculation.

V. CONCLUSION

Parallel robots such in human-systems interaction such as medical, rehabilitation, exoskeleton robots depend on robustness, precision, and dynamic workspace computation, as the ultimate aspects of their safe and successful interaction with humans. This paper presents fundamental guidelines for optimal design of 3 DOF translation parallel robots based on evolutionary approach. Genetic algorithms (GA) are so far generally the best and most robust representative in a suite of evolutionary algorithms.

Design optimization via Genetic Algorithms (GA) was performed with regards to transmission quality index, stiffness and workspace. The workspace optimization program was implemented in Matlab. The optimum design and performance evaluation of the mini parallel robot represents the key issue in efficient use of parallel robots. The presented optimization methodology presented a framework for the complex tasks of optimum design of parallel robots with regards to basic characteristics of workspace, stiffness and transmission quality. The obtained results demonstrated how the use of GA enhances the robustness and the quality of the optimization outcome, providing a better and more realistic support for the decision maker.

Future work entails employment of fuzzy intelligent control for addressing dynamic robot movements, and neural network control for dynamic learning of workspace for autonomous robot deployment.

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